\frac{1}{2}

Discrete **particles** and **collisions** systems in cellular automata

CA

N. Ollinger, 19-21 février 2007

1 particles? collisions?

Cellular automata

- In this talk CA will be of dimension 1, first neighbors, a few states, using the following notations:

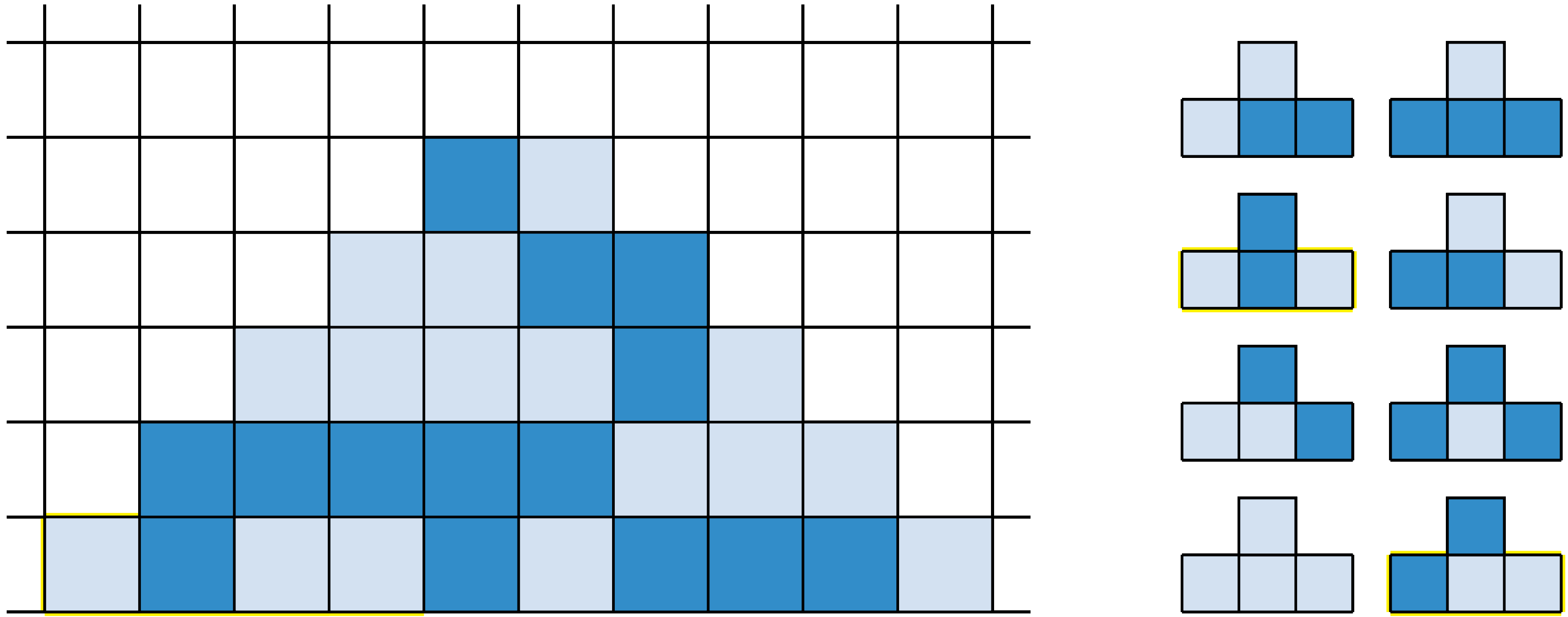
$$\mathcal{A} = (S, f) \qquad S = \{\blacksquare, \square, \blacksquare\} \qquad f : S^3 \to S$$

$$C \in S^{\mathbb{Z}} \qquad G(C)_i = f(C_{i-1}, C_i, C_{i+1})$$

- Ideally, we would like to use *fully* 2D space-time diagram, *i.e.* limit set configurations:

$$\Delta \in S^{\mathbb{Z}^2} \qquad \Delta_{t+1} = G(\Delta)_t \qquad \Omega = \bigcap_{k \in \mathbb{N}} G^k(S^{\mathbb{Z}})$$

Space-time diagram



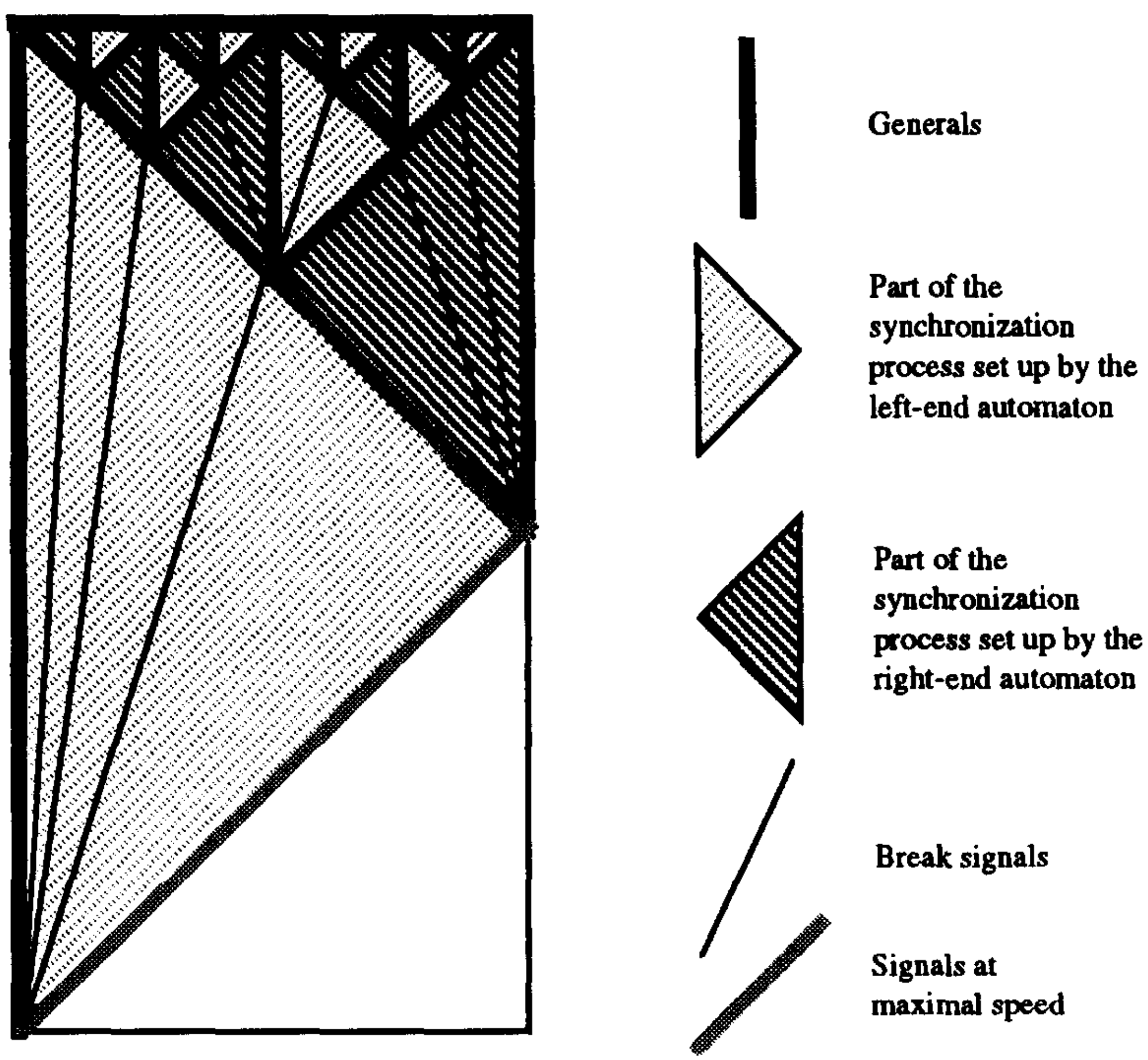
Point of view

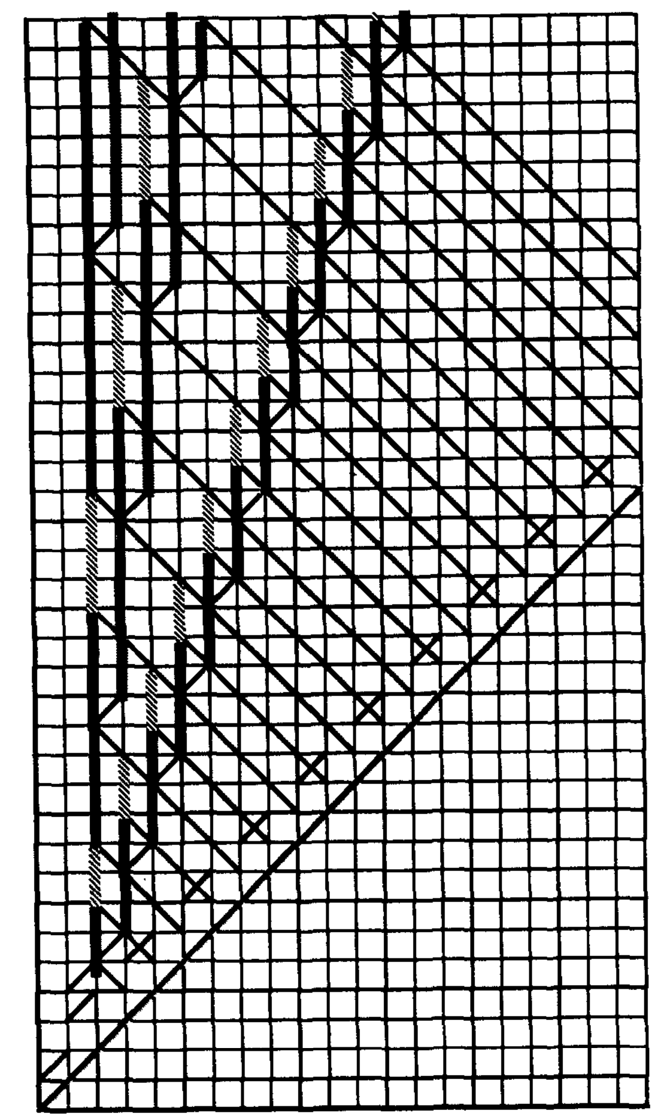
- In this talk we take the point of view of algorithmic constructions on CA:
- We discuss efficient (read few states) directed information propagation in space-time diagrams.

Undecidability everywhere

- In practice, *almost all* properties of CA are undecidable in the general case:
- From being nilpotent to the membership of a word to the limit set of a given CA.
- More theorems than algorithms!

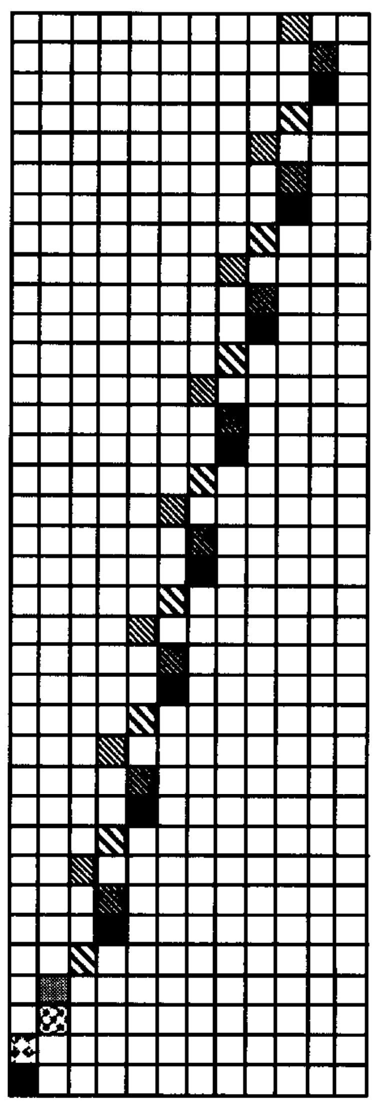
FSSP constructions

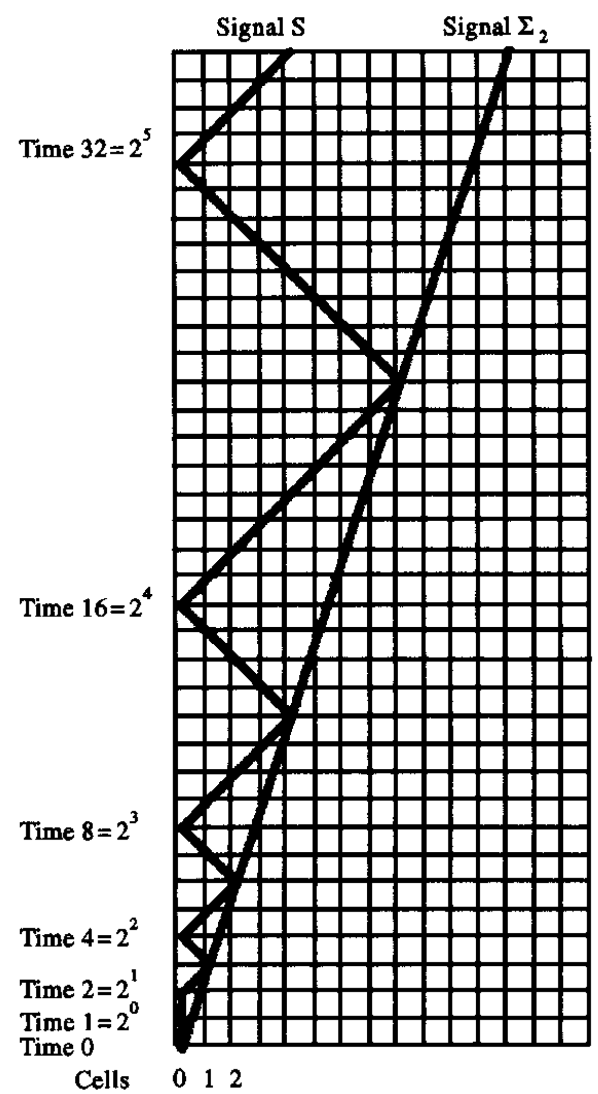


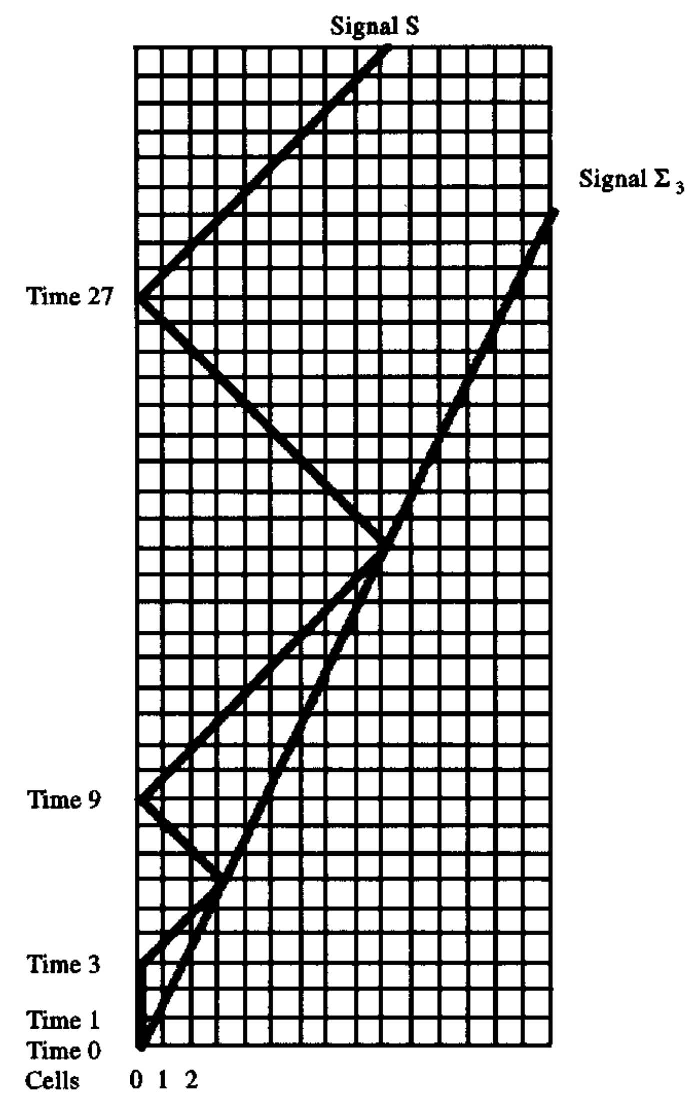


Pictures from Mazoyer 1996

Signals *à la* Fischer







Pictures from Mazoyer & Terrier 1999

Universalities

- B. Durand and Z. Róka, *The game of life: universality revisited*, Cellular automata (Saissac, 1996) (Kluwer Acad. Publ., Dordrecht, 1999), (pp. 51 74).
- Different notions of universality:
  - T-universality = **simulate** *Turing machines*
  - I-universality = **simulate** *any other CA*

Order on CA

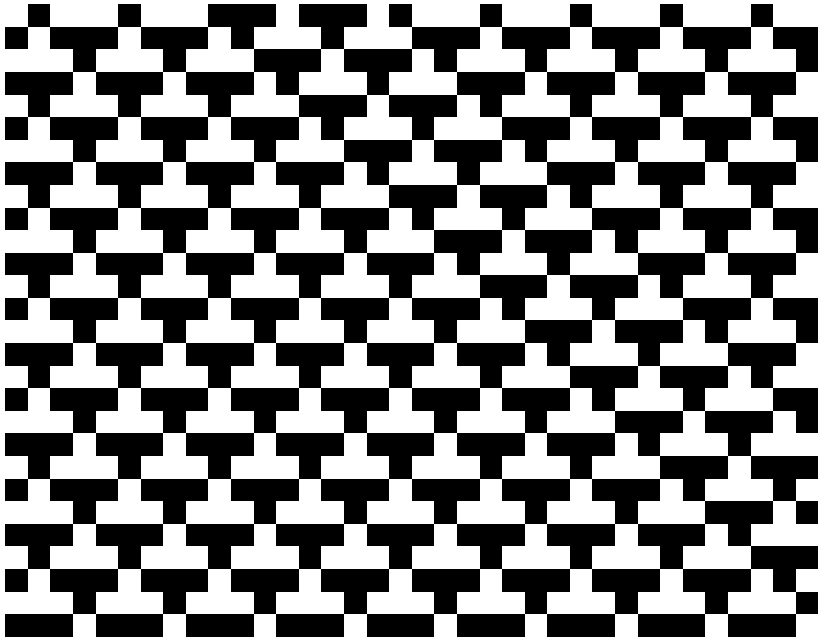
Idea. A CA $\mathcal{A}$ is **less complex** than a CA $\mathcal{B}$ if, up to some renaming of states and some rescaling, every space-time diagram of $\mathcal{A}$ is a space-time diagram of $\mathcal{B}$.

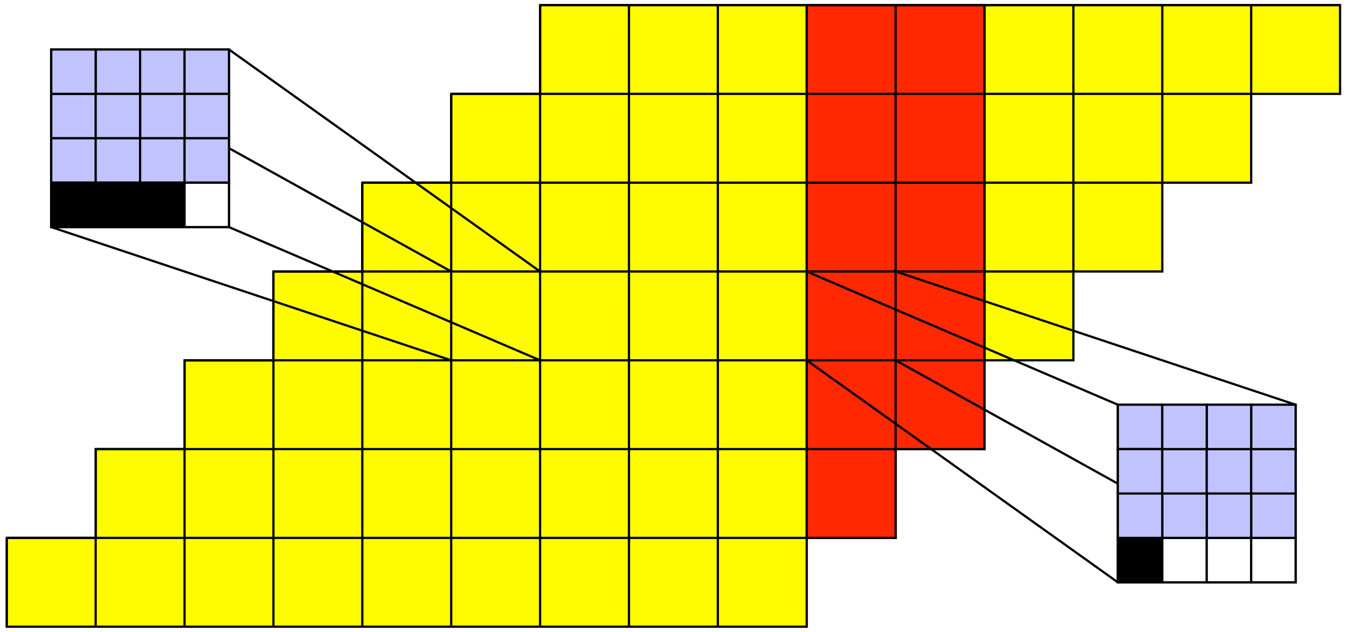
$\mathcal{A} \subseteq \mathcal{B}$ if there exists an injective mapping φ from $S_{\mathcal{A}}$ into $S_{\mathcal{B}}$ such that this diagram commutes:

$$\begin{array}{ccc} C & \xrightarrow{\ \varphi\ } & \overline{\varphi}(C) \\ {\scriptstyle G_{\mathcal{A}}}\big\downarrow & & \big\downarrow{\scriptstyle G_{\mathcal{B}}} \\ G_{\mathcal{A}}(C) & \xrightarrow[\ \varphi\ ]{} & \overline{\varphi}(G_{\mathcal{A}}(C)) \end{array}$$

Rescaling

$$G_{\mathcal{A}}^{\langle m,n,k\rangle} = \sigma^k \circ o^m \circ G_{\mathcal{A}}^n \circ o^{-m}$$

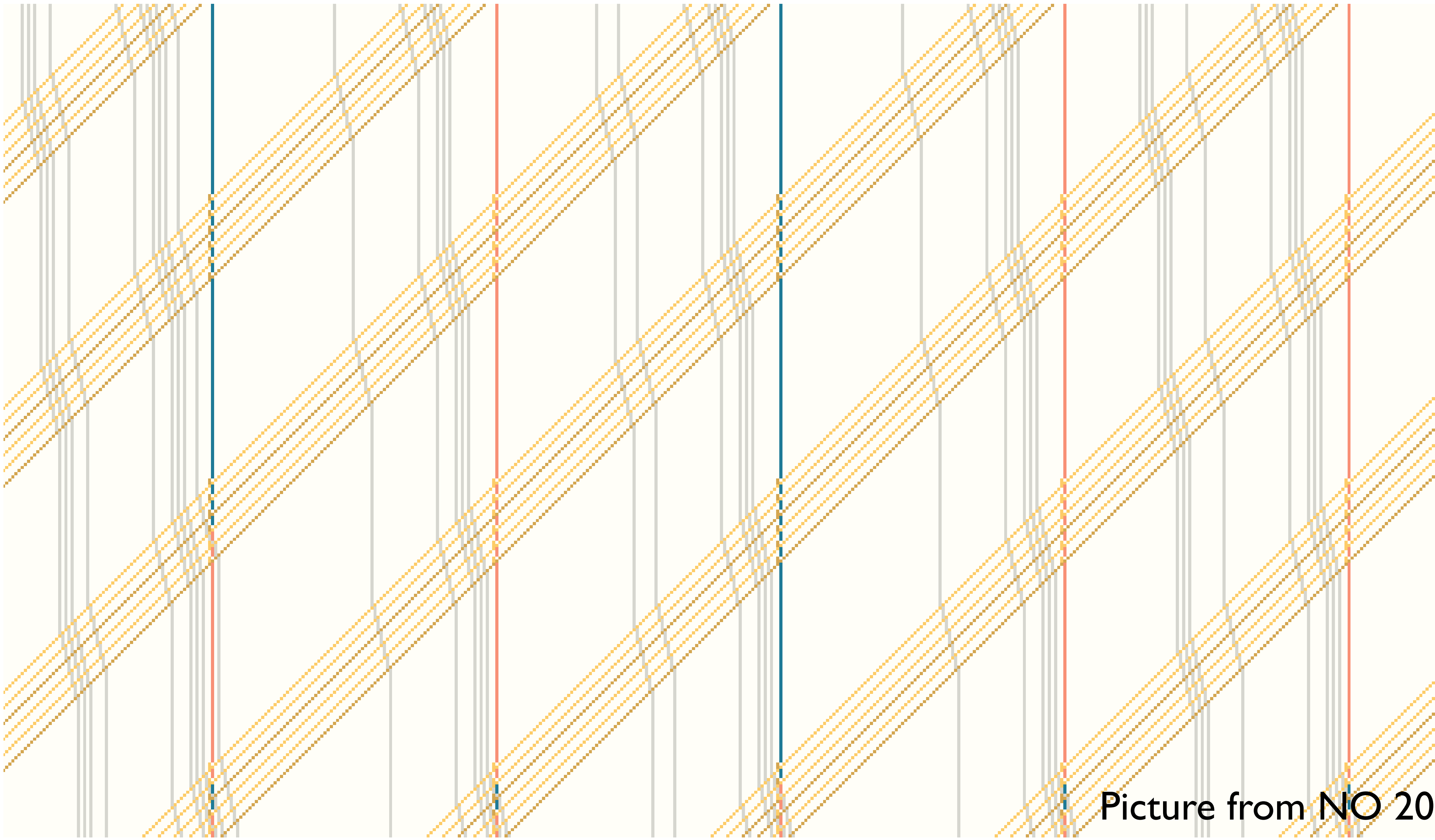




$\mathcal{A} \leqslant \mathcal{B}$ if there exist $\langle m, n, k\rangle$ and $\langle m', n', k'\rangle$ such that

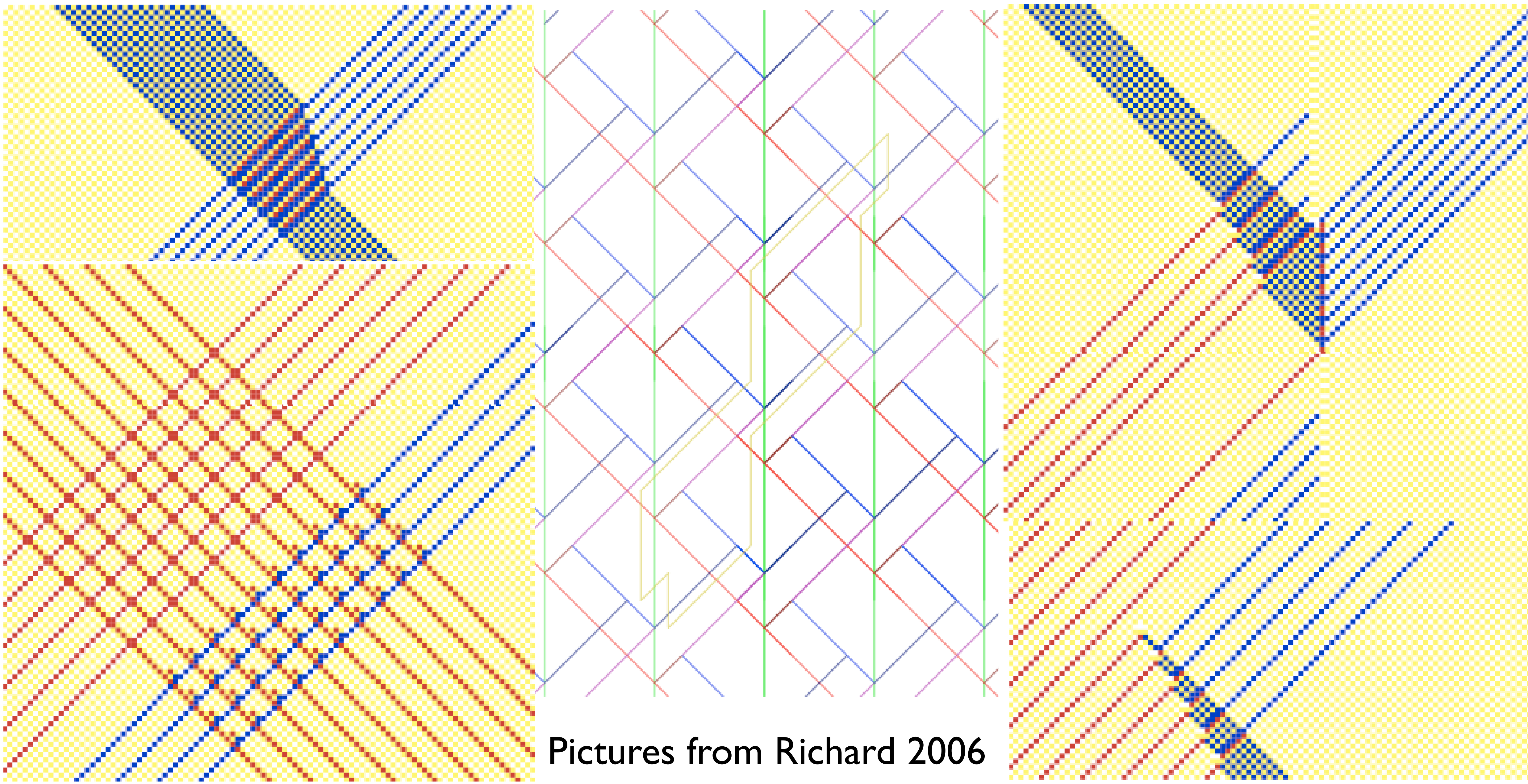
$\mathcal{A}^{\langle m,n,k\rangle} \subseteq \mathcal{B}^{\langle m',n',k'\rangle}$.

Signals and universality: 6st



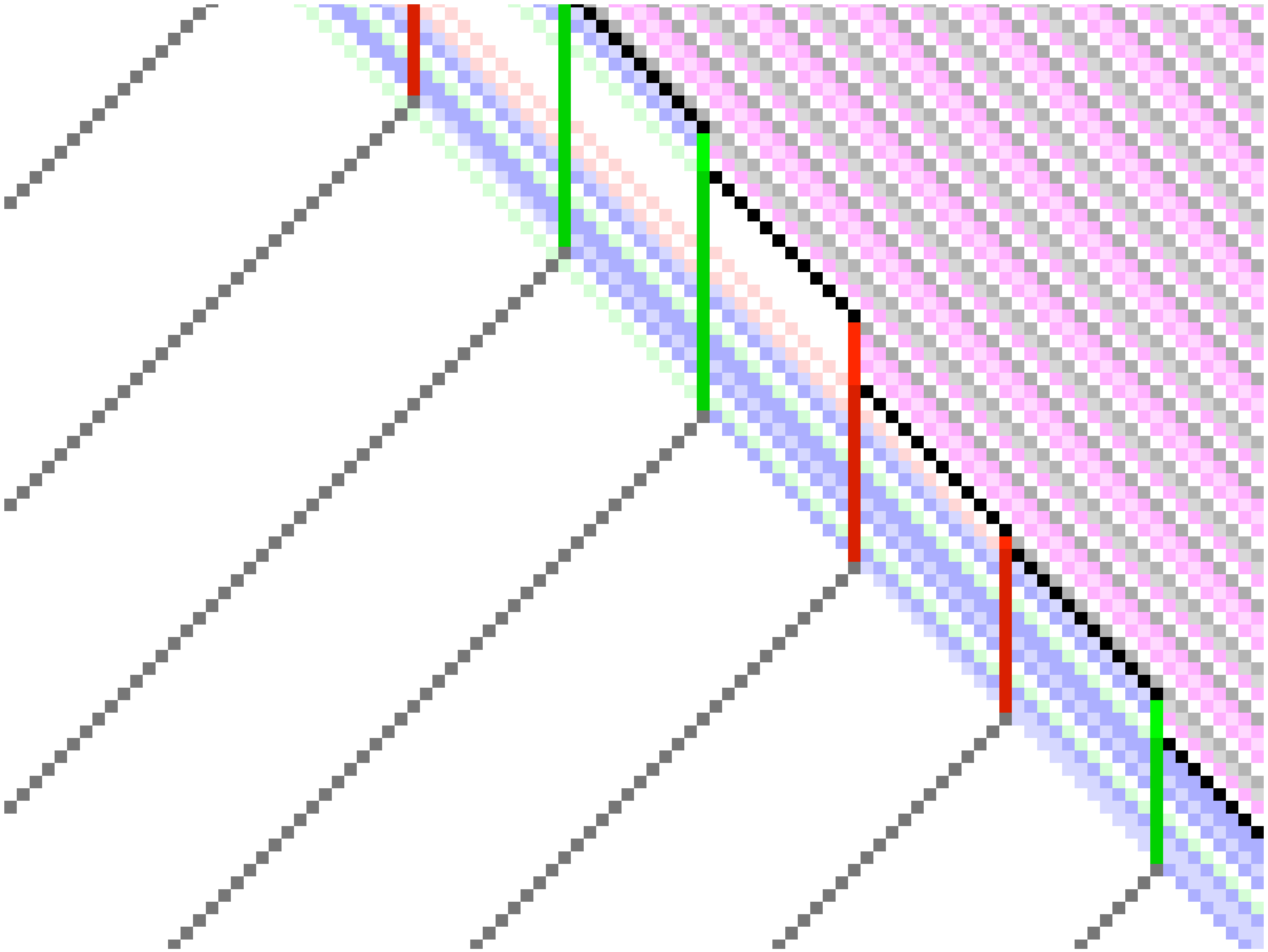
Picture from NO 2002

Signals and universality: 4st



Pictures from Richard 2006

Signals and universality: 110

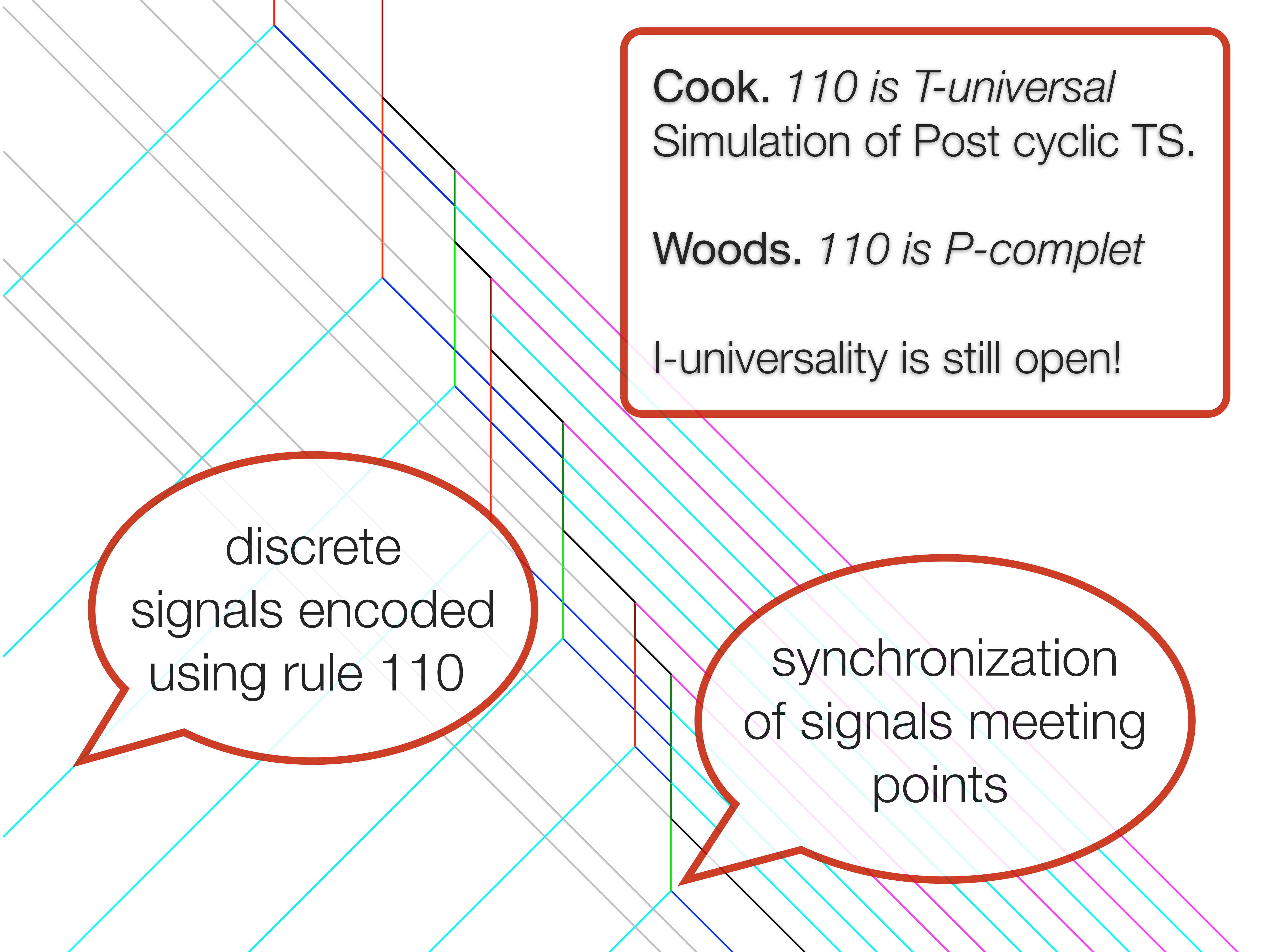


a 16 state universal CA simulating Post cyclic Tag Systems.

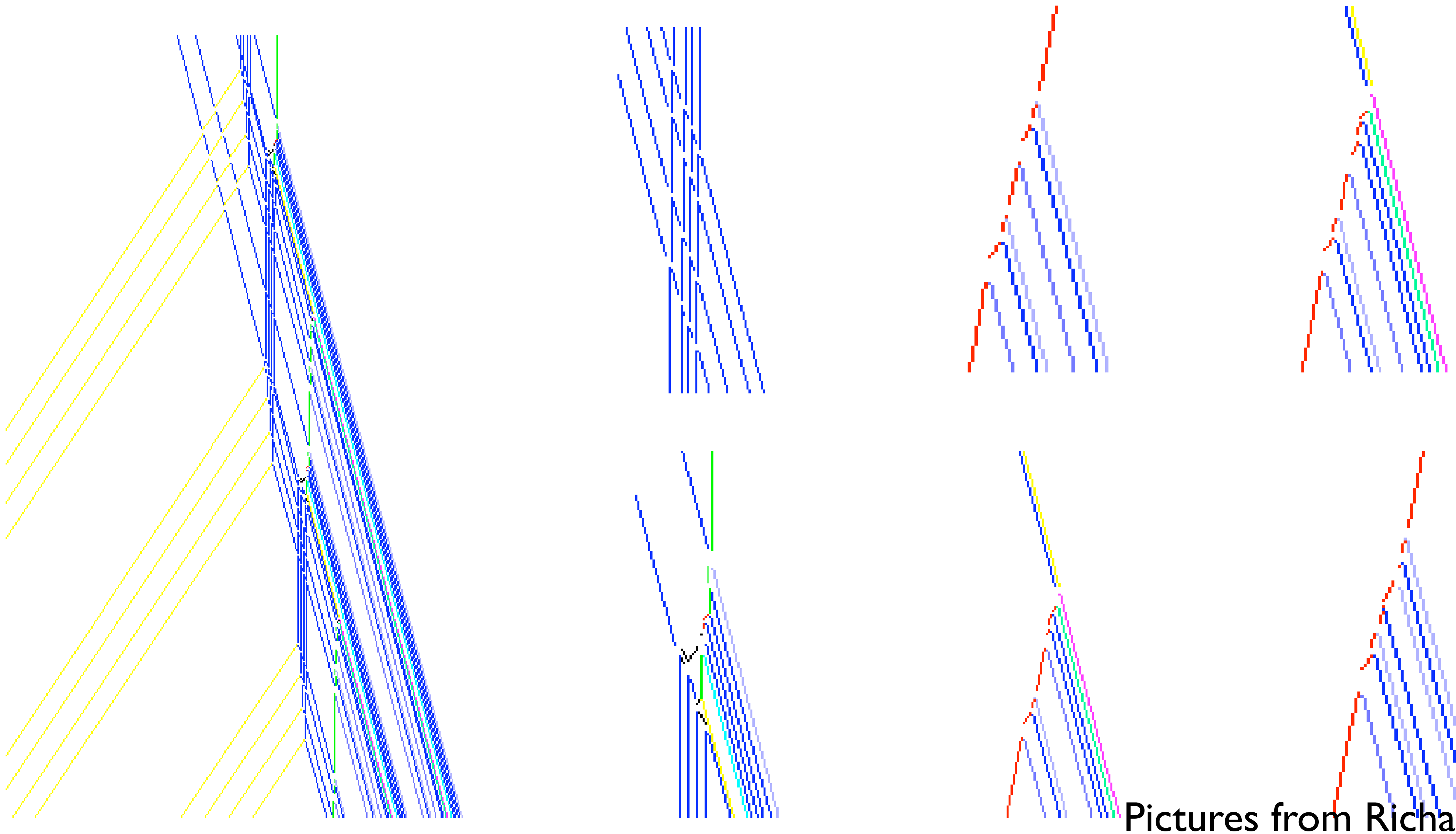
Cook. *110 is T-universal*
Simulation of Post cyclic TS.

Woods. *110 is P-complet*

I-universality is still open!

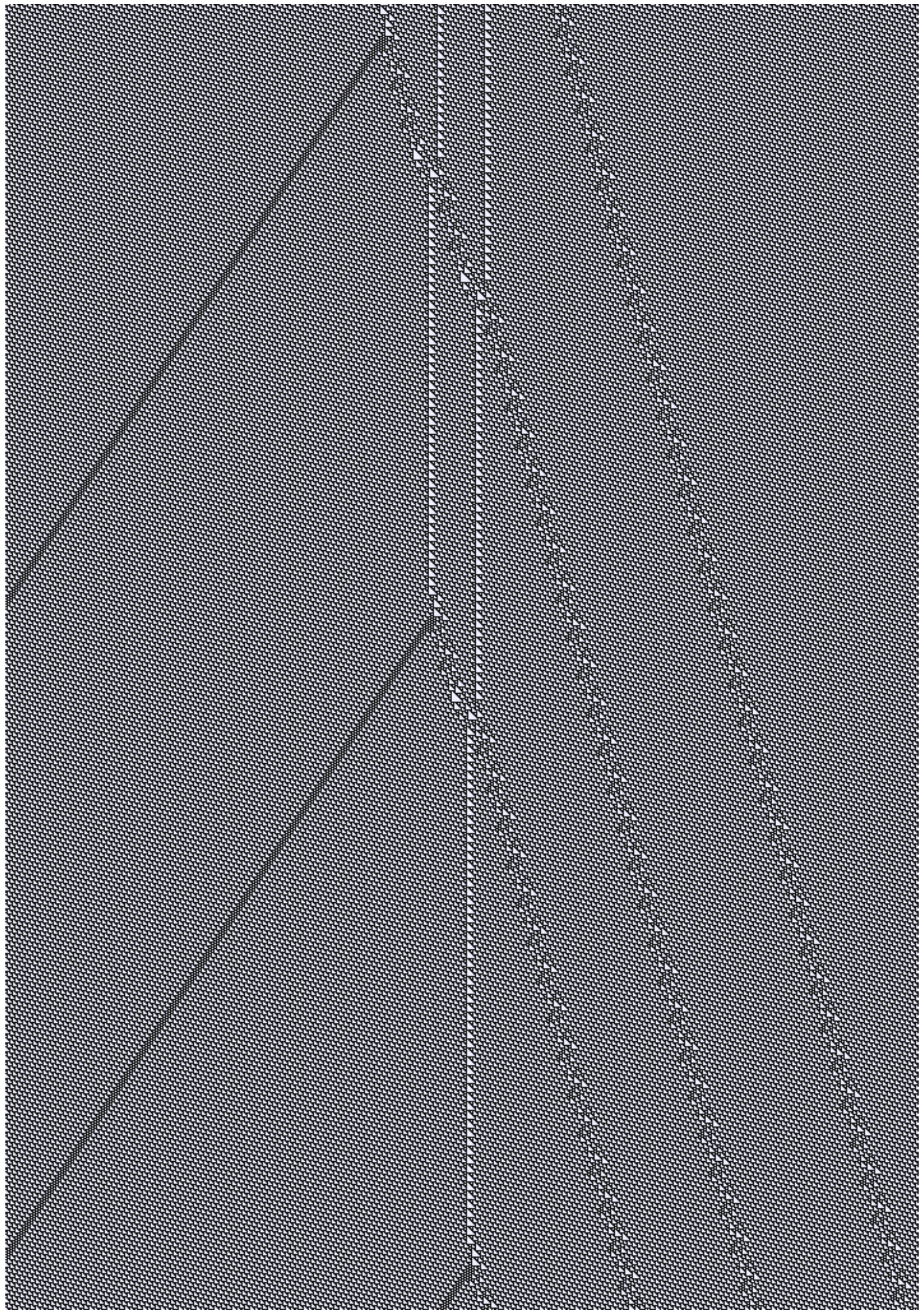


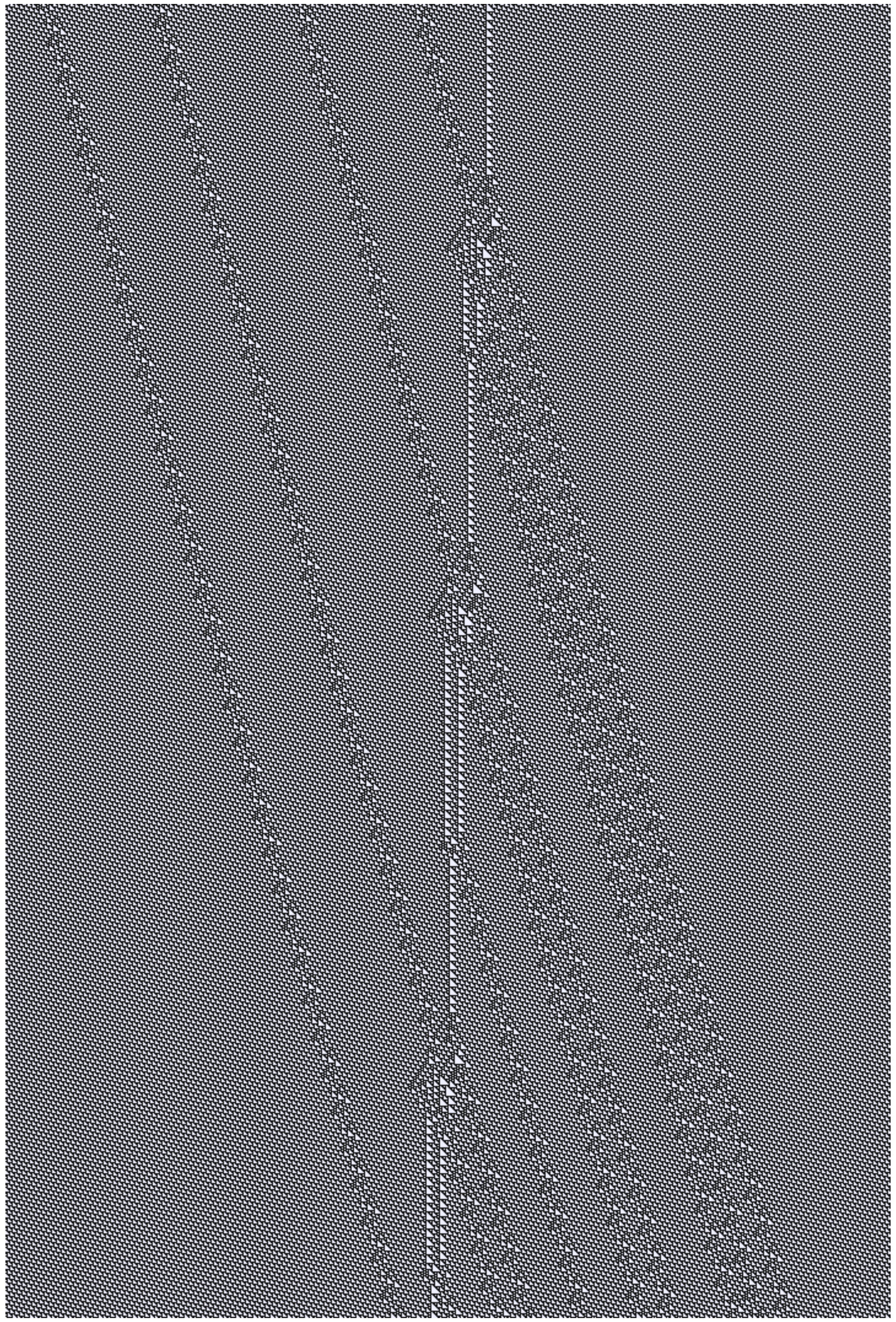
Encoding into rule 110



Pictures from Richard 2003

Signals encoding in rule 110



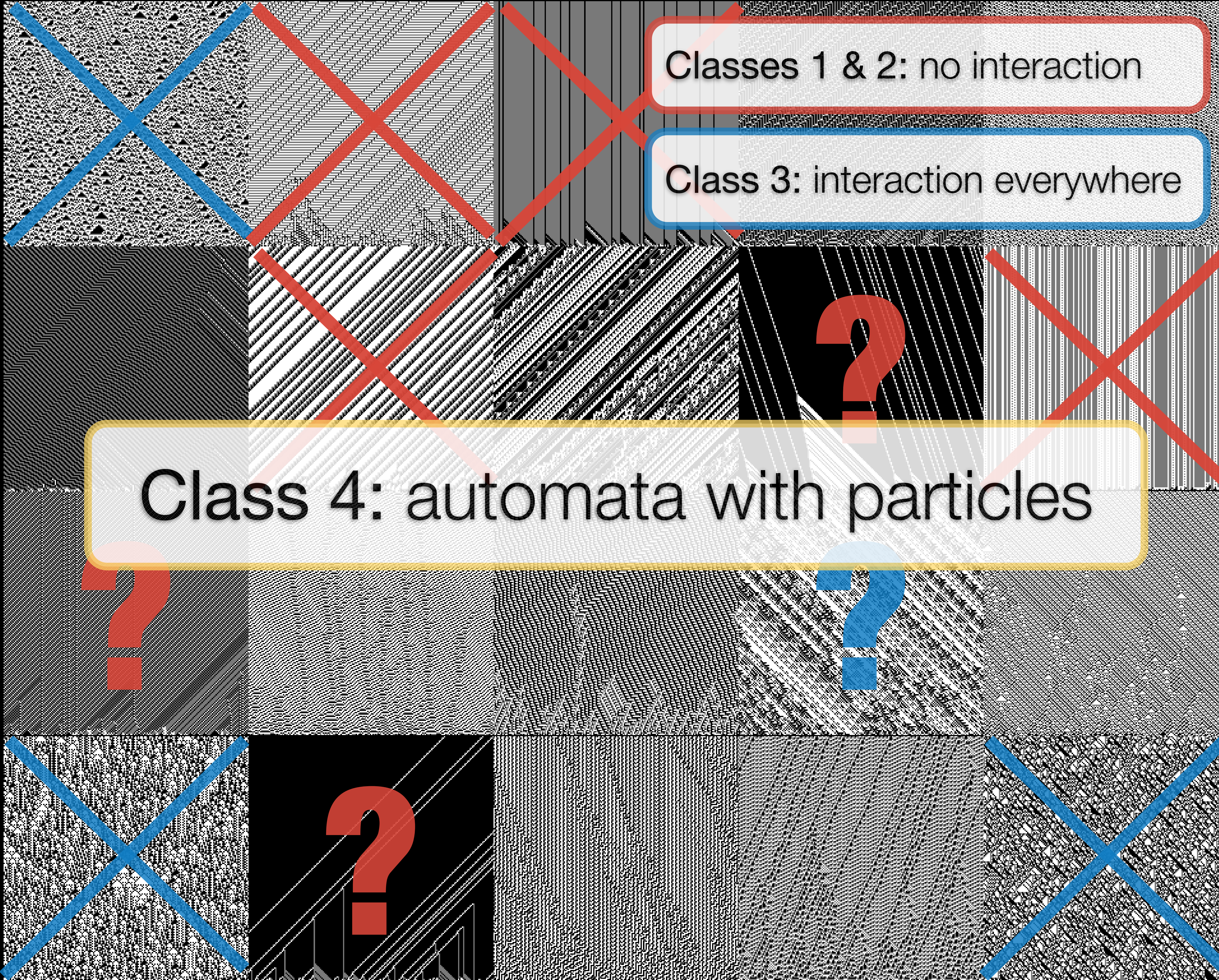


How common are signals?

- Below 4 states, it is difficult to **construct** rules.
- The rule space is huge:

$$n^{n^3} \qquad (1, 256, 7\,625\,597\,484\,987, \ldots)$$

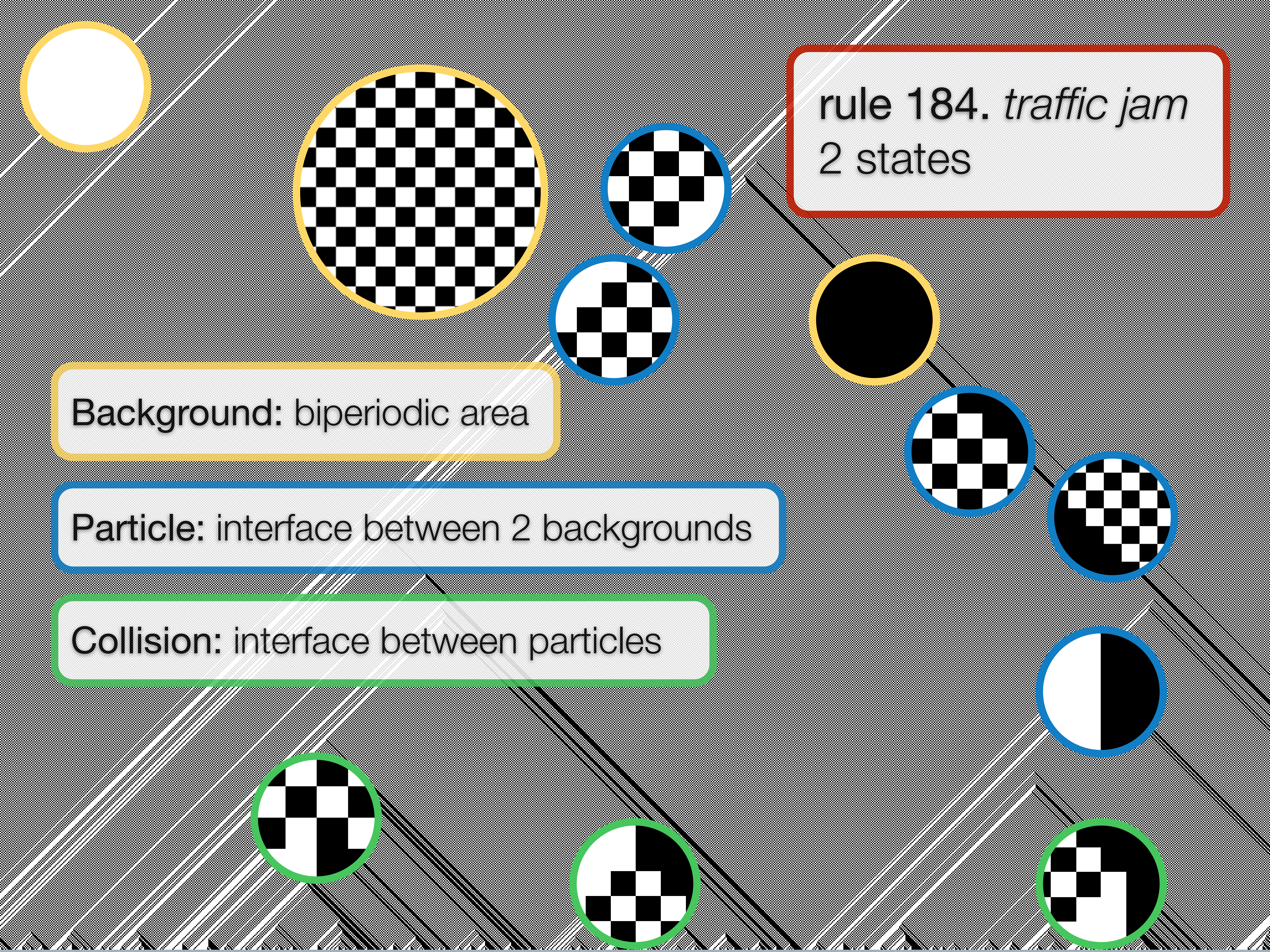
- How to find a rule with nice signals?

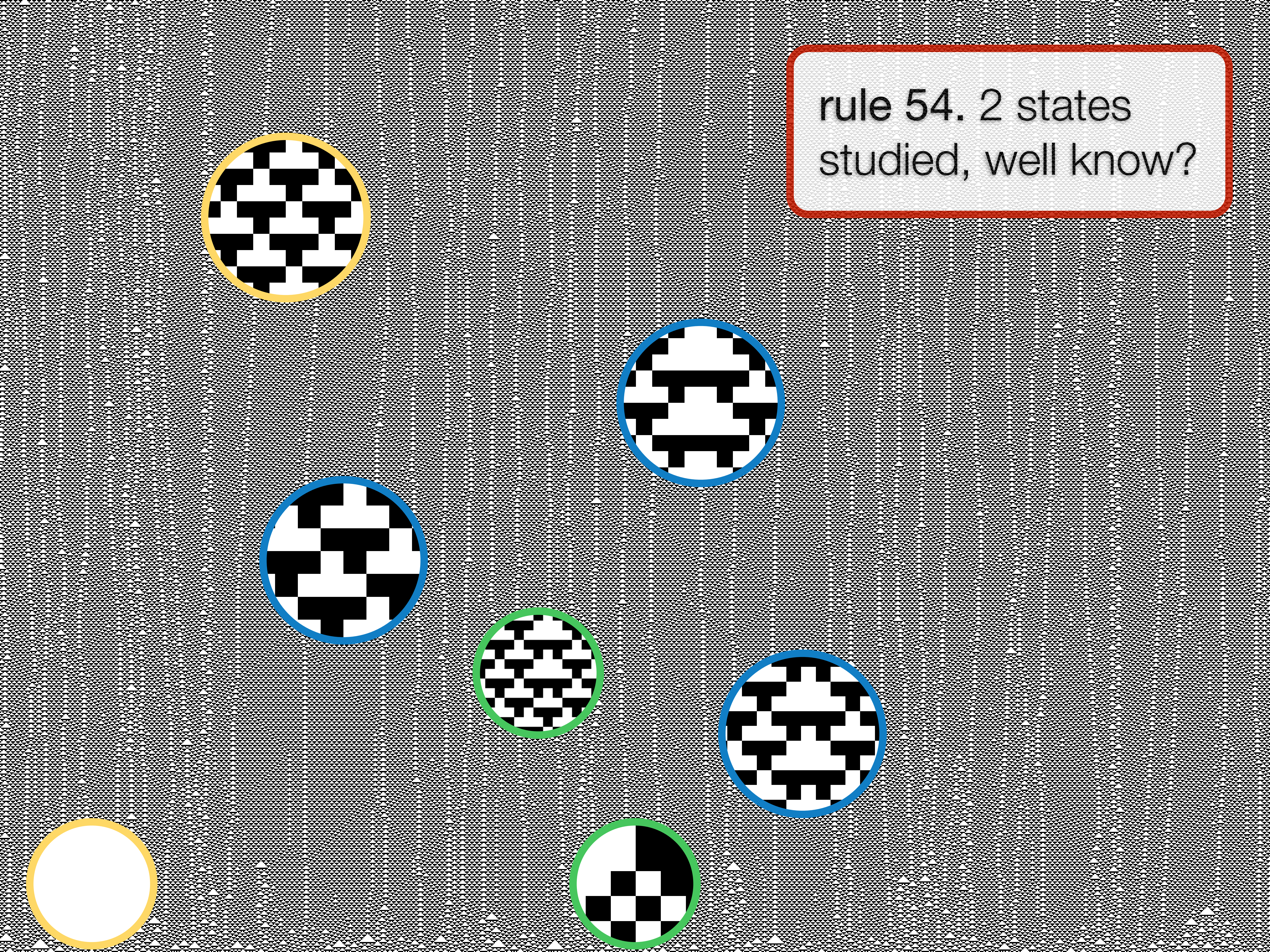

Classes 1 & 2: no interaction
Class 3: interaction everywhere
Class 4: automata with particles

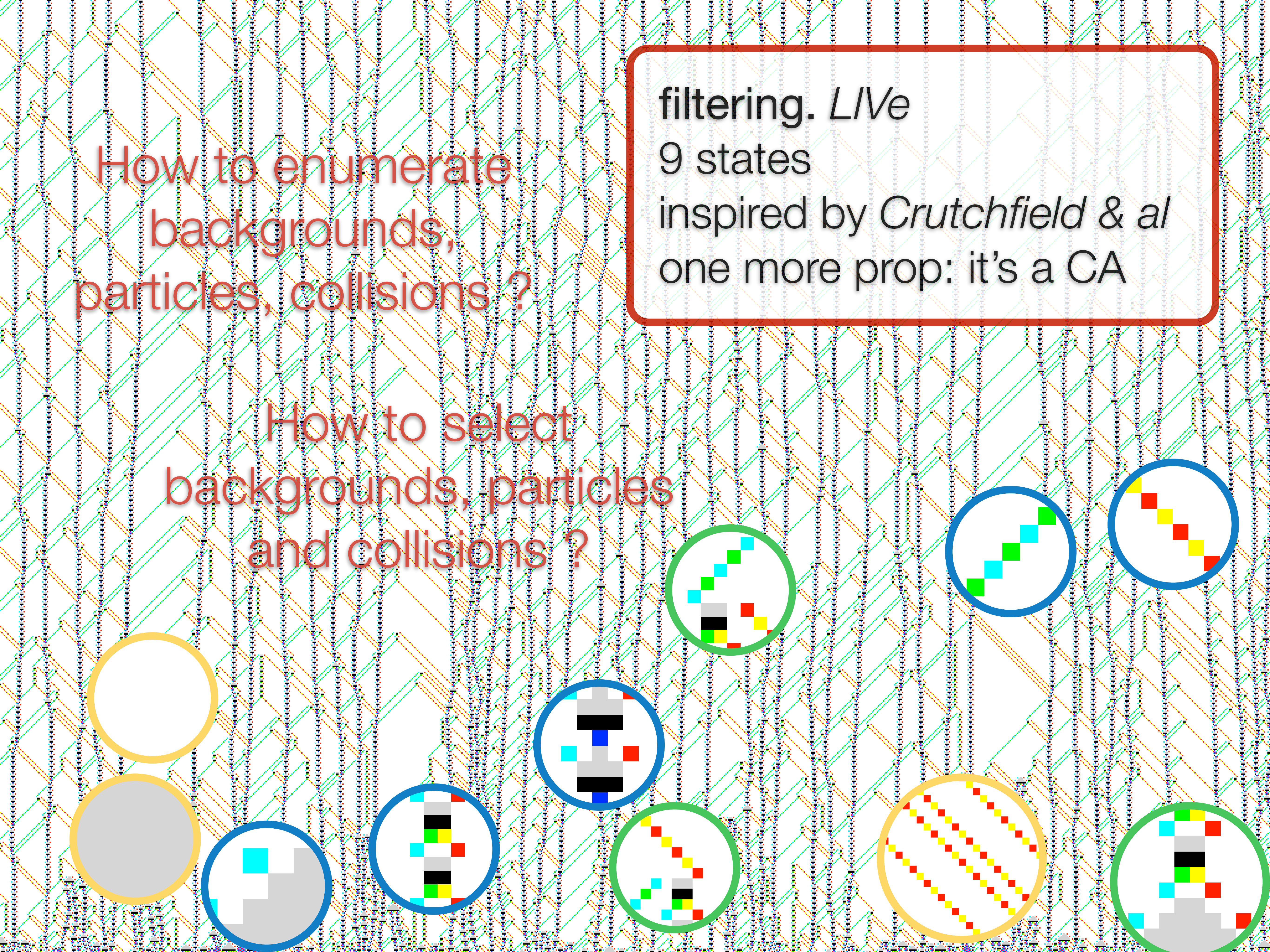
Automata with particles

*« And finally […] class 4 involves a mixture of order and randomness: **localized structures** are produced which on their own are fairly simple, but these structures **move around and interact** with each other in very complicated ways. […] »*

S. Wolfram, ANKOS


rule 184. *traffic jam*
2 states
Background: biperiodic area
Particle: interface between 2 backgrounds
Collision: interface between particles


rule 54. 2 states
studied, well know?



How to enumerate backgrounds, particles, collisions ?

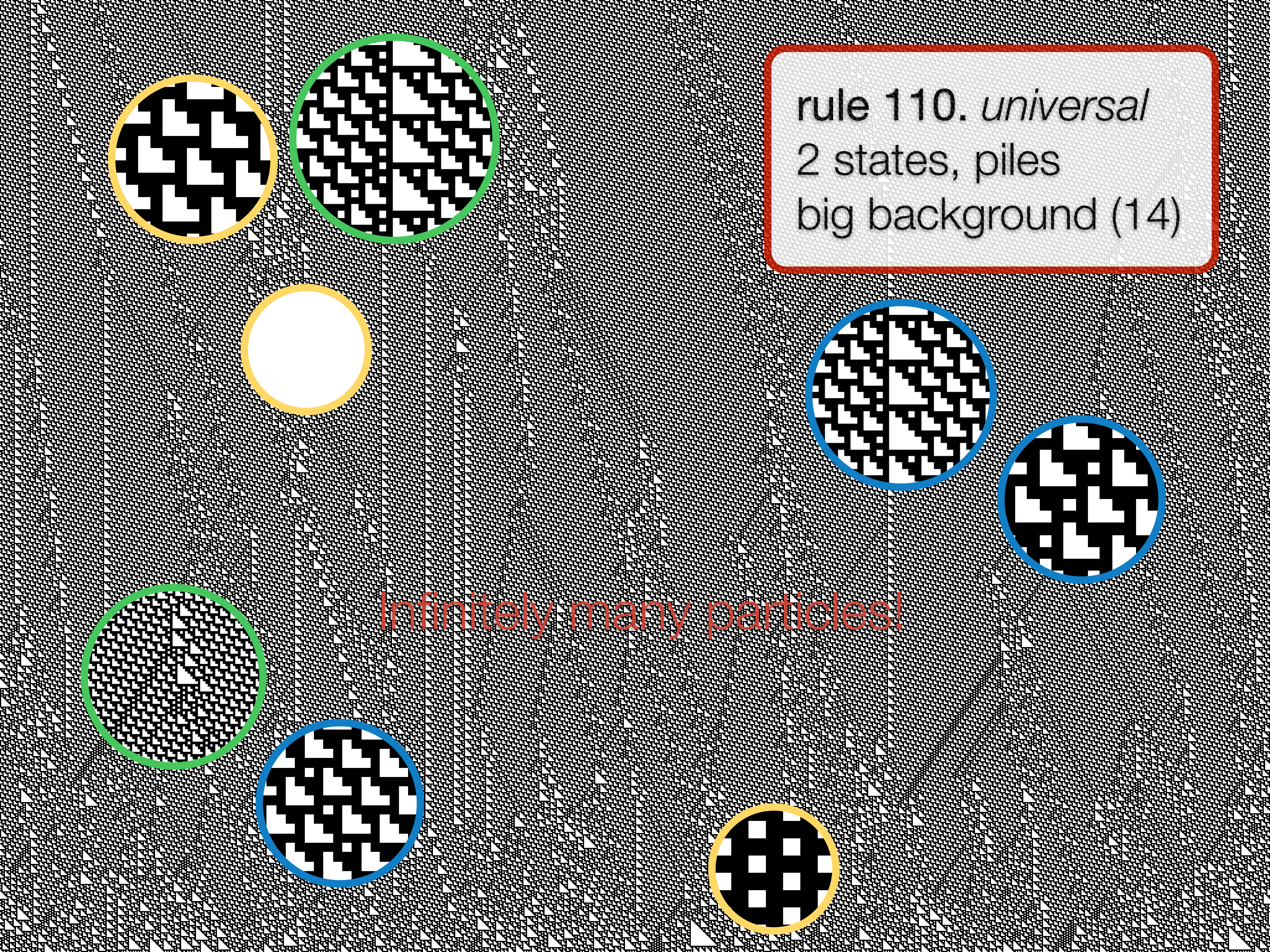
filtering. *LIVe*

9 states

inspired by *Crutchfield & al*

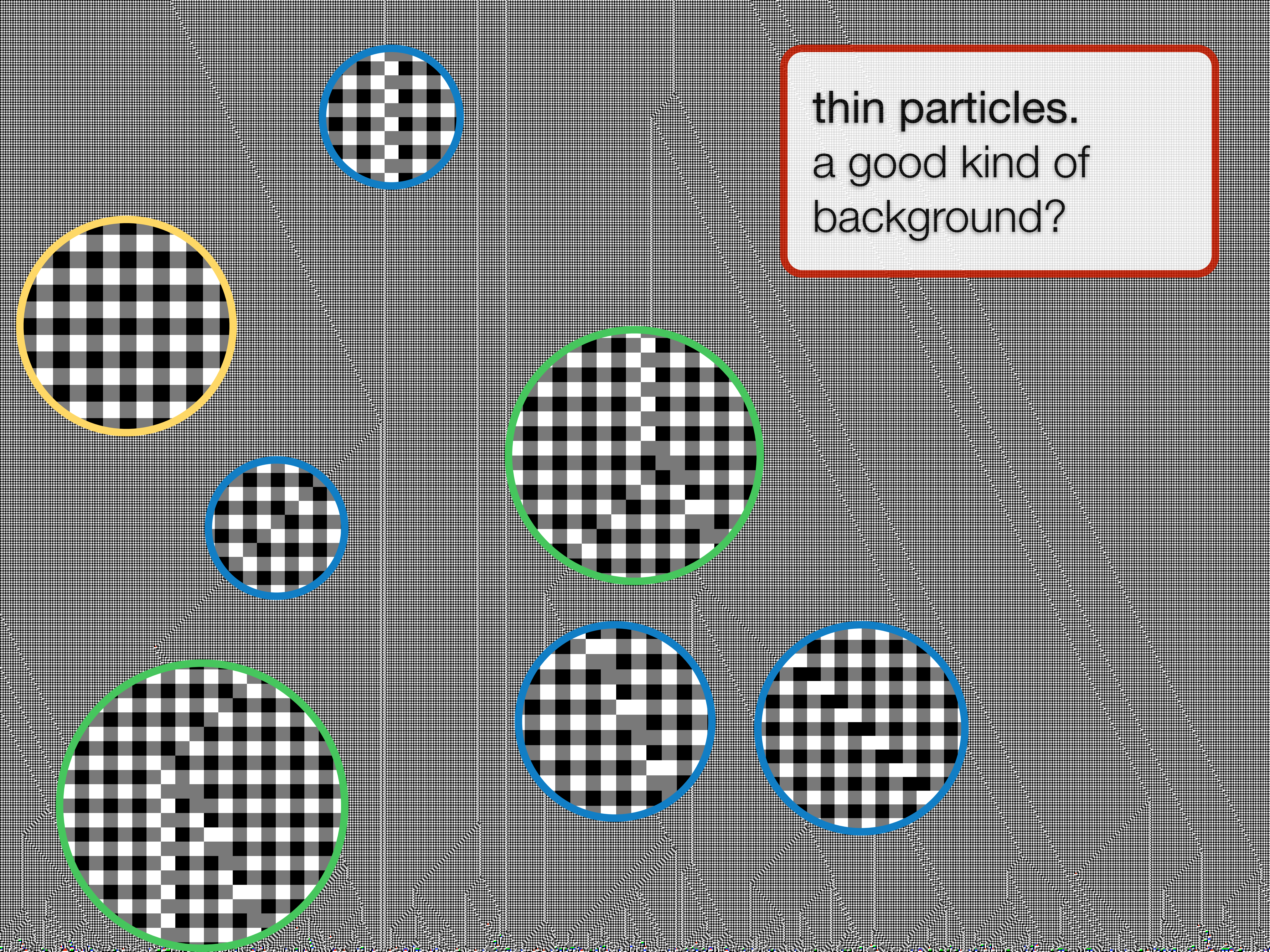
one more prop: it's a CA

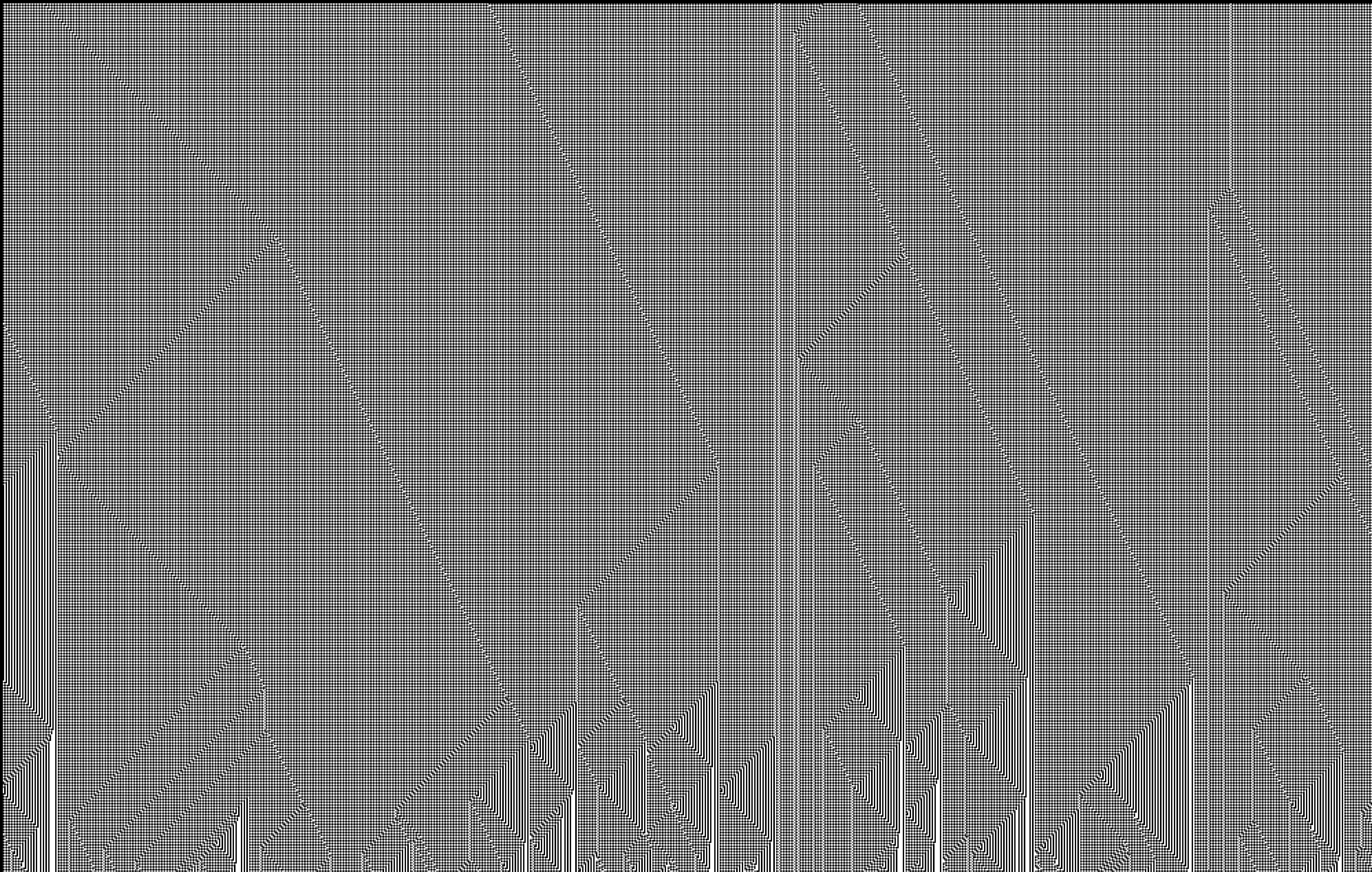
How to select backgrounds, particles and collisions ?

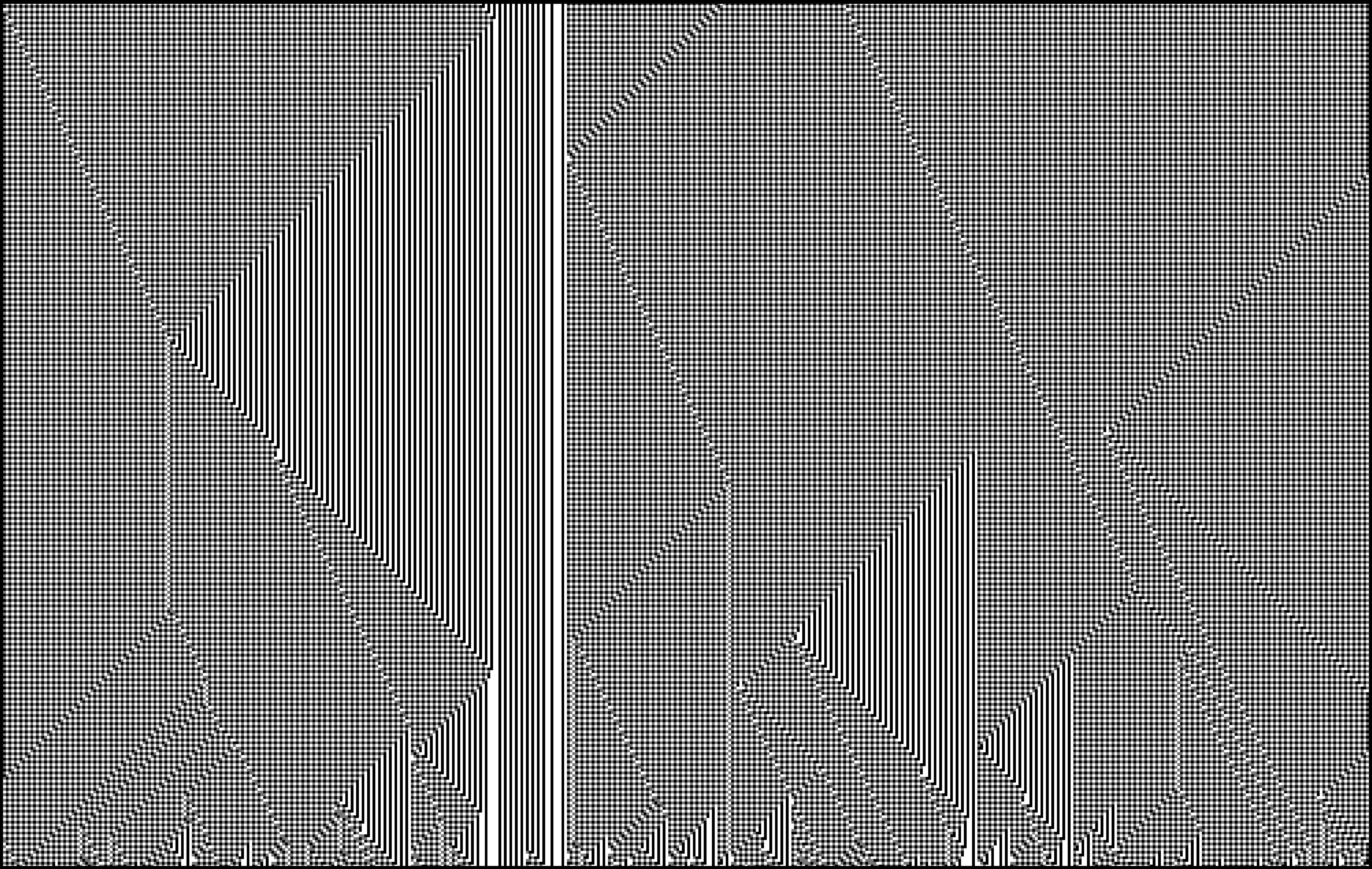

rule 110. *universal*
2 states, piles
big background (14)
Infinitely many particles!

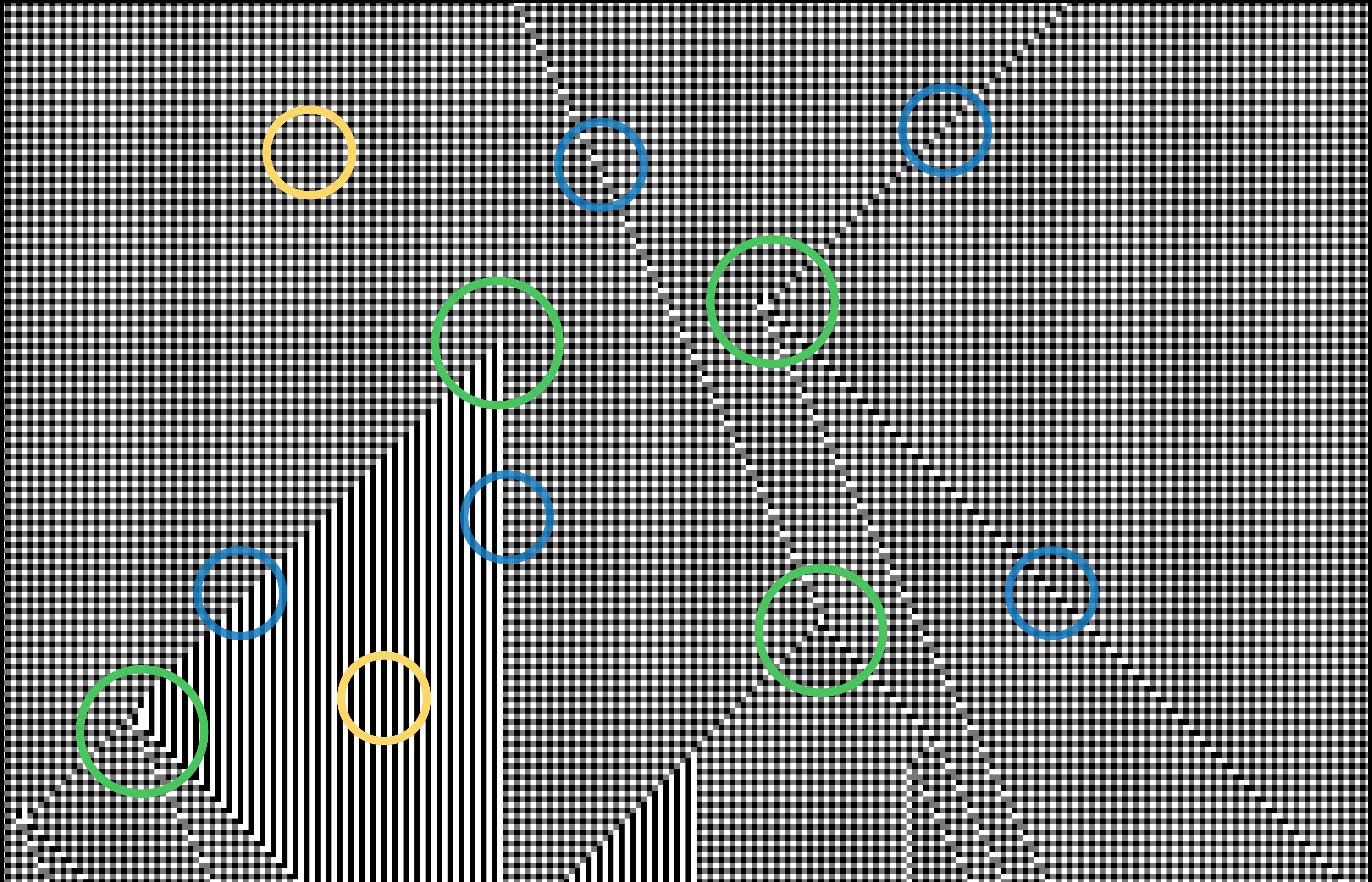
How to construct particle CA

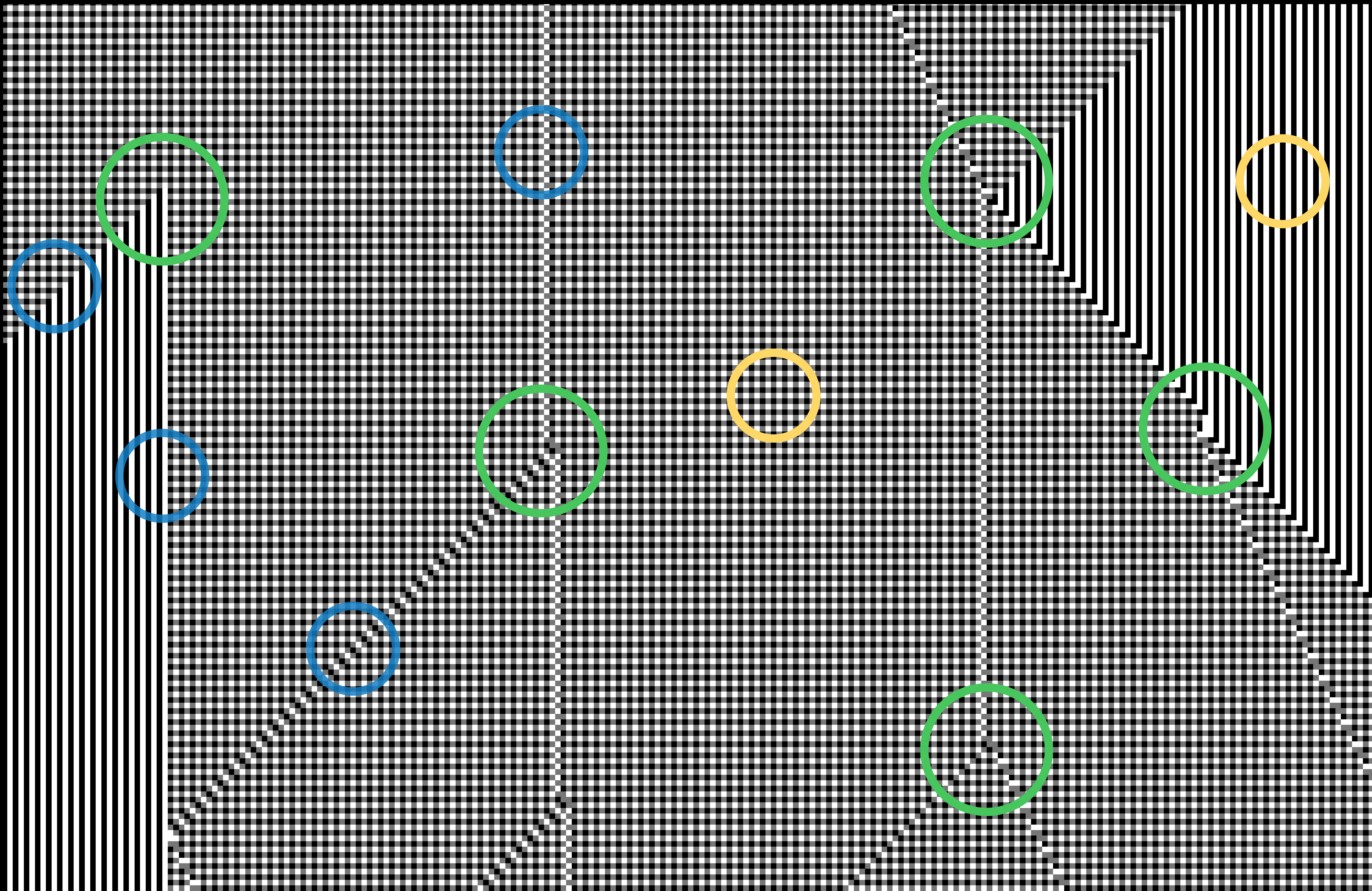
- it is easy to construct particle CA
- random rules regularily exhibit such behavior
- as an example for 3 states and first neighbors
- such particle CA are quite common


thin particles.
a good kind of
background?

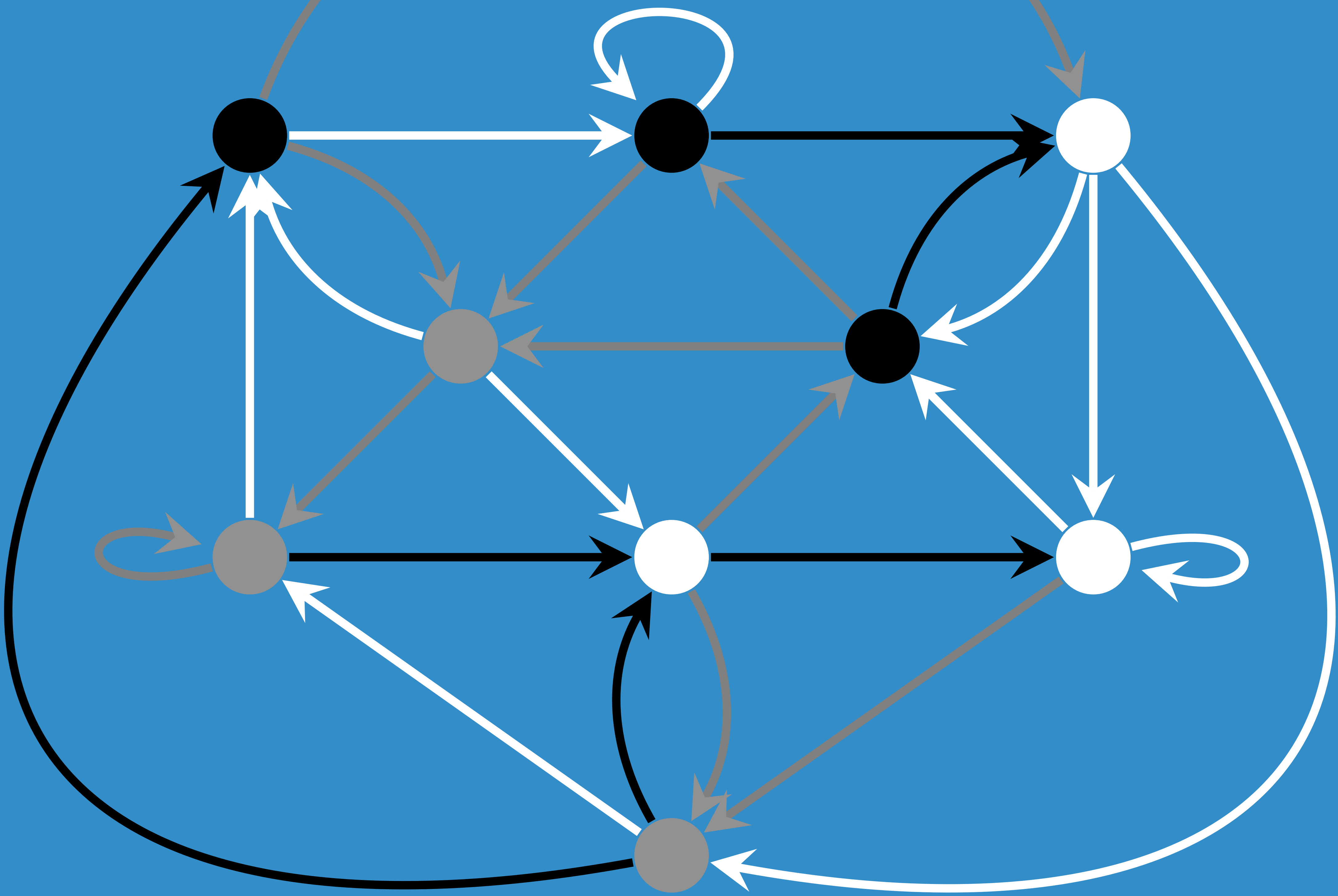


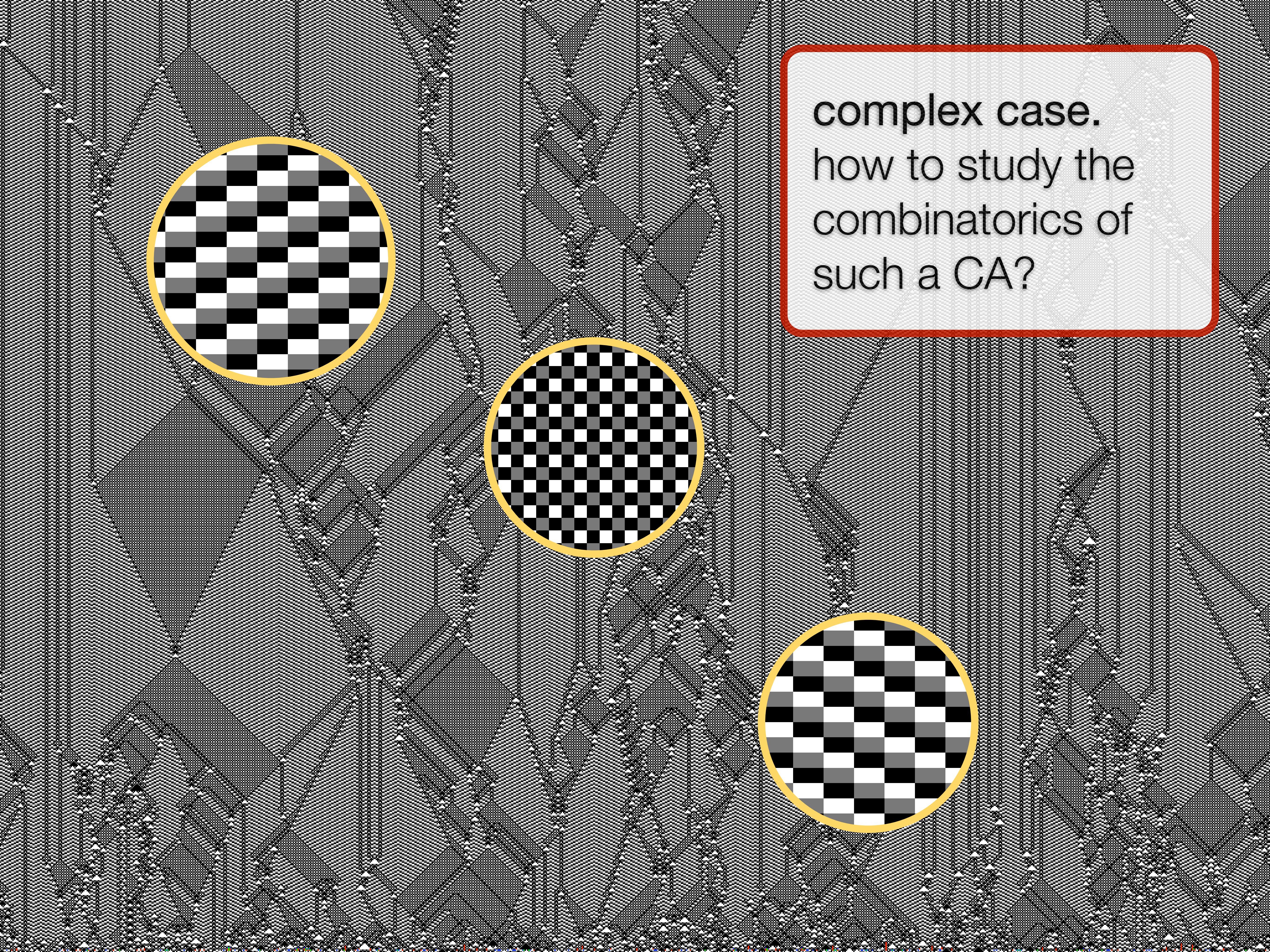




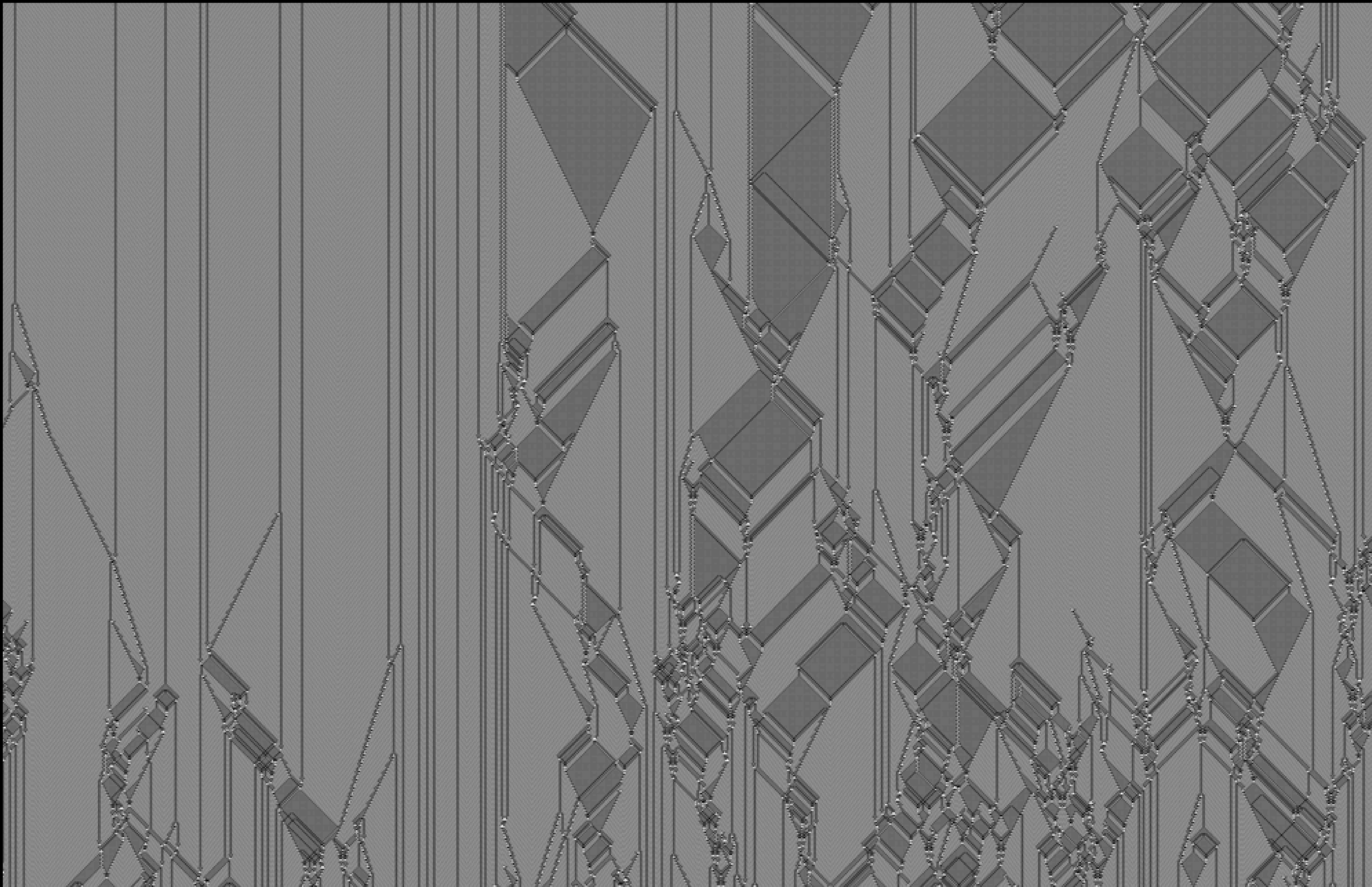


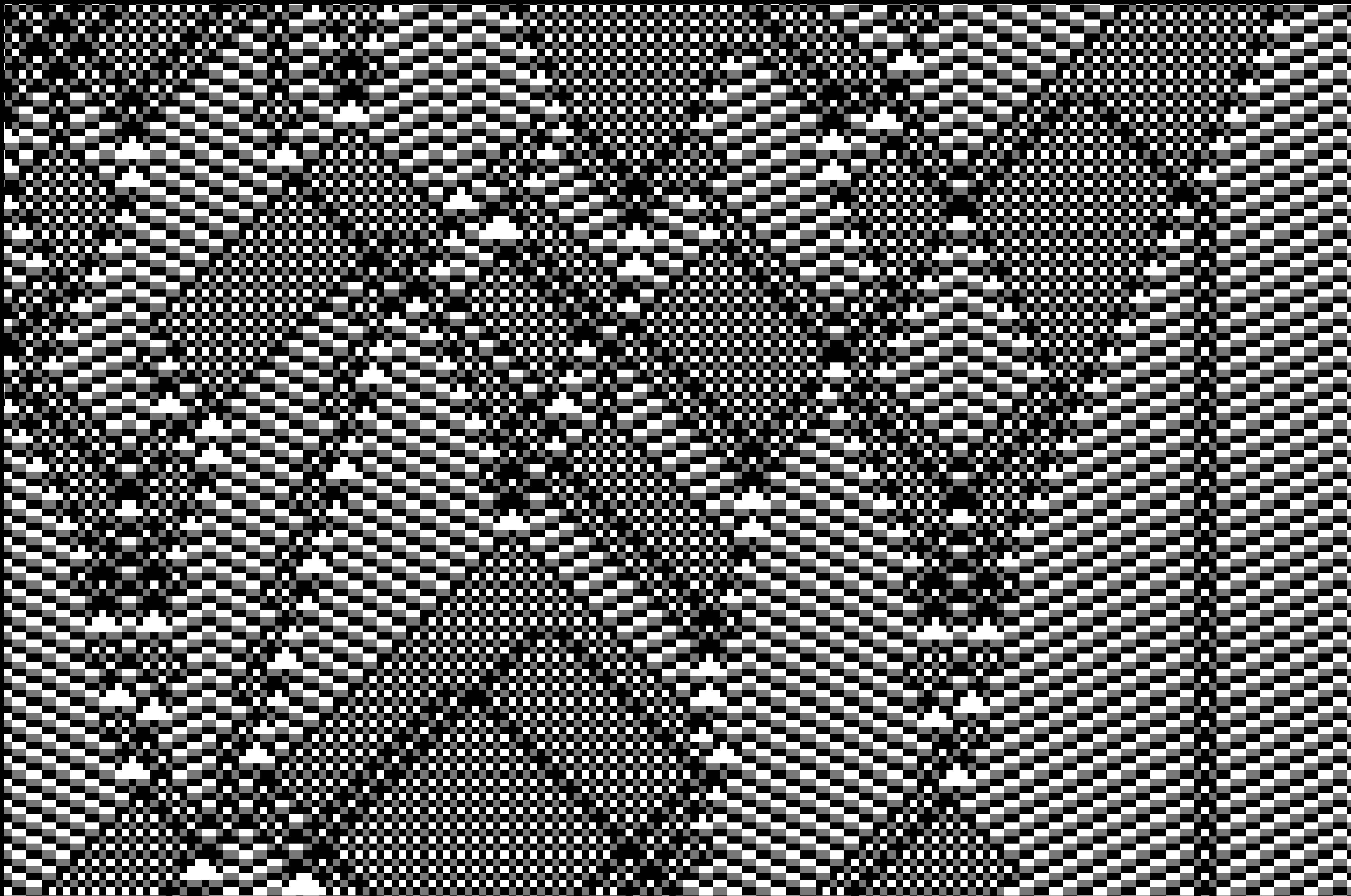
De Bruijn graph $\Gamma_{3,2}$

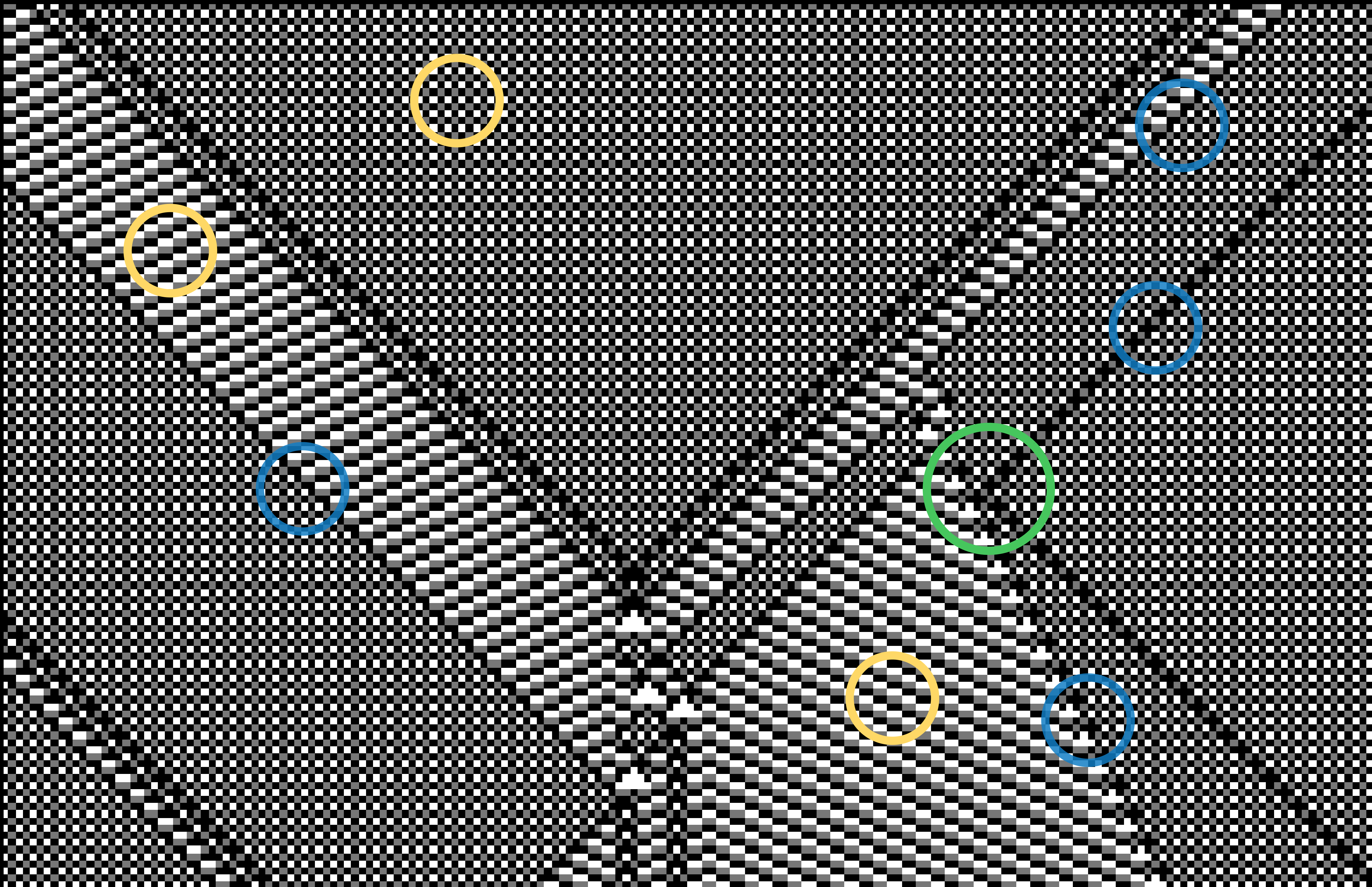


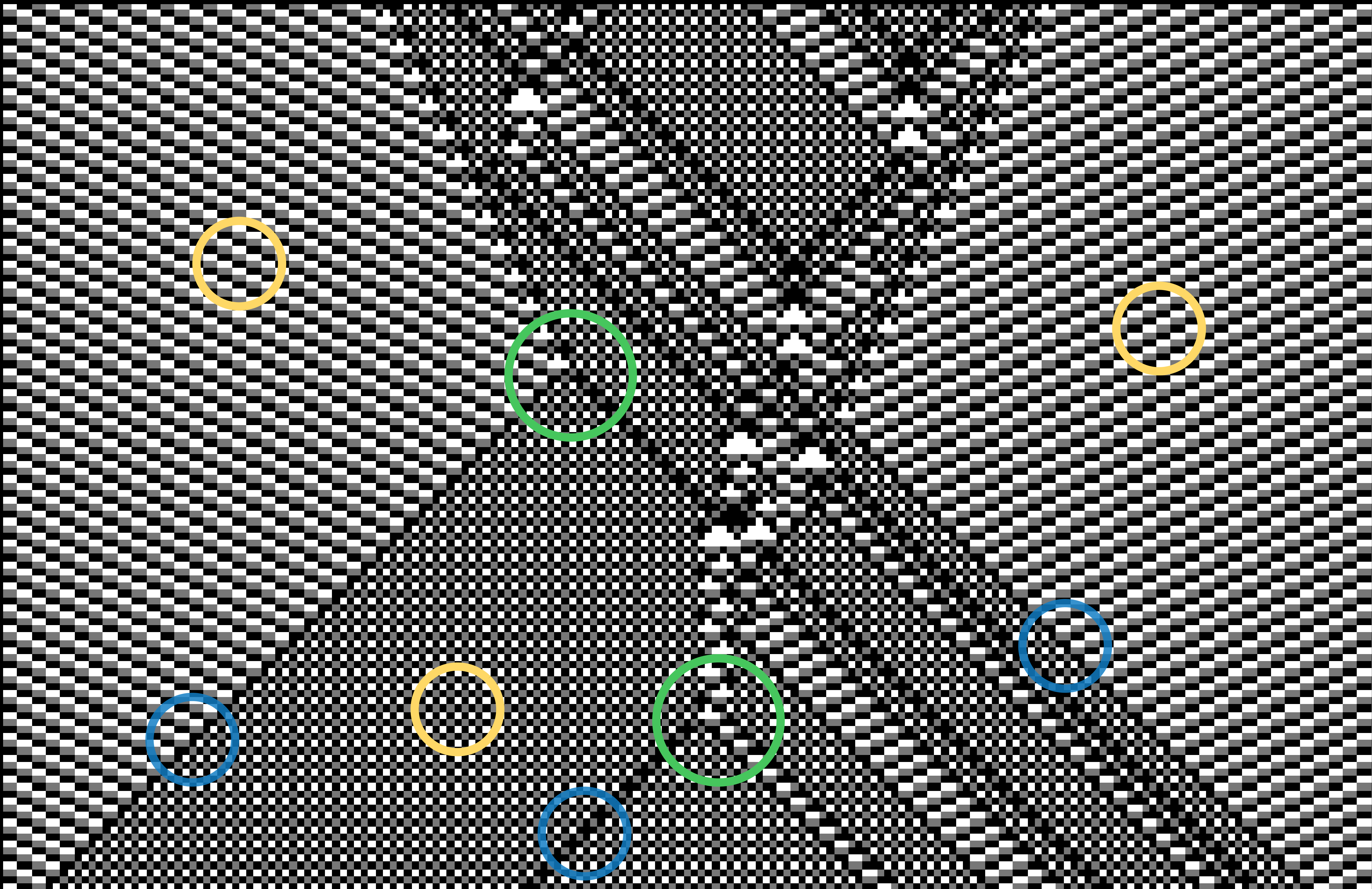


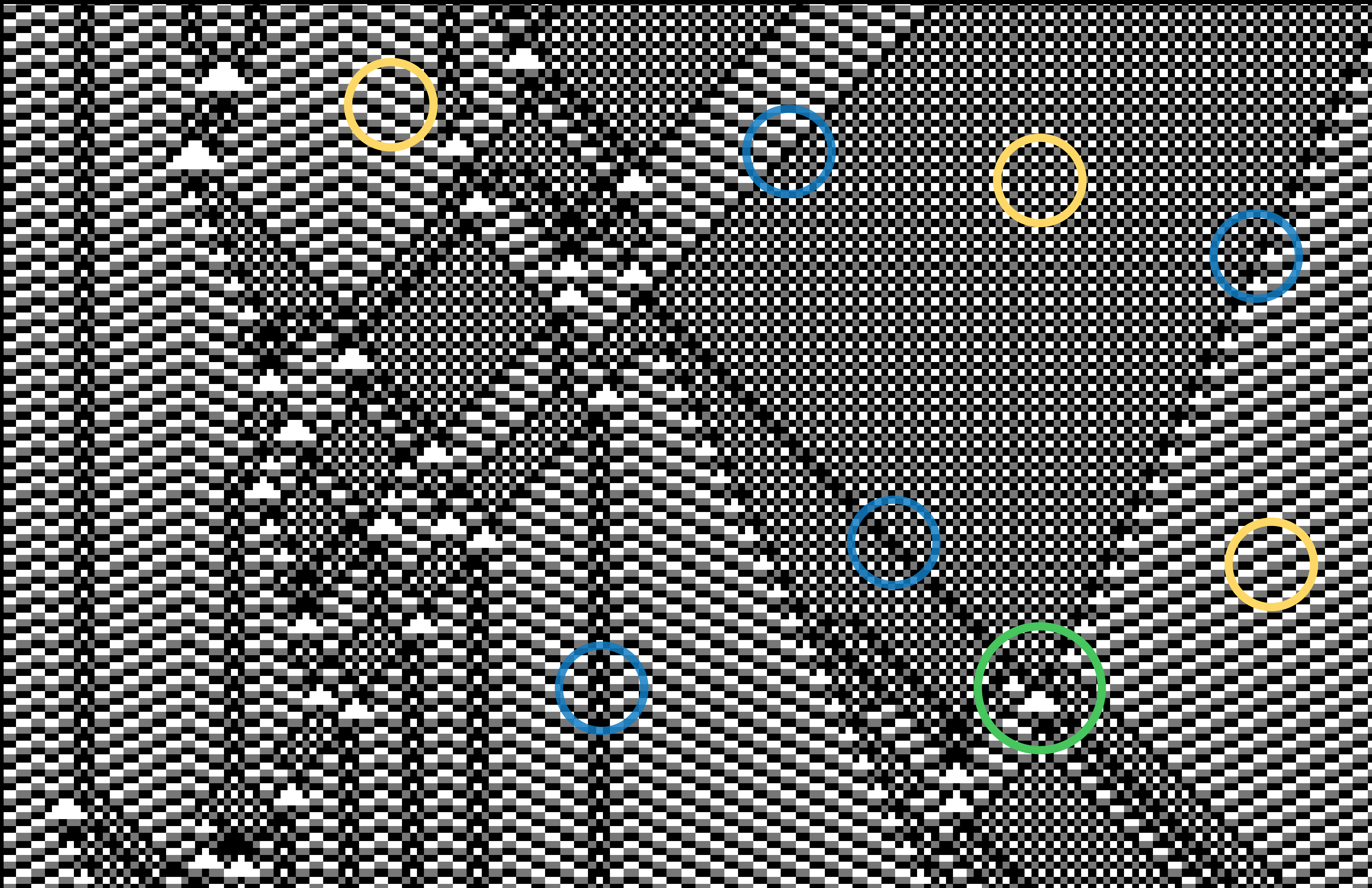
complex case.
how to study the combinatorics of such a CA?



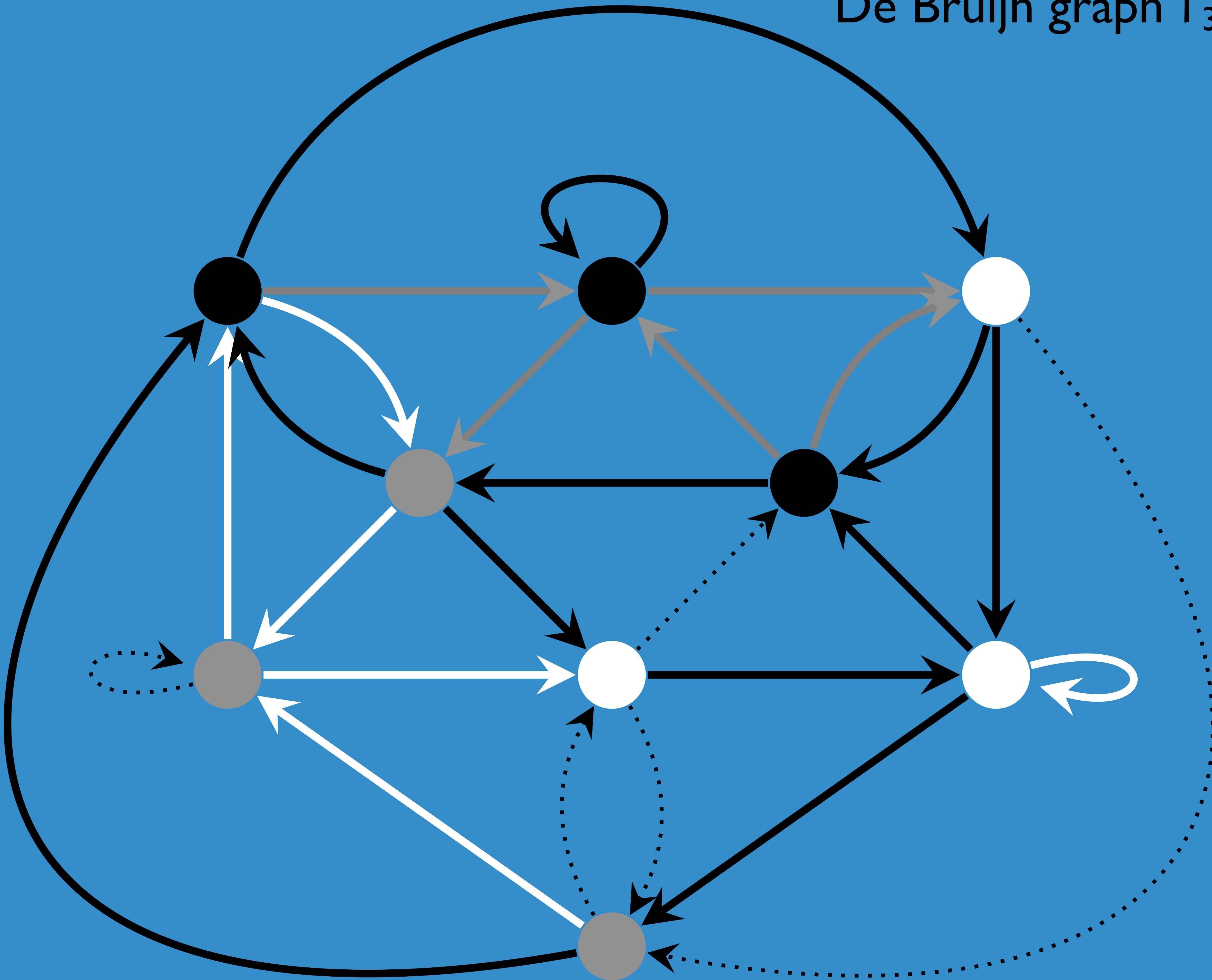








De Bruijn graph $\Gamma_{3,2}$



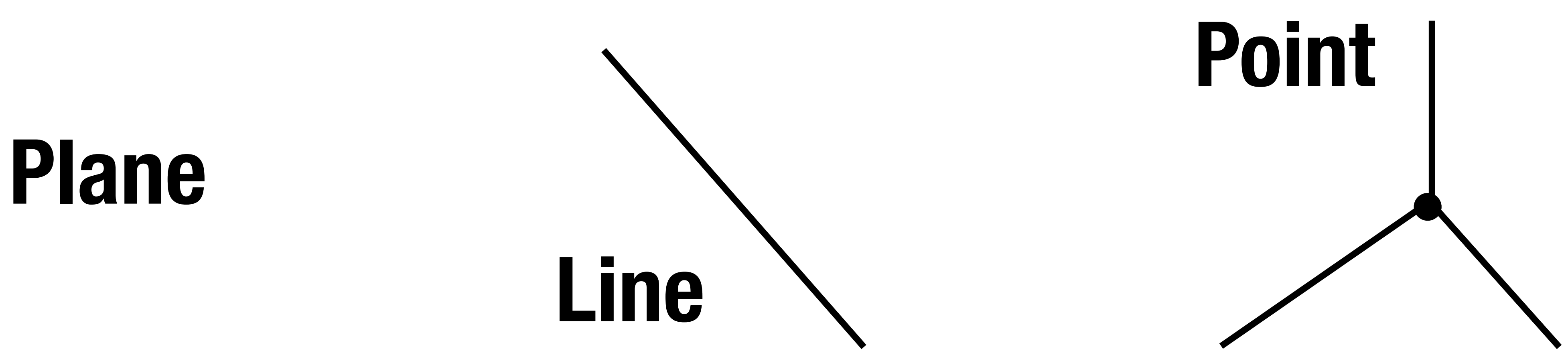
Questions

- What kind of tool can we hope to find?
- How to explain such a self-organization?
- How to study a given family of particles?
- How to exhibit complex particle CA?

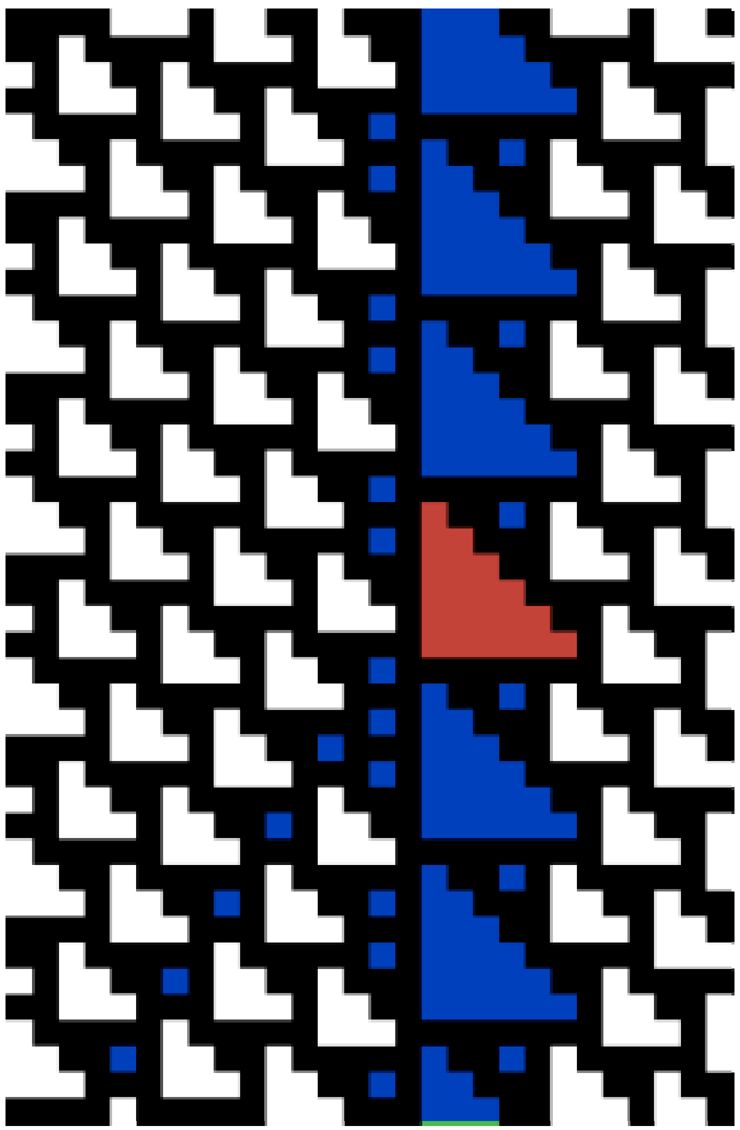
2 PaCo systems

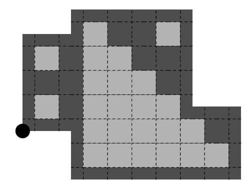
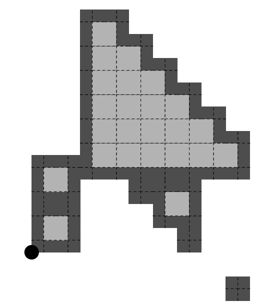
Back to basics

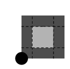
- We manipulate space-time diagrams
- **Background** (2D): bipériodic finite pattern ;
- **Particle** (1D): finite periodic pattern + bg ;
- **Collision** (0D): finite pattern + particles.



PaCo 110



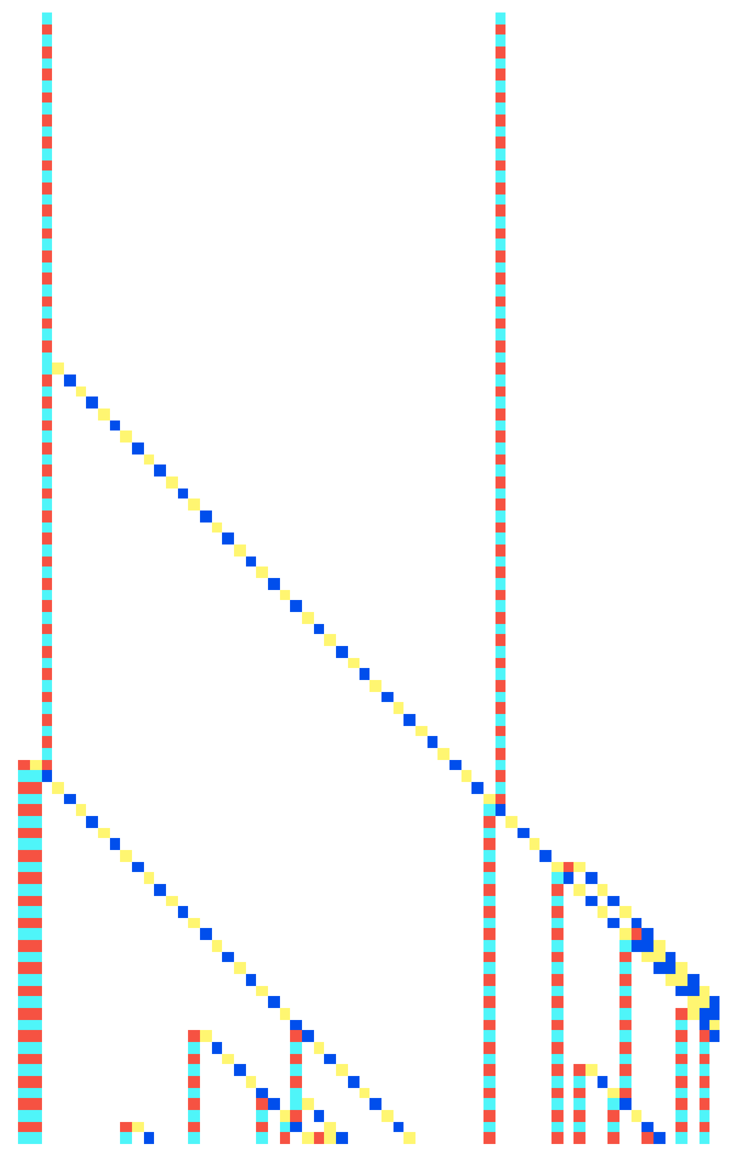
$A = \Big\langle \binom{0}{7},$  $\Big\rangle$ $\quad B = \Big\langle \binom{0}{7},$  $\Big\rangle$

$C = \langle \binom{2}{3},$  $\rangle$

$\Gamma: \quad \binom{0}{0}C + \binom{0}{-4}A \vdash \binom{0}{5}B$

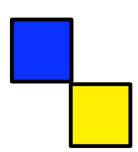
A simpler example

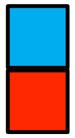
l	m	r	$\delta(l,m,r)$
–	red	yellow	white
–	red	blue	blue
–	cyan	yellow	cyan
cyan	yellow	–	white
–	cyan	blue	yellow
cyan	blue	–	red
–	red	–	cyan
–	cyan	–	red
–	–	yellow	blue
–	–	blue	yellow
–	–	–	white



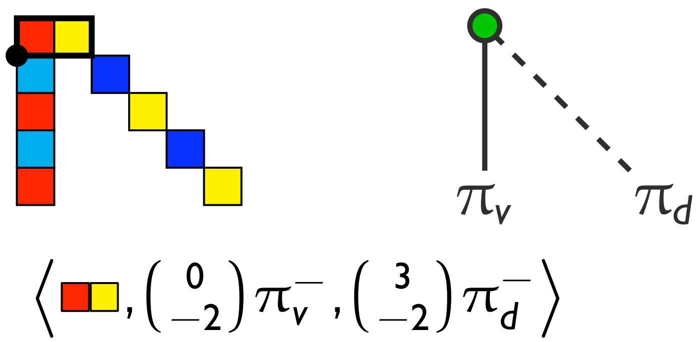
A simpler example (cont')

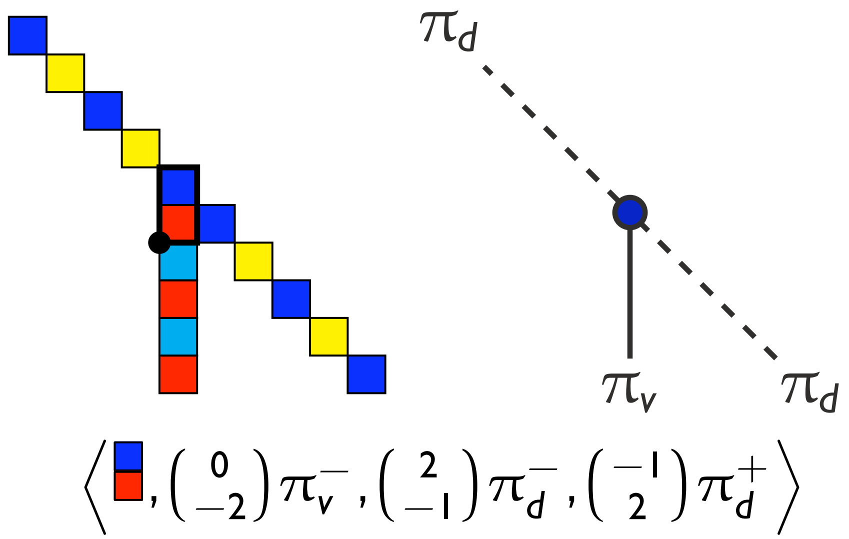
$$\left\langle \square, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\rangle$$

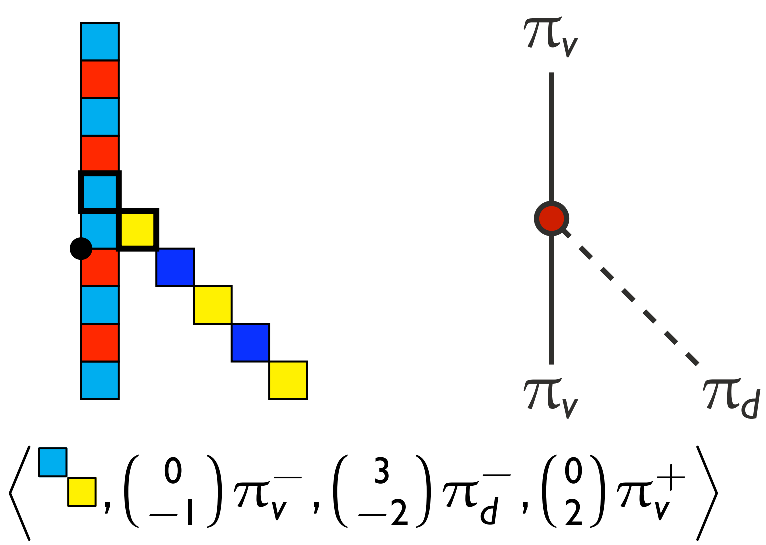
$\pi_d = \Big\langle$  $, \begin{pmatrix} -2 \\ 2 \end{pmatrix} \Big\rangle$

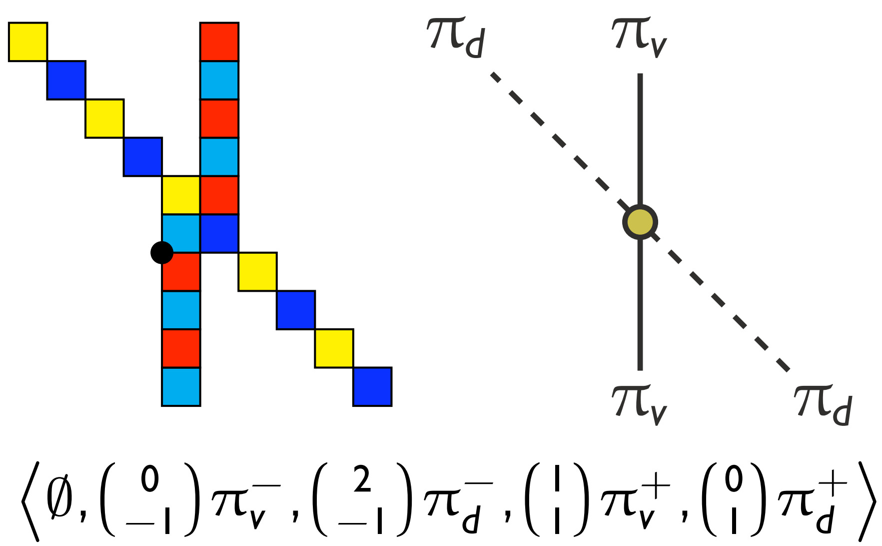
$\pi_v = \Big\langle$  $, \begin{pmatrix} 0 \\ 2 \end{pmatrix} \Big\rangle$

A simpler example (cont')









Simplicity ⇒ Properties

- These objects are kept simple on purpose
- Finite & periodic = regular, rational
- Elements can be composed easily (algo)

Already too much complicated!

Composing PaCo

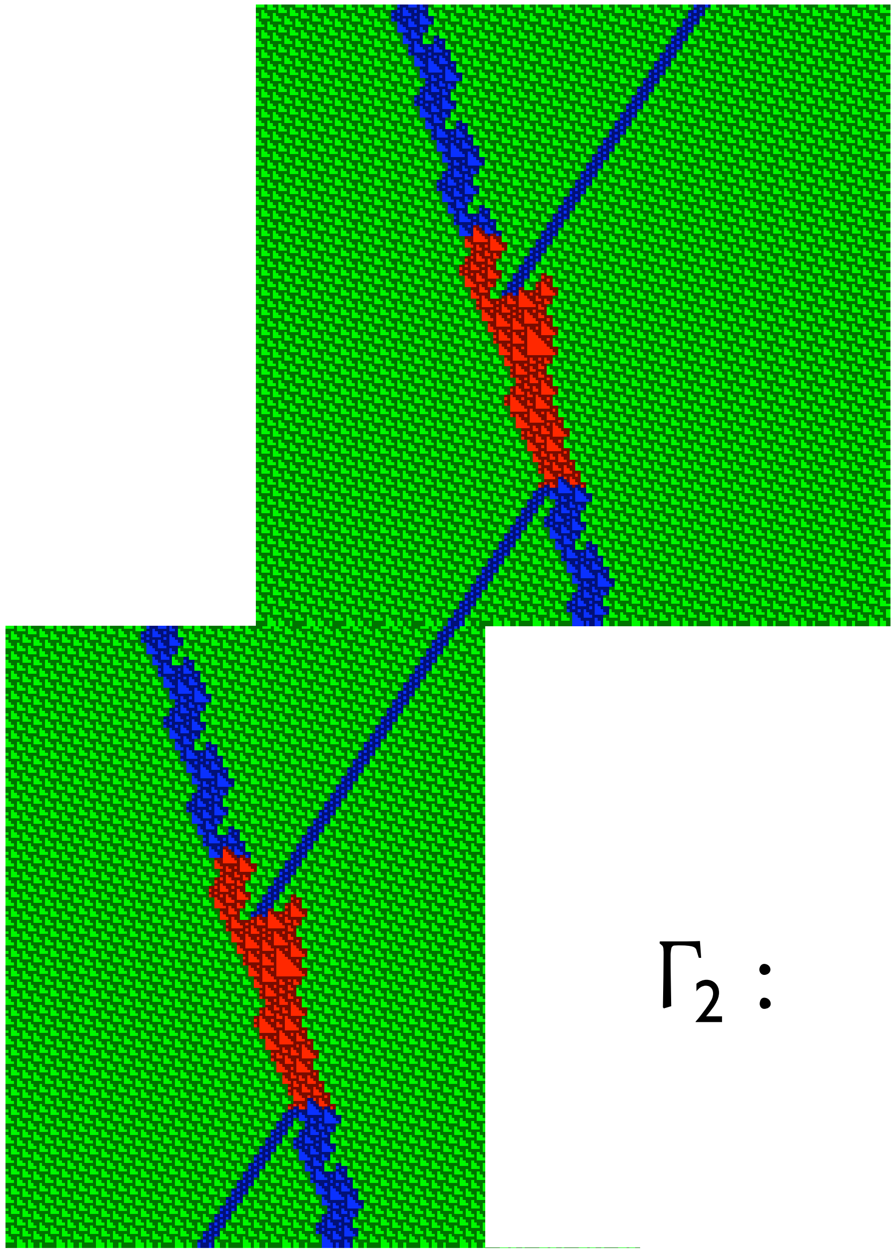
- One can bind collisions together.

Principle Merge incoming and outgoing particles when possible. Some bindings are not valid !

$$\Gamma' = \left(\binom{\alpha_1}{\beta_1} \Gamma_1 + \binom{\alpha_2}{\beta_2} \Gamma_2 + \cdots + \binom{\alpha_n}{\beta_n} \Gamma_n \right)_{\text{bind}}$$

- Binding is easy to construct and validate.

Binding in 110



$$\Gamma_c : \quad \alpha L + \beta R \vdash \gamma L + \delta R$$

$$\Gamma_2 = (\Gamma_c + \nu \Gamma_c)_{R_1^{\text{out}} + R_2^{\text{in}}}$$

$$\Gamma_2 : \quad \alpha L + \beta R + \nu \alpha L \vdash \gamma L + \nu \gamma L + \nu \delta R$$

Simple not simplistic

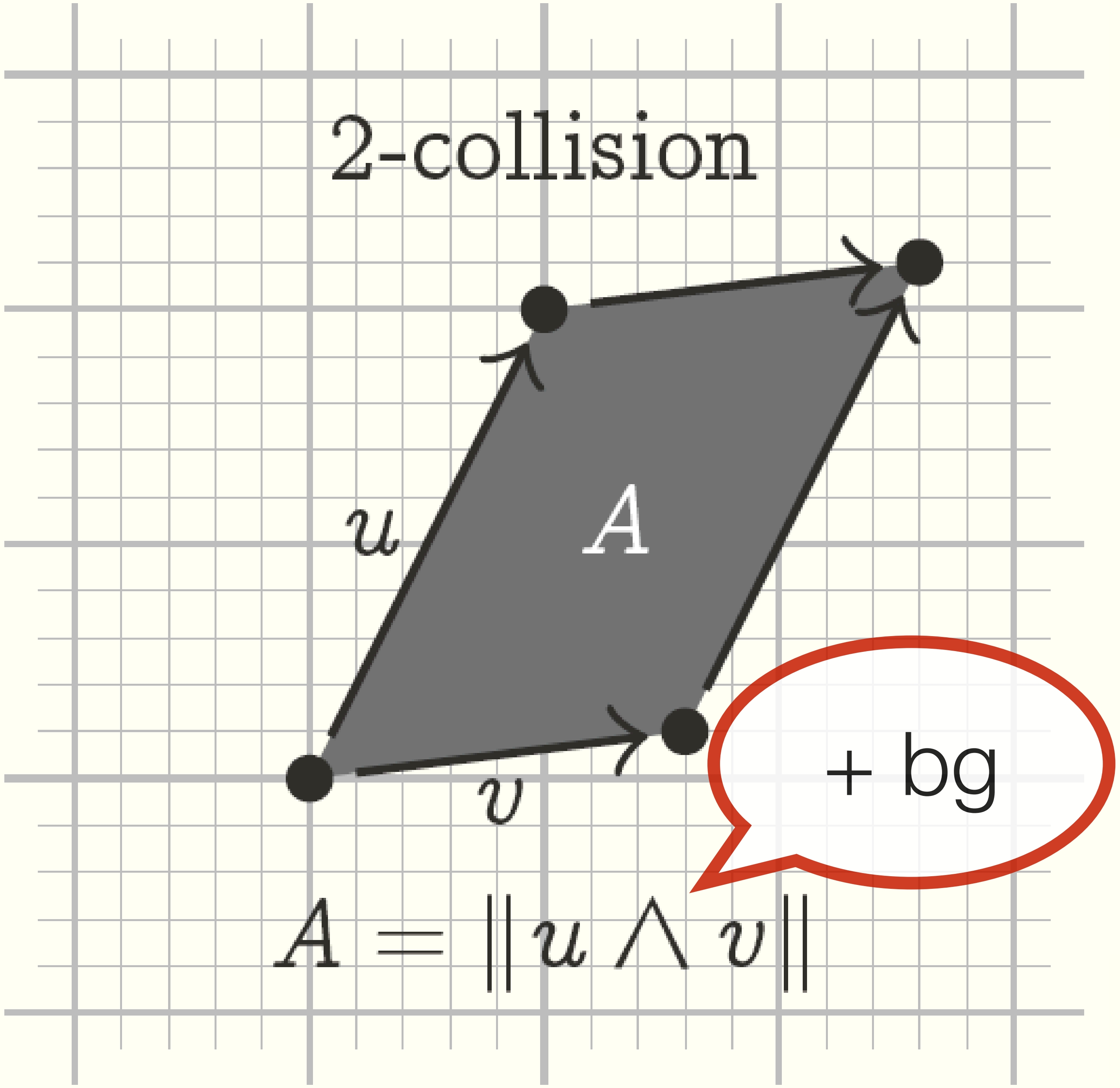
- **Glider-Gun** are not a problem
- Most constructions fit (*e.g.* 110, 4st)
- Match with intuition *à la Wolfram...*

From CA to PaCo

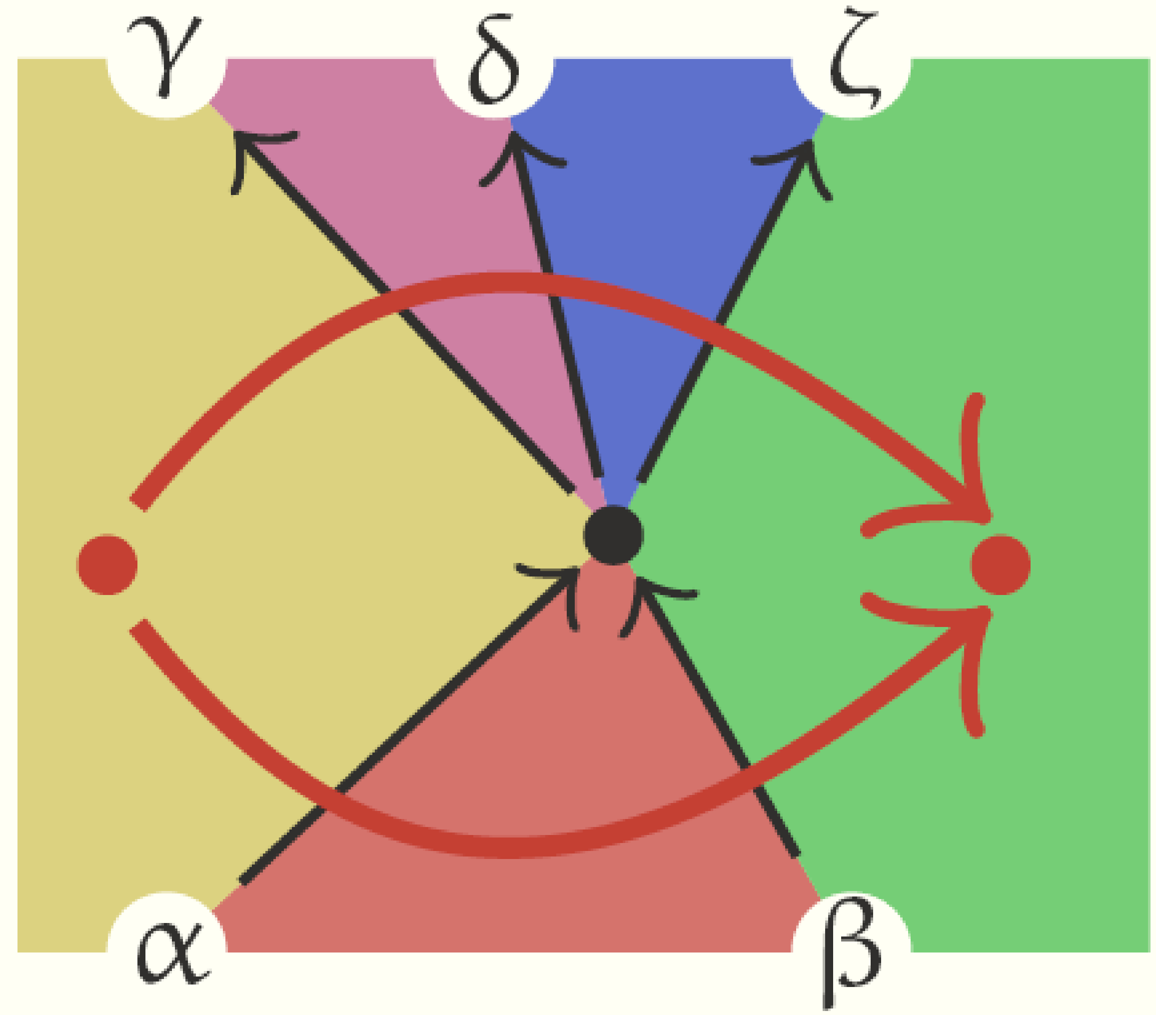
- How to select background, particles, collisions?
- Completness if difficult to achieve...
- By now, we just choose acording to goal.

How to bound the collisions?

- Let’s fix both background and particles.
- What collisions may happen?
- Why are there only collisions occuring?



à la Crutchfield

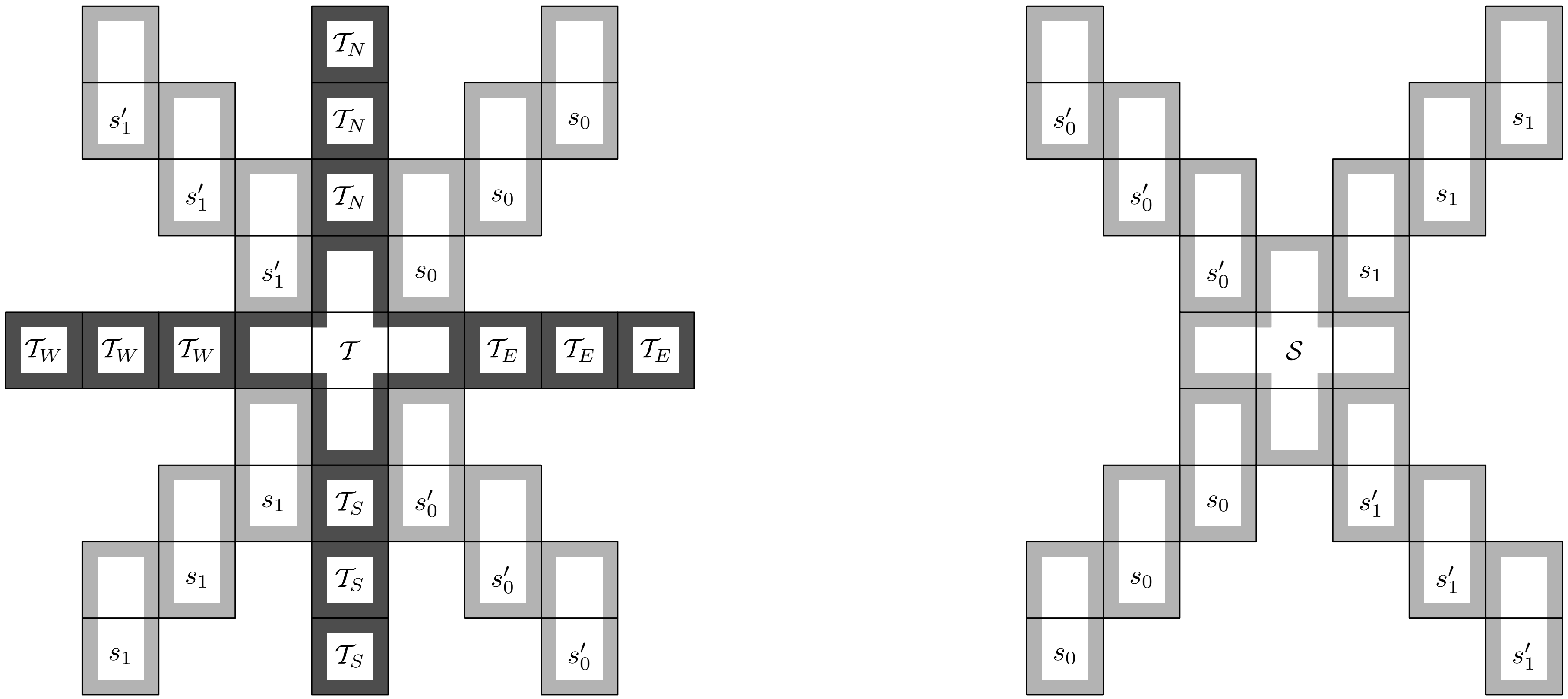


à la B. Martin II

Dealing with constructions

- Think in 2 dimensions
- PaCo ⇔ Wang tileset:
  - PaCo diagrams are tilings (straightforward) ;
  - Tileset can be encoded into PaCo (easy):

Gadgets

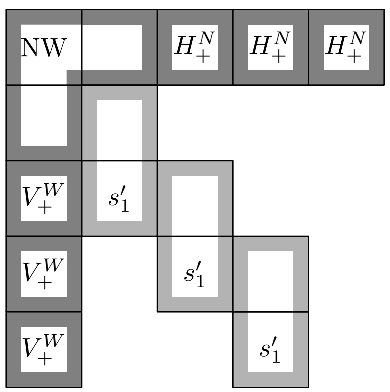


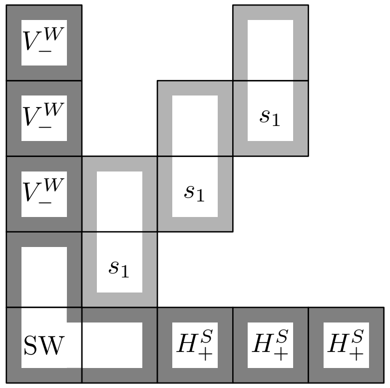
$\mathcal{T}_N$
s_1'
$\mathcal{T}_N$
s_0
s_1'
$\mathcal{T}_N$
s_0
s_1'
s_0
$\mathcal{T}_W$
$\mathcal{T}_W$
$\mathcal{T}_W$
$\mathcal{T}$
$\mathcal{T}_E$
$\mathcal{T}_E$
$\mathcal{T}_E$
s_1
$\mathcal{T}_S$
s_0'
s_1
$\mathcal{T}_S$
s_0'
s_1
$\mathcal{T}_S$
s_0'
s_0'
s_1
s_0'
s_1
s_0'
s_1
$\mathcal{S}$
s_0
s_1'
s_0
s_1'
s_0
s_1'

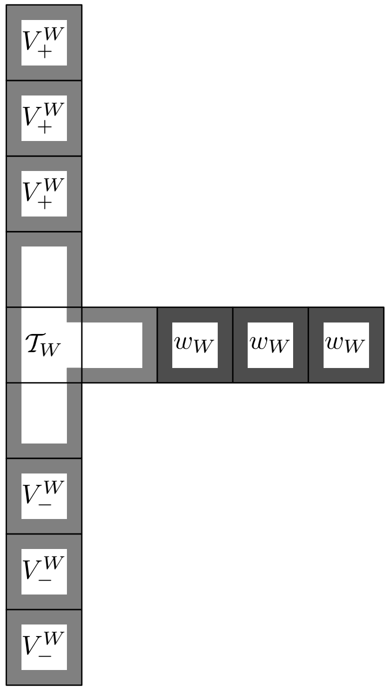
Finite binding is undecidable

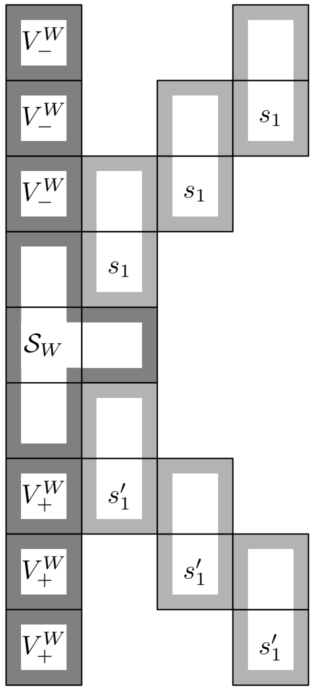
- With tilings come undecidability
- Finite collision binding can be reduced to the problem of finite tiling of the plane.

Gadgets

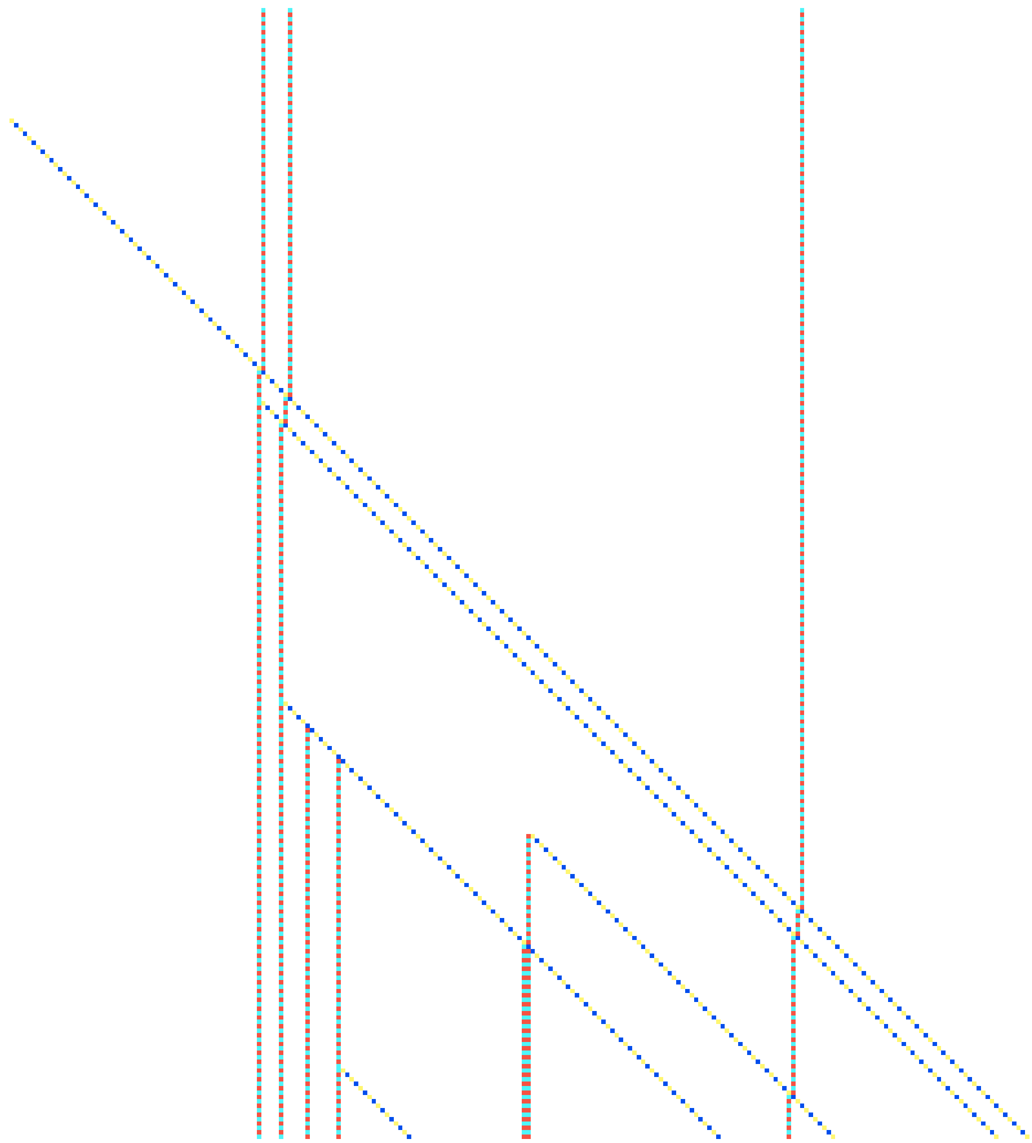

NW
H_+^N
H_+^N
H_+^N
V_+^W
s_1'
V_+^W
s_1'
V_+^W
s_1'

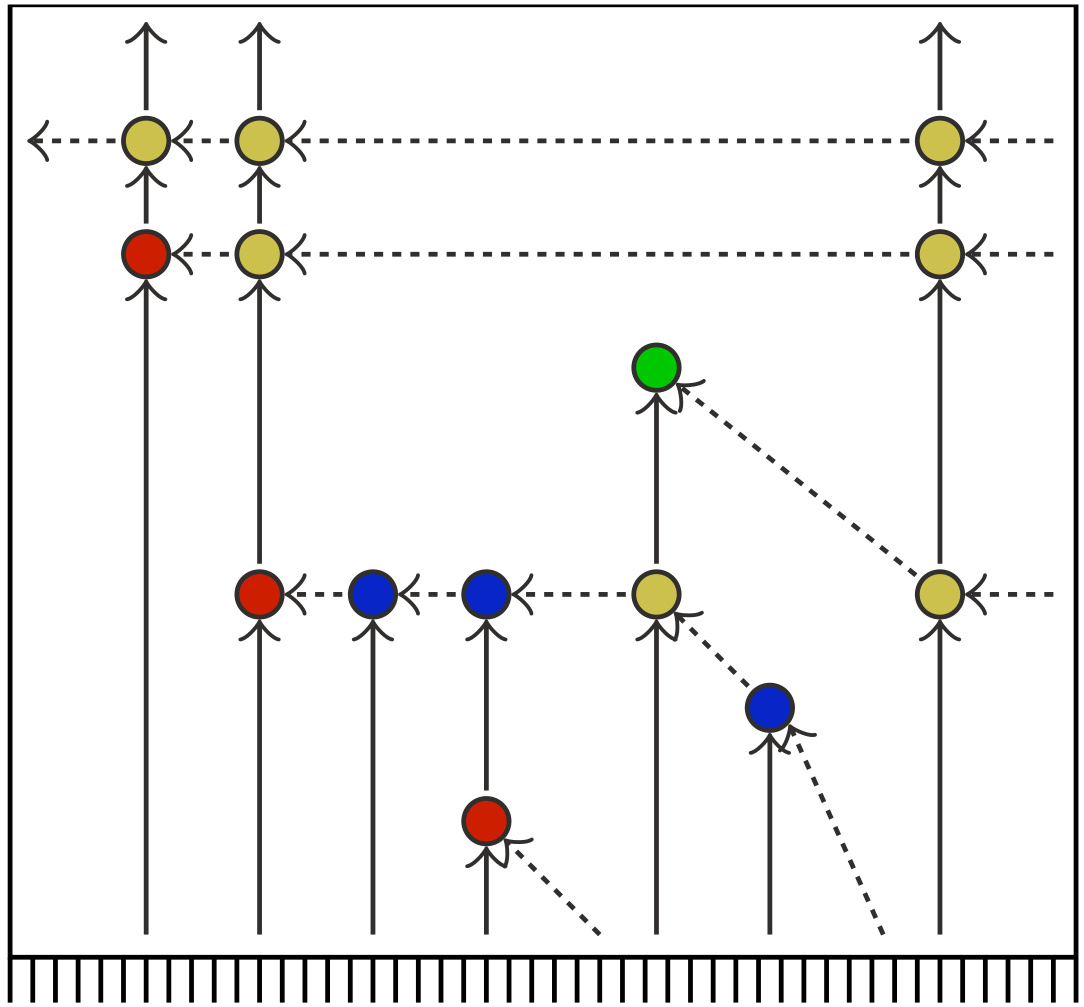

V_-^W
V_-^W
s_1
V_-^W
s_1
s_1
SW
H_+^S
H_+^S
H_+^S


V_+^W
V_+^W
V_+^W
$\mathcal{T}_W$
w_W
w_W
w_W
V_-^W
V_-^W
V_-^W


V_-^W
V_-^W
s_1
V_-^W
s_1
s_1
$\mathcal{S}_W$
V_+^W
s_1'
V_+^W
s_1'
V_+^W
s_1'

Space-time diagrams?



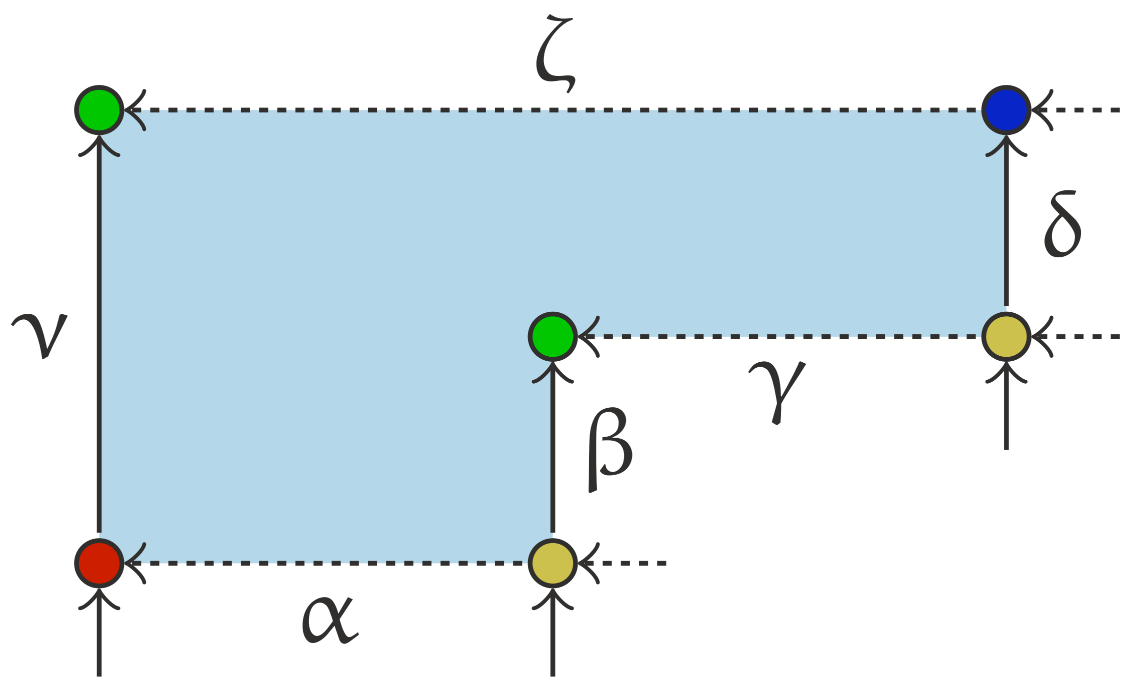


Planar maps

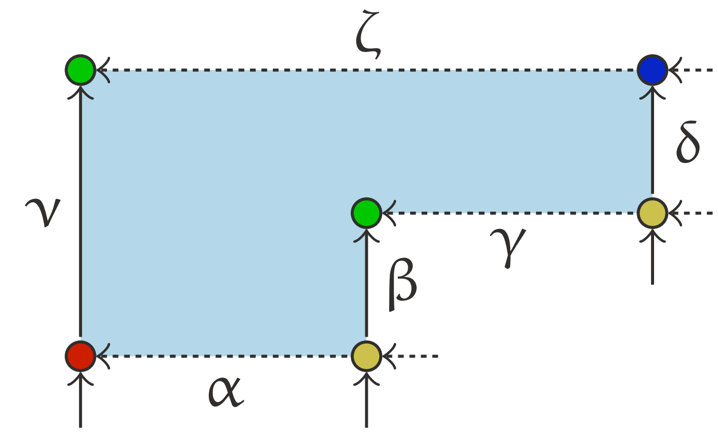
- Networks of collisions...
- ...are graphs
- **Better:** planar graphs
- and planar graphs have **Faces**!

Faces

- Here a **face** corresponds to a polyomino bounded by particles and collisions.



Constraints on repetitions

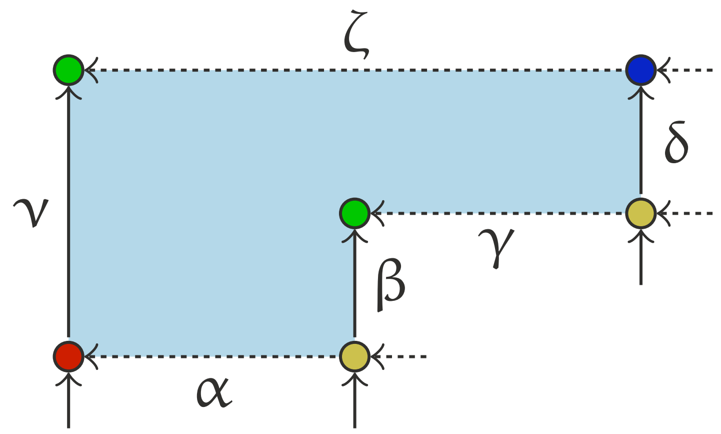


System of linear equations over integers
searching for integer solutions

can be encoded in Presburger arithmetic
solutions are semi-linear sets

[illegible]

We cheated!



We just checked for a closed loop.

We need to care about near particles
We need to care about particles crossing
= We need to check for polyomino!

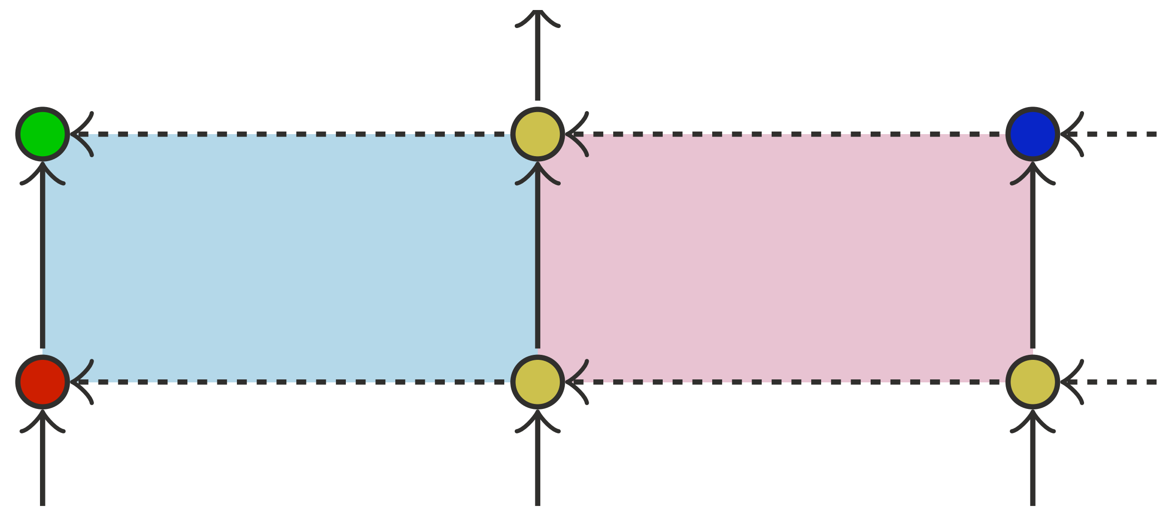
Encode the frontier of the polyomino over ***N, S, E, W***

$$SSESE \cdot (SESE)^{\alpha} \cdot E \cdot (NN)^{\beta} \cdot NESE \cdot (SESE)^{\gamma} \cdots$$

Encode lack of collision using Presburger arithmetic

$$\bigwedge_{m} \bigwedge_{n} \neg \exists i \exists j \, (i \in [1, \alpha_m] \wedge j \in [1, \beta_n] \wedge \varphi_m(i) = \varphi_n(j))$$

Combining faces



To solve combined faces, apply a synchronization product on the Presburger formulas:

$$\varphi(\alpha) \land \psi(\beta) \land \alpha = \beta$$

Solutions are still **semi-linear sets** (which admit finite encoding)

That's all

- Are there other constraints for finite bindings?
- Take care of infinite faces on the border
- And that's it!
- Allowed finite bindings are caracterized.

Some questions

- How to describe the set of all finite bindings?
- How to caracterize the complexity of this set?
- How to construct complex PaCo?

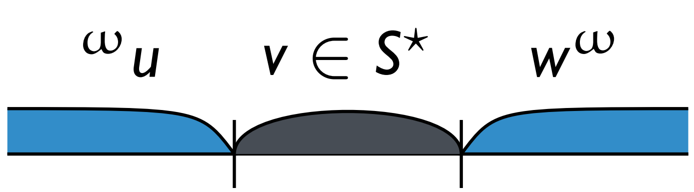
3 more questions

Backgrounds and particles

- Backgrounds are periodic configurations
- Particles are ult periodic configurations
- Collisions are transient phenomena

Ult periodic configurations

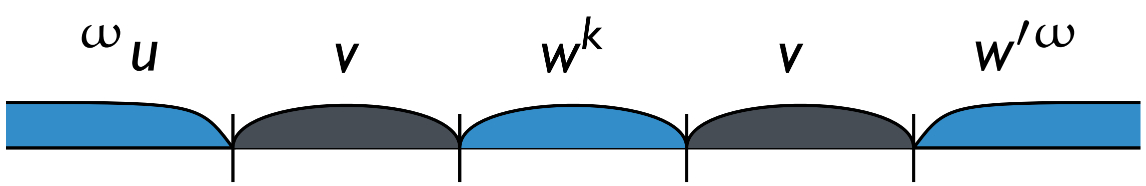
- Finite and periodic configurations are well studied (injectivity, surjectivity, etc)
- What can we say about 1D ult per conf?



- Study growth rate of the nonperiodic part

Combining ult per confs

- Put two ult per confs side by side



- What about the growth rate?
- How to describe the dynamic?
- limit set stable sublanguage?

the end