Heterogeneous Fundamentalists and Imitative Process

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Outline

- Heterogeneous Agent Models
- The Model 1
- The Model 2
- Main Results
- Conclusion and Further Analysis
Heterogeneous Agent Models in Finance

1. Heterogeneous Agents:
   - First Attempt: Zeeman 1974
   - The canonical models (Day and Huang, 1990; Chiarella, 1991)
   - Three different agents:
     - Fundamentalists look at the distance between current price and a fundamental value extract from information about the economic system; [Demand = (F-P_t)]
     - Chartists worked out observed price patterns from the past trying to take advantage of bull and bear market; [Demand=(P_t-P_{t-1})]
     - Noise traders reply to no strictly economic thoughts such as rumours
Heterogeneous Agent Models

2. A market maker model

\[ P_{t+1} = P_t + \beta \left[ q^f + q^c + q^n \right] \]

- with a positive (negative) excess demand market makers dismiss their inventory increasing (decreasing) the price.

3. Interacting Agents
Heterogeneity vs Homogeneity

Kriman (1992):

“...My basic point is to explain that this reduction of the behaviour of a group of heterogeneous agents even if they are all themselves utility maximizers is not simply an analytical convenience as often explained, but is both unjustified and leads to conclusions which are usually misleading and often wrong”
The Model

- As in Brock and Hommes (1998) and in Hommes et al. (2005) we explore a model with two assets: one risky and one risk free.

- There are two experts (gurus) who act as fundamentalists.

- Agents imitate the two experts and can switch from one expert to the other following an adaptive belief system.

- Agents’ switch is driven by experts’ ability, approximated by the distance between fundamental value and price.

- The switch mechanism is based on errors square: the less is the error square the more is the quota of agents that emulate that expert.
Gurus

- As in Follmer et al. (2005) our model “involves agents who may use one of a number of predictor which they might obtain from financial “gurus”
- Agents switch from one guru to the other evaluating gurus’ performance over time
- Finally, we use two different switching mechanisms
  - The first can lead agents to concentrate themselves on one guru, driving out of the market the other
  - The second lead to the coexistence of both gurus
The Model: general equations

\[ q_{i,t} = \alpha(F_i - P_t) \]  
Demand for expert \( i \) \((i=1,2)\)  
(microfound in the paper)

\[ P_{t+1} = P_t + \beta \left[ w_{t+1} q_{1,t+1} + (1 - w_{t+1}) q_{2,t+1} \right] \]  
Market Maker
Switching Process – Model 1

\[ w_{t+1} = 1 - \frac{(F_1 - P_t)^2}{(F_1 - P_t)^2 + (F_2 - P_t)^2} \]

Adaptive Rational Mechanism

- The switching process is based on the heterogeneity in expertise, represented by both the distance between the fundamentals and \( P_t \), between the two experts. Particularly, agents are more likely to imitate the expert whose prediction is closer to \( P_t \).

- Similarly to Kaizoji (2003) and He and Westerhoff (2005) the switching mechanism is based on the accuracy of prediction. However their mechanism is built looking at differences between chartists and fundamentalist.

- Moreover this mechanism is real clear-cut: when the fundamental value is equal to current price, in the next period all agents follow the corresponding expert. This implies that the quota varies from zero to one.
Dynamic Price Equation

\[ P_{t+1} = P_t + \alpha \beta \left[ \frac{(F_1 - P_t)(F_2 - P_t)[F_1 + F_2 - 2P_t]}{(F_1 - P_t)^2 + (F_2 - P_t)^2} \right] \]

- It is a one dimensional nonlinear map. The dynamics is triggered by a cubic one-dimensional map follow-on the switching mechanism.

- The function points out the relation between the excess quantity (demanded or supplied) and the price change where is the speed of adjustment to change quantity demanded and is the proportion of agents that imitate expert 1 and depend on the switching mechanism. According to (4) with a positive (negative) excess demand market makers dismiss their inventory increasing (decreasing) the price.
Steady States

\[ P_{t+1} = P_t = P^* \]

Hence, from the dynamic equation, steady states are points that solve:

\[
\frac{(F_1 - P^*)(F_2 - P^*)}{(F_1 - P^*)^2 + (F_2 - P^*)^2} \left[ F_1 + F_2 - 2P^* \right] = 0
\]

There are three steady states:

- The two fundamental values: \( F_1 \) and \( F_2 \)

- And the arithmetic mean of the fundamental values; hence it is encompassed between the two fundamental values:

\[
P^* = \frac{F_2 + F_1}{2}
\]
Local Stability

- **Given the map (6), the steady states** $F_1$ **and** $F_2$ **are stable if**

  $$\alpha \beta < 2$$

- **On the other hand the steady state is always unstable given that:**

  $$\frac{dP_{t+1}}{dP_t} \bigg|_{P_t = P^*} = 1 + \alpha \beta$$
Flip Bifurcation

- The map exhibits in $F_i$ a period doubling bifurcation when

\[ \alpha \beta = 2 \]

- The behaviour is symmetric: it happens around each of the fundamental values.

- Economically, starting from an excess in demand (for example $P_0 < F_2$, agents’ overreaction leads to a large price increase in such a way that the price becomes higher than $F_2$. An excess in demand is transformed into an excess in supply. Even in this case, given overreaction, agents that follow expert 2 supply a bulky quantity that leads the price down, particularly less than $F_2$. Hence the system fluctuates between excess of demand and excess of supply at round.
A Period-doubling Bifurcation

\[ F_1 = 1, \quad F_2 = 4, \quad \beta = 1 \quad \text{and} \quad \alpha = 2.1 \]
Note 3 - $F_1 = 1$, $F_2 = 4$, $\beta = 1$ and $\alpha = [1.8,3]$
Homoclinic Bifurcation

- Increasing either agents’ or market makers’ reaction coefficients when a period-doubling bifurcation arises, there are two symmetric stable cycles of period two. According to Dieci et al. (2001), flip bifurcation does not affect the global structure of the basins: cycles of period two simply substitute the two steady states. However, further growth of these parameters leads initially to a new attractive period-four cycles, which is followed by a two symmetric chaotic attractors. Finally for an homoclinic bifurcation emerges. The new structure of the basins produced implies the synthesis between the basins of the two fundamental values: bull and bear price fluctuations appear.
**Homoclinic Bifurcation**

Note 5 - \( F_1 = 1, \ F_2 = 4, \ \beta = 3 \) and \( \alpha = 1 \)
Homoclinic Bifurcation
Homoclinic Bifurcation
Switching Process – Model 2

\[ w_{t+1} = \frac{\exp \left[ -\gamma (F_i - P_t)^2 \right]}{\exp \left[ -\gamma (F_1 - P_t)^2 \right] + \exp \left[ -\gamma (F_2 - P_t)^2 \right]} \]

Adaptive Rational Mechanism

- This mechanism is not a clear-cut: when the fundamental value \( F_i \) is equal to current price, in the next period a share of agents still follow the expert j. This implies that the quota varies from zero to one, with extremes not included.
Dynamic Price Equation

\[ P_{t+1} = P_t + \alpha \beta \left\{ (F_2 - P_t) - \frac{\Delta F}{1 + \exp[\gamma \Delta F (2P_t - F_1 - F_2)]} \right\} \]

\[ \Delta F = F_2 - F_1 \geq 0 \]

\( \Delta F \) represents the degree of heterogeneity
Steady States

\[ P_{t+1} = P_t = P^* \]

Hence, from the dynamic equation, steady states are points that solve:

\[
(F_2 - P^*) - \left( \frac{\Delta F}{1 + \exp[\gamma (2P^* - F_1 - F_2)]} \right) = 0
\]

There is at least one steady state:

- the arithmetic mean of the fundamental values:

\[
P^* = P_M = \frac{F_2 + F_1}{2}
\]
Steady States (cont.d)

- The set of steady states belong to the interval \((F_1, F_2)\);

- The map can be rewritten as follows:

  \[- \left( \frac{F_1 - P^*}{F_2 - P^*} \right) = \exp[y\Delta F \left( 2P^* - F_1 - F_2 \right)]\]

- the LHS crosses the x-axis in and has an asymptote for and the RHS is a positive increasing exponential function. Hence, whatever happens RHS crosses the LHS for a value that is less than the asymptote, : there is at least a steady state (Figure 2).
Stability

if there is a unique steady state, \( P_M \), given the degree of heterogeneity and intensity of switching there is a value\(^1\) such as the map is globally stable:

\[
\alpha \beta \in (0, \alpha \beta)
\]

such as the map is globally stable;

Given the following first derivate of map:

\[
\frac{dP_{t+1}}{dP_t} = 1 - \alpha \beta + 2\gamma \alpha \beta \frac{\exp[\gamma \Delta F(2P_t - F_1 - F_2)] \Delta F^2}{[1 + \exp[\gamma \Delta F(2P_t - F_1 - F_2)]]^2}
\]

a low enough degree of heterogeneity and intensity of switching, such as there is a unique steady state, there exists an interval for which the dynamic map is a contraction, and therefore the steady state is globally stable.
Figure 3 A low degree of heterogeneity

Note: $\gamma = 0.8; F_2 = 8; F_1 = 7; \alpha = 1.1; \beta = 1$
Pitchfork Bifurcation

Figure 4: Pitchfork Bifurcation through an increase of the degree of heterogeneity (a) or through an increase in the transfer speed (b).

\[ a) \gamma = 0.8; F_2 = 8; F_1 = 6; \alpha = 1.1; \beta = 1 \]

\[ b) \gamma = 3; F_2 = 8; F_1 = 7; \alpha = 1.1; \beta = 1 \]
Figure 5 Flip Bifurcation

Note $\gamma = 0.8; F_2 = 8; F_1 = 6; \alpha = 1.1; \beta = 3$
Figure 6 Homoclinic Bifurcation

Note $\gamma = 0.8; F_2 = 8; F_1 = 6; \alpha = 1.1; \beta = 4.6$
Main Results

- market instability and periodic, or even, chaotic price fluctuations can be generated even if there is homogeneity in the kind of trading rule;
- there are some conditions under which an expert can drive out of the market the other;
- the two experts can survive, generating complex bull and bear market fluctuations and a global (homoclinic) bifurcation may arise;
- a central role is played by the reaction to misalignment of both market makers and agents.
Further Analysis

- Strongly related with this model:
  - it would be interesting analyse the dynamics in the case in which the imitative process is based on profitability.