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Borrowing to Finance Public Investment? The 'Golden Rule of Public Finance' Reconsidered in an Endogenous Growth Setting*

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- ***la question de la règle d'or des finances publiques***

- Règle d'or : permettre aux Etats de financer les dépenses publiques d'infrastructure par endettement
  - ✓ faire supporter le coût des dépenses publiques aux générations qui en profitent effectivement
  - ✓ éviter le biais anti-investissement des critères du PSC (les dépenses d'investissement sont plus faciles à contracter que les salaires, l'impôt est difficile à augmenter)

- Exemple de l'initiative européenne de croissance
  Dans le cadre du processus de Lisbonne (gagner 1 point de productivité des facteurs) : infrastructures de communications trans-européennes, financement par une taxe CO2. Italie : plutôt projets « asphalte » France-Allemagne, plutôt recherche et télécom.

- La règle est adoptée en Grande Bretagne, en Allemagne et aux Etats-Unis, mais avec 2 interprétations :
  - soit une règle qui limite les déficits et empêche le financement des dépenses improductives (règle restrictive)
  - soit une règle qui permet de dépasser un critère (budget équilibré, limite des 3%) pour les dépenses productives (règle laxiste)

- ***effet sur la croissance dans une modélisation théorique***

- Il faut un modèle avec croissance à long terme (croissance « endogène »)
- Il faut des dépenses publiques d'investissement
- Il faut de la dette publique

- Dans un modèle qui satisfait les canons théoriques usuels

  Modélisation la plus simple possible avec
  i) un critère de bien être,
  ii) une contrainte budgétaire pour l'Etat

1. Présentation du modèle

1.1. Les ménages

- Critère de bien être : $U = \int_0^{\infty} u(c_t)\exp(-\beta t)dt$

$$u(c_t) = \begin{cases} \frac{S}{S-1}\left((c_t)^{\frac{S-1}{S}} - 1\right) & , \quad for \quad S \neq 1 \\ Log(c_t) & , \quad for \quad S = 1 \end{cases}$$

- Fonction de production macroéconomique : $y_t = k_t^{1-\alpha} g_t^{\alpha}$

- Contrainte budgétaire

$$\dot{a} \equiv \dot{k}_t + \dot{b}_t = r_t b_t + (1-\tau) y_t - c_t - \delta^k k_t$$
$$a_t = k_t + b_t$$

- Résolution par la procédure du contrôle optimal

$$H_c = u(c_t) + \lambda_t \left[r_t b_t + (1-\tau) y_t - c_t - \delta^k k_t \right] + \mu_t \left[a_t - k_t - b_t \right]$$

Conditions nécessaires et suffisantes:

- Relations d'Euler :

$$\frac{\partial H_c}{\partial c_t} = 0, \frac{\partial H_c}{\partial k_t} = 0, \frac{\partial H_c}{\partial b_t} = 0$$

$$\dot{\lambda}_t = \beta \lambda_t - \frac{\partial H_c}{\partial a_t}$$

- Standard Transversality Condition (STVC): $\lim_{t \to \infty} \left[\exp(-\beta t) \lambda_t a_t \right] = 0$

NB: STVC équivalente à une condition No Ponzi Game $\gamma < r$ à long terme.

- La solution donne le taux de croissance de la consommation (qui déterminera partiellement le taux de croissance de l'économie). « Règle de Keynes-Ramsey ».

$$\gamma_{ct} = \frac{\dot{c}_t}{c_t} = S\left[(1-\alpha)(1-\tau)\left(\frac{g_t}{k_t}\right)^{\alpha} - \delta^k - \beta \right]$$

1.2. Le gouvernment

- Contrainte budgétaire

$$\dot{b}_t = r_t b_t + \dot{g}_t + \delta^g g_t + g_t^c - \tau y_t$$

- Règle d'or des finances publiques :

$$\dot{b}_t = \theta \dot{g}_t$$
$$\tau y_t = r_t b_t + (1-\theta)\dot{g}_t + g_c + \delta^g g_t$$

$\theta = 0$: balanced budget rule (BBR).

$\theta = 1$: GRPF stricte

Ici, formulation plus générale $0 \le \theta \le 1$ qui permet d'étudier l'effet d'un accroissement de θ

2. Equilibre du modèle

2.1. Système dynamique en $c_k \equiv c/k$, $g_k = g/k$ et $b_k = b/k$:

$$
\begin{cases}
(a) \quad \dfrac{\dot{c}_k}{c_k} = \dfrac{\dot{c}}{c} - \dfrac{\dot{k}}{k} = S\left[(1-\alpha)(1-\tau)g_k^{\alpha} - \delta^k - \beta\right] + (1+\eta)\left(\gamma_g + \delta^g\right)g_k + c_k + \delta^k - g_k^{\alpha} \\
(b) \quad \dfrac{\dot{b}_k}{b_k} = \dfrac{\dot{b}}{b} - \dfrac{\dot{k}}{k} = \dfrac{\theta g_k}{b_k}\gamma_g + (1+\eta)\left(\gamma_g + \delta^g\right)g_k + c_k + \delta^k - g_k^{\alpha} \\
(c) \quad \dfrac{\dot{g}_k}{g_k} = \dfrac{\dot{g}}{g} - \dfrac{\dot{k}}{k} = \gamma_g + (1+\eta)\left(\gamma_g + \delta^g\right)g_k + c_k + \delta^k - g_k^{\alpha} \\
(d) \quad \gamma_g = \dfrac{1}{1+\eta-\theta}\left[\tau g_k^{\alpha-1} - (1+\eta)\delta^g - r\dfrac{b_k}{g_k}\right], \ with\ r = (1-\alpha)(1-\tau)g_k^{\alpha} - \delta^k
\end{cases}
\tag{12}
$$

2.2. Etat stationnaire

Calcul d'une solution stationnaire de croissance endogène :

$\dot{c}_k = \dot{b}_k = \dot{g}_k = 0$ dans le système (12), d'où des valeurs constantes de c_k, b_k et g_k

c, k, et g croissent au même taux constant γ^* $\left(\gamma_c^* = \gamma_k^* = \gamma_g^* = \gamma^*\right)$

2.2.1. Existence :

2 relations :

- taux de croissance de la consommation

$$\gamma^* = S\left[(1-\alpha)(1-\tau)g_k^{*\alpha} - \delta^k - \beta\right] \quad (13)$$

- contrainte budgétaire du gouvernement

$$(1+\eta-\theta)\gamma^* = \tau g_k^{*\alpha-1} - (1+\eta)\delta^g - \theta\left[(1-\alpha)(1-\tau)g_k^{*\alpha} - \delta^k\right] \quad (14)$$

- (13-14) is a two-dimensional system $\left(\gamma^*, g_k^*\right)$, with parameters $\left(S, \alpha, \beta, \eta, \delta^k, \delta^g\right)$ and fiscal exogenous instruments (τ, θ).

- Since γ^* is an increasing function of g_k^* in (13) and a decreasing function of g_k^* in (14), a (single) steady state solution γ^* exists in most cases.

2.2.2 Stabilité :

If $\theta = 0$, the *Jacobian* matrix of system (12) is:

$$J = \begin{bmatrix} c_k^* & -r^* c_k^* & \left\{(1+\eta)\left(\gamma^* + \delta^g\right) + \left[(1-\alpha)\left[\alpha S(1-\tau) - \tau\right] - \alpha\right]\left(g_k^*\right)^{\alpha-1}\right\} c_k^* \\ 0 & -\gamma^* & 0 \\ g_k^* & -r^*\left(\frac{1}{1+\eta} + g_k^*\right) & \left\{(1+\eta)\left(\gamma^* + \delta^g\right) - \left[\alpha + \tau(1-\alpha)\right]\left(g_k^*\right)^{\alpha-1} - \frac{(1-\alpha)\tau}{1+\eta}\left(g_k^*\right)^{\alpha-2}\right\} g_k^* \end{bmatrix}$$

- $Det(J) = \left[\alpha S(1-\alpha)(1-\tau) g_k^* + \frac{(1-\alpha)\tau}{1+\eta}\right]\left(g_k^*\right)^{\alpha-1} c_k^* \gamma^* > 0$. Therefore, the steady-state is associated with either one or three positive eigenvalues.
- But we can immediately see in J that one eigenvalue is equal to $-\gamma^* < 0$, thus only one eigenvalue is positive.
- Since system (12) admits only one jump variable (c_k) and two predetermined variables (the two stock variables b_k and g_k), we conclude that the steady-state is a saddle path.

Si $\theta > 0$, simulations numériques

Table 2: Local stability of the steady state (eigenvalues of J)

	$\theta = 0$	$\theta = 0.1$	$\theta = 0.5$	$\theta = 1$
$S = 0.3$	0.4962 -0.2520 -0.0849	0.3561 -0.2547 -0.1040	0.3072 -0.4710 -0.0912	0.2796 -0.9164 -0.0805
$S = 0.5$	0.4310 -0.2889 -0.1035	0.2880 -0.294 -0.1471	0.2587 -0.4833 -0.1370	0.2362 -0.9073 -0.1270
$S = 1$	0.3679 -0.3400 -0.1303	0.1686 -0.3580 -0.2284	0.1511 -0.5112 -0.2276	0.1311 -0.8943 -0.2266

For: $\beta = 0.01$, $\delta^g = \delta^k = 0.05$, $\eta = 1$, $\alpha = \tau = 0.4$

	$\theta = 0$	$\theta = 0.1$	$\theta = 0.5$	$\theta = 1$
$S = 0.3$	0.2810 -0.0980 -0.0453	0.2708 -0.1066 -0.0441	0.2390 -0.1411 -0.0400	0.2131 -0.1832 -0.0355
$S = 0.5$	0.2450 -0.1178 -0.0544	0.2401 -0.1243 -0.0535	0.2232 -0.1507 -0.0500	0.2070 -0.1846 -0.0459
$S = 1$	0.2137 -0.1475 -0.0677	0.2118 -0.1521 -0.0671	0.2046 -0.1712 -0.0645	0.1966 -0.1965 -0.0613

For: $\beta = 0.1$, $\delta^g = \delta^k = 0$, $\eta = 10$, $\alpha = \tau = 0.5$

	$\theta = 0$	$\theta = 0.1$	$\theta = 0.5$	$\theta = 1$
$S = 0.3$	0.3018 -0.1525 -0.0503	0.2845 -0.1666 -0.0483	0.2360 -0.2219 -0.0416	0.2018 -0.2878 -0.0351
$S = 0.5$	0.2532 -0.1660 -0.0575	0.2454 -0.1755 -0.0562	0.2195 -0.2148 -0.0512	0.1963 -0.2674 -0.0458
$S = 1$	0.2113 -0.1845 -0.0673	0.2084 -0.1904 -0.0667	0.1972 -0.2160 -0.0640	0.1846 -0.2534 -0.0608

For: $\beta = 0.05$, $\delta^g = \delta^k = 0.01$, $\eta = 5$, $\alpha = 0.3$, $\tau = 0.4$

All simulations conduct to positive long-run rates of economic growth, which respect the STVC.

3. Propriétés

3.1 Long terme

- Une augmentation de la part de l'investissement public financée par endettement détériore la croissance à long terme

Par la différentielle totale du système (13)-(14) on obtient :

$$\frac{d\gamma^*}{d\theta} = \frac{-\alpha(1-\tau)S(r^* - \gamma^*)}{\tau g_k^{*-1} + \theta\alpha(1-\tau) + \alpha S(1-\tau)(1+\eta-\theta)} < 0$$

because the STVC requires: $r^* > \gamma^*$.

- Interprétation
  - *En régime permanent, le flux permanent de nouvelles ressources procurées par l'endettement (le déficit public* $\dot{b}$*) ne peut jamais dépasser le flux permanent de nouvelles dépenses engendré par la charge de la dette* $(rb > \dot{b})$
  - *Les déficits publics ne sont donc pas une ressource mais un coût pour les finances publiques à long terme.*
  - *Pour financer le coût supplémentaire des déficits à long terme, il faudra soit augmenter les impôts (exclu par hypothèse) soit diminuer les dépenses de consommation (exclu par hypothèse) soit réduire les dépenses d'investissement, avec un effet néfaste sur la croissance.*

3.2. Dynamique transitoire

To study transitional dynamics, we linearize system (12) in the neighborhood of the steady-state $\left(c_k^*, b_k^*, g_k^*\right)$:

$$
\begin{aligned}
c_k - c_k^* &= a_1 \exp\left(\lambda_1 t\right) v_{11} + a_2 \exp\left(\lambda_2 t\right) v_{21} + a_3 \exp\left(\lambda_3 t\right) v_{31} \\
b_k - b_k^* &= a_1 \exp\left(\lambda_1 t\right) v_{12} + a_2 \exp\left(\lambda_2 t\right) v_{22} + a_3 \exp\left(\lambda_3 t\right) v_{32} \\
g_k - g_k^* &= a_1 \exp\left(\lambda_1 t\right) v_{13} + a_2 \exp\left(\lambda_2 t\right) v_{23} + a_3 \exp\left(\lambda_3 t\right) v_{33}
\end{aligned}
$$

v_{ij} are the coordinates of the eigenvectors associated to each eigenvalue and a_i are constants to be computed. Defining $\lambda_3 > 0$ as the unstable eigenvalue, we must choose $a_3 = 0$. Further, we can normalize eigenvectors' first coordinate: $v_{11} = v_{21} = v_{31} = 1$. Let us write the *Jacobian* matrix of system (12) in the neighborhood of the high steady-state, as:

$$
J^H = \begin{bmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ g_1 & g_2 & g_3 \end{bmatrix}
$$

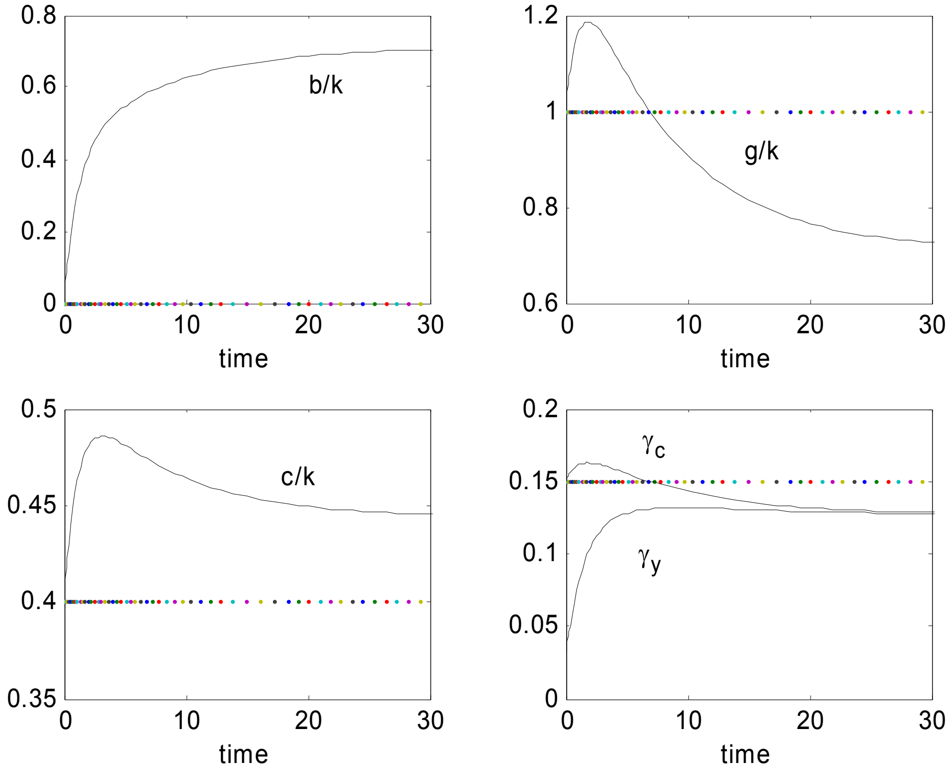
Knowing that eigenvectors' coordinates are solutions of: $J^H V_i = \lambda_i V_i$, for $V_i = \begin{pmatrix} 1 \\ v_{i2} \\ v_{i3} \end{pmatrix}$, we can easily compute, for $i = 1,2$: $v_{i2} = \dfrac{-b_1 c_3 - b_3\left(\lambda_i - c_1\right)}{c_3\left(b_2 - \lambda_i\right) - b_3 c_2}$, $v_{i3} = \dfrac{b_1 c_2 + \left(b_2 - \lambda_i\right)\left(\lambda_i - c_1\right)}{c_3\left(b_2 - \lambda_i\right) - b_3 c_2}$; and the constants a_i are: $a_1 = \dfrac{v_{23}\left(b_{k0} - b_k^*\right) - v_{22}\left(g_{k0} - g_k^*\right)}{v_{23} v_{12} - v_{13} v_{22}}$, $a_2 = \dfrac{-v_{13}\left(b_{k0} - b_k^*\right) + v_{12}\left(g_{k0} - g_k^*\right)}{v_{23} v_{12} - v_{13} v_{22}}$

which defines $c_{k0} = c_k^H + a_1 + a_2$ for predetermined b_{k0} and g_{k0}.

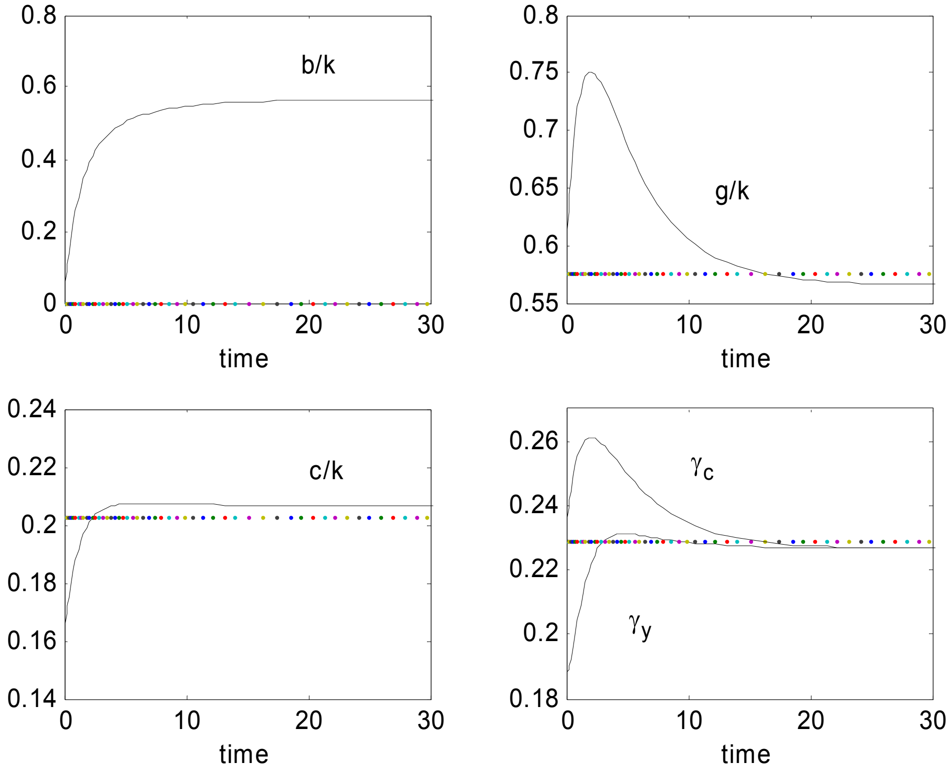
3.2.1. Effet du deficit public sur la croissance à court et moyen termes

Fig.1: Dynamic adjustment of variables following a jump in the deficit rule from $\theta = 0$ *to* $\theta = 100\%$ *(for:* $\beta = 0.01$*,* $\delta^g = \delta^k = 0.05$*,* $\eta = 1$*,* $\alpha = \tau = 0.4$*)*

$S = 0.5$



$S = 1$



3.2.2 Effet de la Règle d'or sur le bien-être intertemporel

On tente d'évaluer l'effet de θ sur $U = \int_0^{\infty} u(c_t) \exp(-\beta t) dt$

Remarques :

i) il y a un arbitrage intertemporel : le taux de croissance de la consommation augmente à CT et diminue à LT (même problème en niveau)

ii) pour cette raison, une évaluation directe du signe de $dU/d\theta$ *est non conclusive*

Procédure:

1/ First, we compute the present value of intertemporal welfare for an economy which stays, from $t=0$ to $t \to +\infty$, on the initial steady-state, namely with the BBR $\theta=0$. In such a steady-state, consumption grows at the constant rate γ^{*0}:

$$c_t^0 = k_0 c_{k0}^0 \exp\left(\gamma^{*0} t\right)$$

We define this consumption path as: $\left\{c_t^0\right\}_0^\infty$, and the present value of welfare associated to this path is:

$$U^0 \equiv \int_0^\infty u\left(c_t^0\right)\exp\left(-\beta t\right)dt$$

2/ Second, we compute the present value of intertemporal welfare for an economy which, starting from the same initial position for predetermined variables: b_0, g_0 and k_0, adopts, from $t=0$ to $t \to +\infty$, a deficit rule $\theta > 0$.

We define the consumption path $\left\{c_t^\theta\right\}_0^\infty$ associated with a θ deficit rule as:

$$c_t^\theta = k_0 c_{k0}^\theta \exp\left(\int_0^t \gamma_{cs}^\theta ds\right)$$

and the present value of welfare associated to this path is:

$$U^\theta = \int_0^\infty u\left(c_t^\theta\right)\exp\left(-\beta t\right)dt$$

3/ Let Δ_t^θ be the present value of the net gain of welfare between the two paths $\left\{c_t^\theta\right\}_0^\infty$ and $\left\{c_t^0\right\}_0^\infty$ at date t, namely:

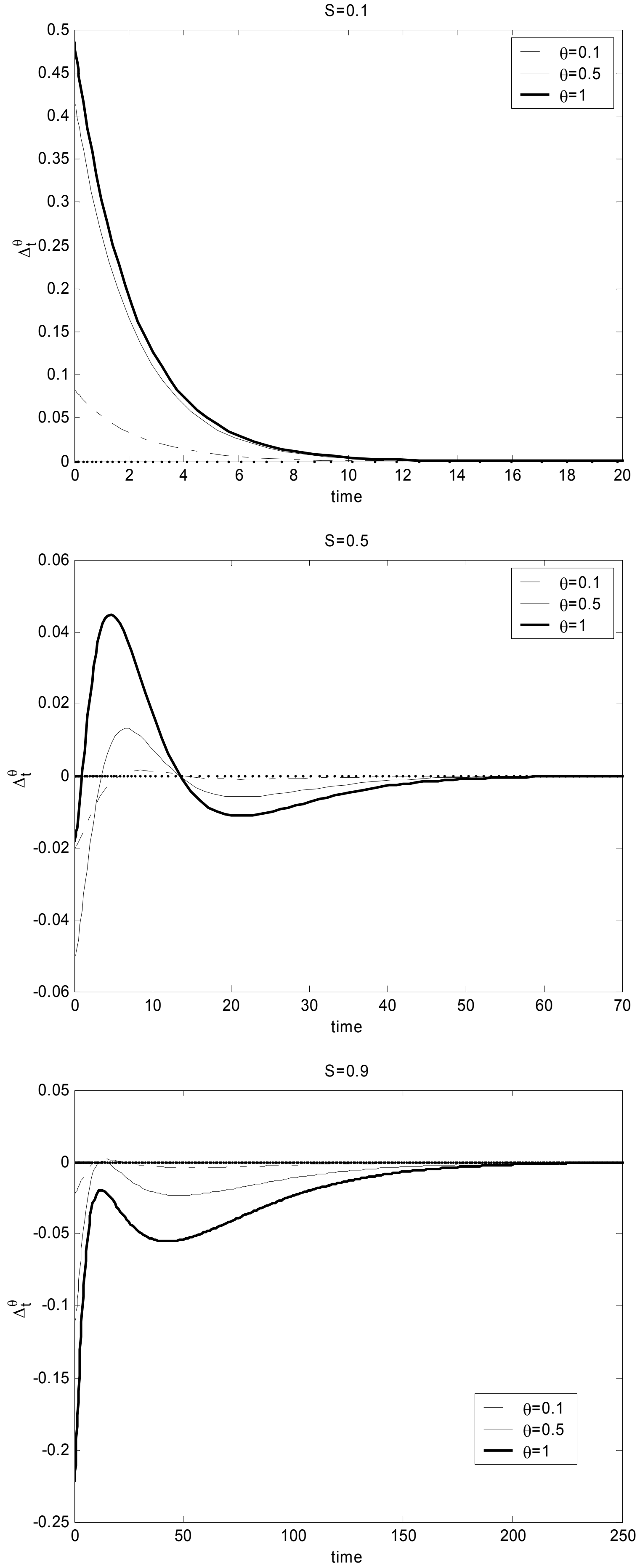
$$\Delta_t^\theta = \left(u\left(c_t^\theta\right) - u\left(c_t^0\right)\right)\exp\left(-\beta t\right)$$

so that the total discounted net gain of welfare associated to a θ deficit rule can be easily defined as:

$$\Delta U\left(\theta\right) \equiv U^\theta - U^0 = \int_0^\infty \Delta_t^\theta dt$$

Figure 2: Intertemporal net gain of welfare associated with different debt rules θ

(for: $\beta = 0.01$, $\eta = 1$, $\delta^g = \delta^k = 0.05$, $\alpha = \tau = 0.4$, $k_0 = 1$*)*



We report in *Table 1* the net gain of welfare (the integral of Δ_t^θ) associated with different debt rules for different values of the consumption elasticity of substitution:

Table 1: Net gain of welfare (in %) with a "golden rule" of $\theta > 0$ compared to the balanced-budget rule $(\theta = 0)$

	$\theta = 0$	$\theta = 10\%$		$\theta = 50\%$		$\theta = 100\%$	
	U^0	U^θ	$\Delta U(\theta)/U^0$	U^θ	$\Delta U(\theta)/U^0$	U^θ	$\Delta U(\theta)/U^0$
$S = 0.1$	10.03	10.22	+1.84%	10.94	+9.01%	11.08	+10.36%
$S = 0.2$	21.02	21.11	+0.43%	22.23	+5.96%	23.38	+11.17%
$S = 0.3$	36.05	36.06	+0.02%	36.71	+1.79%	37.77	+4.73%
$S = 0.4$	56.33	56.29	-0.06%	56.53	+0.34%	57.18	+1.49%
$S = 0.5$	84.34	84.28	-0.07%	84.23	-0.15%	84.45	+0.10%
$S = 0.6$	125.14	125.04	-0.07%	124.75	-0.32%	124.53	-0.50%
$S = 0.7$	189.73	189.59	-0.07%	189.02	-0.38%	188.29	-0.77%
$S = 0.8$	307.24	307.03	-0.07%	306.02	-0.40%	304.58	-0.88%
$S = 0.9$	587.43	587.04	-0.07%	585.07	-0.40%	581.91	-0.94%
$S = 1$	2076	2075	-0.07%	2068	-0.40%	2056	-0.97%

For: $\beta = 0.01$, $\eta = 1$, $\delta^g = \delta^k = 0.05$, $\alpha = \tau = 0.4$

- In accordance to *Figure 2*, "golden" rules dominate balanced-budget rule if S is low, and inversely if S is high. For medium values of the intertemporal consumption elasticity, only relatively high golden rules (high values of θ) dominate the balanced-budget rule.
- The welfare effect of different golden rules is quite small compared to the balanced-budget rule. Nevertheless, this effect is non-negligible for small values of the consumption elasticity of substitution, since for high values of θ the net gain of welfare exceeds 10%.
- The crucial role of the intertemporal elasticity of substitution comes from the initial adjustment of consumption at $t=0$. If S is low, consumption jumps up initially, generating an upward jump in welfare (see *Figure 2* for $S=0.1$). If S is high ($S=0.9$ in *Figure 2*, for example) on the contrary, consumption jumps downs initially, with an adverse effect on welfare.
- Unfortunately, this question cannot be solved by empirical evidences, since most of empirical investigations locate S values near or smaller than one, which, in our model, is consistent with either an improvement or a weakening in intertemporal welfare.

4. Conclusion

Fiscal rules that allow borrowing for public investment, such as the GRPF, are not a good prescription from the long-run growth perspective, but can improve economic growth in the short-run.

Since short-run and long-run growth effects are conflicting, the effect of deficit rules on household's intertemporal welfare is uncertain, depending on parameters and in particular on the consumption elasticity of substitution, about which we have little empirical evidence.