Colloque d'Analyse Harmonique d'Orsay
9–12 July 2008

Some aspects of Noël Lohoué's work

Jean–Philippe Anker

9 July 2008

CV

Personal data

- Birth : 25 December 1939 in Bayangam (Koung–Khi Department, West Province, Cameroun)
- Ethnicity : Bamiléké
- Family : Wife : Julienne
  Son : Jean–Louis (1971)
  Daughter : Renée (1975)

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  - Complex analysis

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Thesis (Doctorat d'Etat, Orsay, 1972) answers several problems in

P. Eymard, *Algèbres A_p et convoluteurs de L^p*, Séminaire Bourbaki 12 (1969-1970), exposé 367, 55–72

Later works use and have popularized the

Herz majoration principles

Let G locally compact group, H closed subgroup
(with left–invariant Haar measures)
λ_G^p, $\lambda_{G/H}^p$ left regular representations on $L^p(G)$, $L^p(G/H)$
μ bounded measure on G

Then $\|\lambda_G^p(\mu)\| \leq \|\lambda_{G/H}^p(|\mu|)\|$
$\|\lambda_{G/H}^p(\mu)\| \leq \|\lambda_G^p(\mu)\|$ if H amenable

Example : $G = KAN$ semisimple, $H = MAN$ minimal parabolic
$\Rightarrow \|\lambda_G^p(\mu)\| = \|\lambda_{G/H}^p(\mu)\| \ \forall\, \mu \geq 0$

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Young : $L^p(\mathbb{R}^n) * L^q(\mathbb{R}^n) \subset L^r(\mathbb{R}^n) \Leftrightarrow \frac{1}{p}+\frac{1}{q}-\frac{1}{r}=1$
Rajagopalan : G compact $\Leftrightarrow$ $L^p(G)$ is a convolution algebra
(for some hence for every) $1<p<+\infty$

Kunze–Stein phenomenon

$L^2(G) * L^2(G) \subset L^{2+\varepsilon}(G)$
Equivalent : $L^{p-\varepsilon}(G) * L^p(G) \subset L^p(G) \ \ \forall\, 1<p\leq 2$

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- Kunze & Stein (1960) : $G=\mathrm{SL}(2,\mathbb{R})$
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Questions going back to Herz (1970), Eymard & Lohoué (1975) :

- real–variable method ?
- endpoint result ?

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Convolution on semisimple Lie groups

Assume that G has real rank 1

Lohoué & Rychener (1984)

$L^{p,1}(G) * L^{p,1}(G/K) \subset L^{p,1}(G/K) \;\; \forall\, 1<p<2$
$|AB| \geq C\,|A|\,|B| \;\; \forall\, A \subset G,\, B \subset G/K$

Cowling, Meda & Setti (1997)

$L^{p,a}(G) * L^{p,b}(G) \subset L^{p,c}(G)$ if $1<p<2$ and $\frac{1}{a}+\frac{1}{b}-\frac{1}{c} \geq 1$.
In particular $L^{p,1}(G)$ is a convolution algebra.

Ionescu (2000)

$L^{2,1}(G) * L^{2,1}(G) \subset L^{2,\infty}(G)$

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$L^{2,1}(G) * L^{2,1}(G) \subset L^{2,\infty}(G)$

Convolution on semisimple Lie groups

Assume that G has real rank 1

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Lohoué (1980)

For every $1 < p < +\infty$, there exists a positive measure μ on G such that μ convolves $L^p(G)$ but not other $L^q(G)$.

Heat kernel

- **Hyperbolic spaces**

Systematic use of the inverse Abel transform

$$\begin{array}{ccc} & \mathscr{S}(\mathfrak{a})^W & \\ \mathscr{H} \nearrow & & \nwarrow \mathscr{F} \\ \mathscr{S}(K\backslash G/K) & \xrightarrow[\mathscr{A}]{} & \mathscr{S}(\mathfrak{a})^W \end{array}$$

Lohoué & Rychener (1982)

$\mathbb{H}^n(\mathbb{R})$: $p_t(r) = 2^{-\frac{n+1}{2}}\,\pi^{-\frac{n}{2}}\,t^{-\frac{1}{2}}\,e^{-(\frac{n-1}{2})^2 t}\,\left(-\frac{1}{\sinh r}\frac{\partial}{\partial r}\right)^{\frac{n-1}{2}} e^{-\frac{r^2}{4\,t}}$

where

$\left(-\frac{1}{\sinh r}\frac{\partial}{\partial r}\right)^{\frac{n-1}{2}} f(r) = \frac{1}{\sqrt{\pi}}\int_r^{+\infty} ds\,\frac{\sinh s}{\sqrt{\cosh s - \cosh r}}\left(-\frac{1}{\sinh s}\frac{\partial}{\partial s}\right)^{\frac{n}{2}} f(s)$

if n is even.

Similar expressions for $\mathbb{H}^n(\mathbb{C})$, $\mathbb{H}^n(\mathbb{H})$, $\mathbb{H}^2(\mathbb{O})$.

Similar expressions for Damek–Ricci spaces

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$$p_t(r) \asymp t^{-\frac{1}{2}}\, e^{-(\frac{n-1}{2})^2 t}\, e^{-\frac{n-1}{2} r}\, e^{-\frac{r^2}{4\,t}} \times \begin{cases} \frac{1+r}{t} & \text{if } t \geq 1+r \\ (\frac{1+r}{t})^{\frac{n-3}{2}} & \text{if } t \leq 1+r \end{cases}$$

- G/K noncompact symmetric space

A & Ji (1998), A & Ostellari (2003)

Same global upper and lower bound. In particular, if $t \geq 1+|x|$, then $p_t(x) \asymp t^{-\frac{d}{2}-|\Sigma_0^+|}\, e^{-|\rho|^2 t}\, \phi_0(x)\, e^{-\frac{|x|^2}{4\,t}}$.

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Same estimate when $t \geq 1 + |x|$

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L any invariant subLaplacian

Alexopoulos & Lohoué (2003)

Upper estimate for $t \geq 1$:

$$p_t(x) \leq C\, t^{-\frac{d}{2} - |\Sigma_0^+|}\, e^{-\sigma t}\, \phi_0(x)\, e^{-c\frac{|x|^2}{t}}$$

Lower estimate for $t \geq 1$ and $|x| = \mathrm{O}(\sqrt{t})$:

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Differential forms

$[\Gamma\backslash]\, G/K$ [locally] symmetric space

Δ_ℓ Hodge–de Rham Laplacian on ℓ–forms

$\mathrm{Tr}_\Gamma(P_t^{\ell,\perp}) = |\Gamma\backslash G/K|\ \mathrm{tr}\, p_t^{\ell,\perp}(e)$ Γ–trace of heat semi–group

$a_\ell = \sup\left\{0 \leq b \leq \infty \mid \mathrm{Tr}_\Gamma(P_t^{\ell,\perp}) = \mathsf{O}(t^{-\frac{b}{2}})\right\}$

Novikov–Shubin (topological) invariants

Lohoué–Mehdi (2000), Olbrich (2002)

$$a_\ell = \begin{cases} \mathrm{rank}\, G - \mathrm{rank}\, K & \text{if } \left|\ell - \frac{\dim G/K}{2}\right| \leq \frac{\mathrm{rank}\, G - \mathrm{rank}\, K}{2} \\ \infty & \text{otherwise} \end{cases}$$

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$a_\ell = \sup\left\{0 \le b \le \infty \mid \mathrm{Tr}_\Gamma(P_t^{\ell,\perp}) = \mathsf{O}(t^{-\frac{b}{2}})\right\}$

Novikov–Shubin (topological) invariants

Lohoué–Mehdi (2000), Olbrich (2002)

$$a_\ell = \begin{cases} \mathrm{rank}\, G - \mathrm{rank}\, K & \text{if } \left|\ell - \frac{\dim G/K}{2}\right| \le \frac{\mathrm{rank}\, G - \mathrm{rank}\, K}{2} \\ \infty & \text{otherwise} \end{cases}$$

Differential forms

$[\Gamma\backslash]\, G/K$ [locally] symmetric space

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$\forall\ 0<\varepsilon<1\,,\ \exists\ c_\varepsilon>0$ and $C_\varepsilon>0$ such that

$$\|p_t^\ell(x)\| \leq C_\varepsilon\, t^{-\varepsilon\frac{D}{2}}\, e^{-\sigma_\ell t}\, \phi_0(x)\, e^{-\frac{1-\varepsilon}{(1+2\varepsilon)^2}\frac{|x|^2}{4\,t}}$$

$\forall\ t\geq 1$ and $\forall\ |x|\geq c_\epsilon$.

Consequences :

- Green function estimate (see also Carron)
- Poisson equation on forms
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