Olivier Faugeras

Inter Spike Intervals probability distribution and Double Integral Processes

The problem of finding the probability distribution of the first hitting time of a Double Integral Process (DIP) such as the Integrated Wiener Proces (IWP) has been an important and difficult endeavor in stochastic calculus. It has applications in many fields of physics (first exit time of a particle in a noisy force field) or in biology and neuroscience (spike time distribution of an integrate-and-fire neuron with exponentially decaying synaptic current). The only results available are an approximation of the stationary mean crossing time and the distribution of the first hitting time of the IWP to a constant boundary. We generalize these results and find an analytical formula for the first hitting time of the IWP to a continuous piecewise cubic boundary. We use this formula to approximate the law of the first hitting time of a general DIP to a smooth curved boundary, and we provide an estimation of the convergence of this method. The accuracy of the approximation is computed in the general case for the IWP and the effective calculation of the crossing probability can be carried out through a Monte-Carlo method. We discuss the application of our method to the determination of the ISI pdf of popular neuron models such as IF and GIF.