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Modelling neuronal membranes with Piecewise Deterministic Processes

We develop a model of an excitable membrane by Piecewise Deterministic Processes (PDPs), which are a class of hybrid Markov processes that are able to capture the dynamics of deterministic continuous motion influenced by discrete random jump processes depending on the deterministic motion themselves. We will ague that this model is close to the biophysical reality, yet still analytically tractable and also relevant in practice. PDPs naturally arise as the model for a space-clamped patch of excitable membrane where the single voltage gated ion channels are modelled by Markov kinetic schemes and the time evolution of the transmembrane voltage potential is governed by an ordinary differential equation. To model non-space clamped membranes we have extended the PDP theory to allow for spatial dynamics. An advantage of the PDP framework is that it already provides a rich theory that can be exploited for further analysis. In particular, via the Kolmogorov backward equation we can derive analytically a fully continuous approximation by diffusion processes to the PDP model by systems of stochastic differential equations (SDEs). As stochastic differential equations are analytically even more tractable this description of the membrane provides further analytical advantages as well as, more practical, algorithms based on SDE models will allow for much faster simulations than algorithms based on the complete Markov kinetics, especially for ever larger systems.