Signal recognition by finite automata

– work in progress –

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Outline

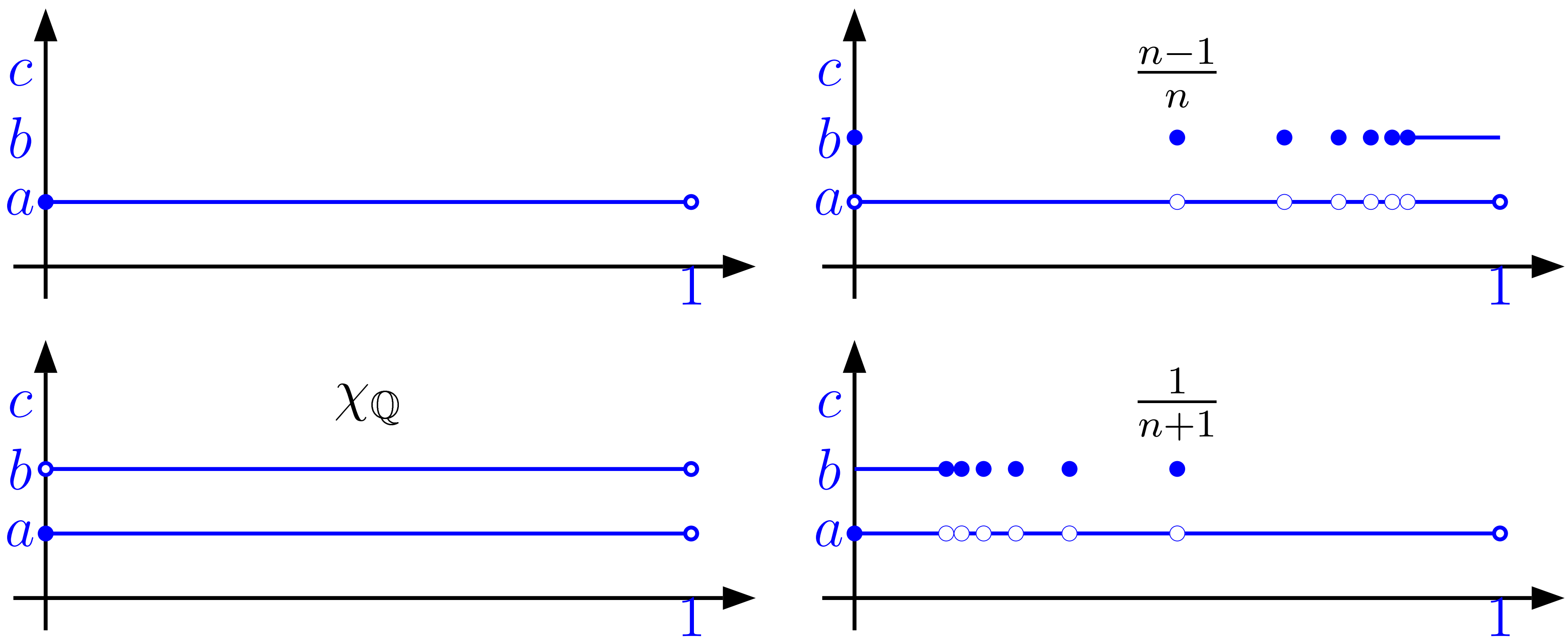
1. Pre-signal and approximation
2. Automata and signals
3. Exercises
4. Cardinality of the set of signals
5. Open questions

Pre-signals

Σ : finite alphabet

Pre-signal $f : [0, 1[\longrightarrow \Sigma$

Represented by its graph



ε-approximation

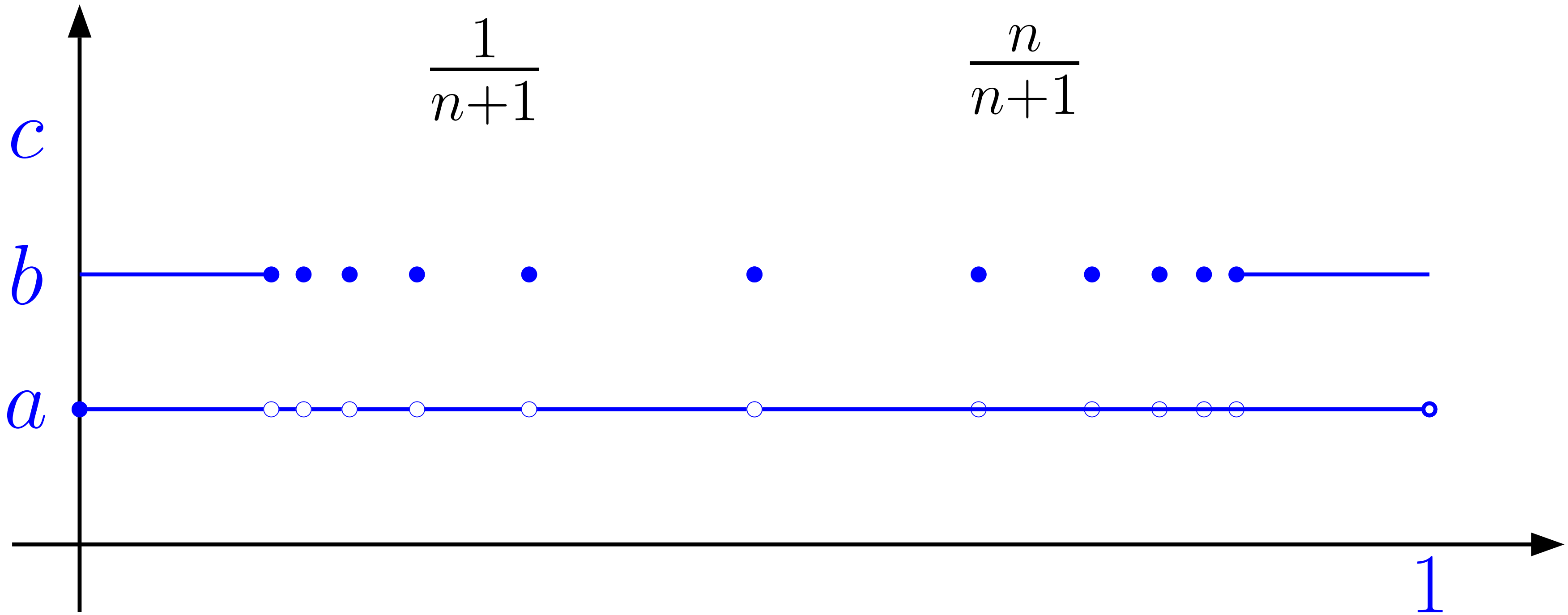
$$\widetilde{\Sigma} = \Sigma \times \left(\mathcal{P}\left(\Sigma\right) \setminus \{\emptyset\}\right)$$

$$w_1 \ldots w_n \in \widetilde{\Sigma} \qquad f : [0, 1[\longrightarrow \Sigma$$

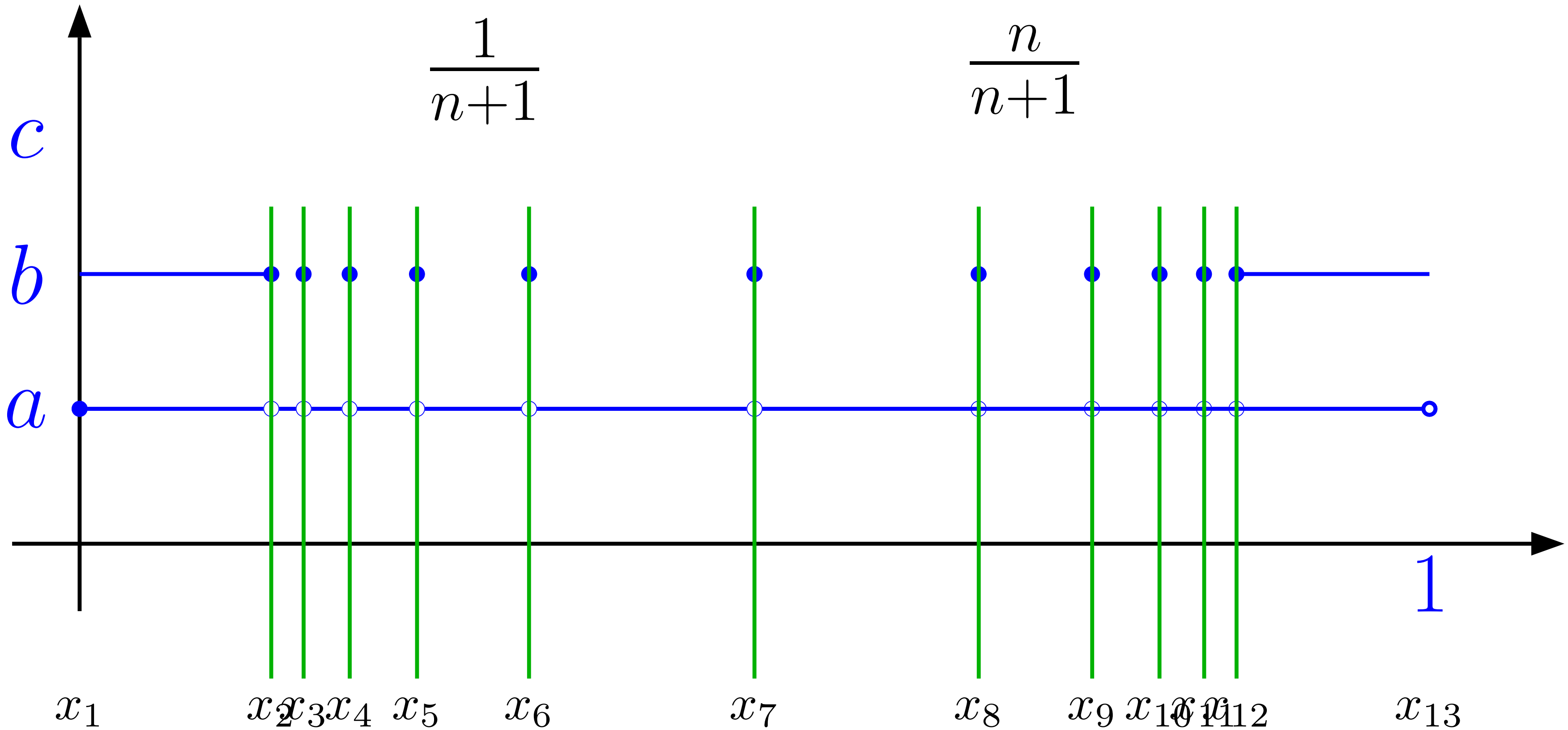
$w_1 \ldots w_n$ *ε-approximates* f

$$\stackrel{\text{def}}{\Longleftrightarrow} \quad \exists x_1, x_2, \ldots, x_{n+1} \begin{cases} x_1 = 0 \\ x_i < x_{i+1} \\ x_{n+1} = 1 \\ w_i = \Big(f(x_i), f\Big(]x_i, x_{i+1}[\Big)\Big) \\ \Big|f\Big(]x_i, x_{i+1}[\Big)\Big| > 1 \Rightarrow |x_i - x_{i+1}| < \varepsilon \end{cases}$$

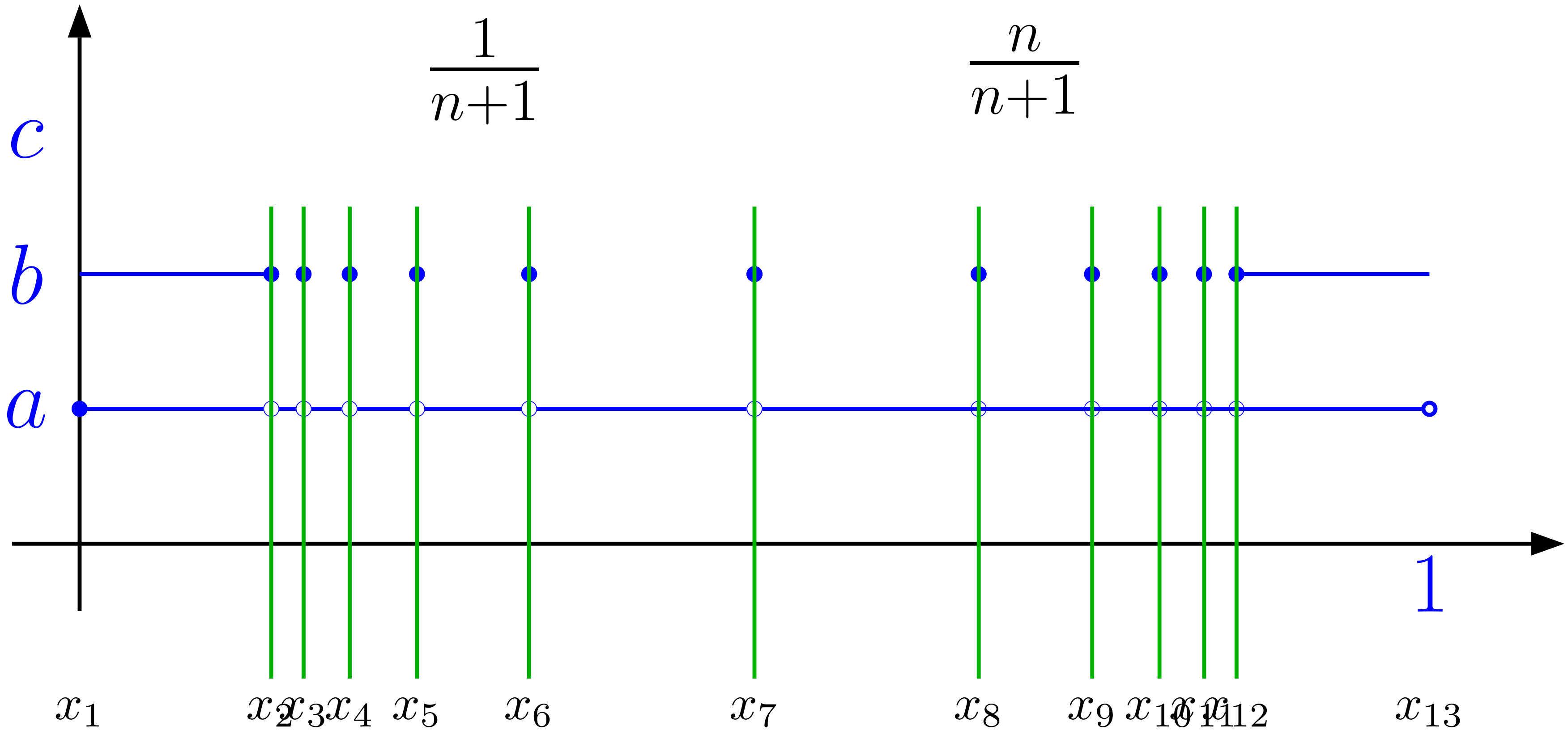
ε-approximation



ε-approximation



ε-approximation



$|x_1 - x_2| < \varepsilon$ and $|x_{12} - x_{13}| < \varepsilon$

$$w = \Big(a, \{a, b\}\Big)\Big(b, \{a\}\Big)\Big(b, \{a\}\Big)\ldots\Big(b, \{a\}\Big)\Big(b, \{a, b\}\Big)$$

Finite automaton for pre-signals

$$\mathcal{A} = \left(\widetilde{\Sigma}, Q, \delta, I, F\right)$$

$\mathcal{L}(\mathcal{A})$ language (on $\widetilde{\Sigma}$) recognized

f in $\Sigma^{[0,1[}$

$\mathcal{A}$ *signal-recognizes* f

$\overset{\text{def}}{\iff}$

$f \in \mathcal{S}(\mathcal{A})$

$\overset{\text{def}}{\iff}$

$\forall \varepsilon > 0, \exists w \in \mathcal{L}(\mathcal{A})$, w ε-approximates f

Equivalence and signals

f, g in $\Sigma^{[0,1[}$

$$f \approx g$$

$$\overset{\text{def}}{\Longleftrightarrow}$$

$$\forall \mathcal{A},\ f \in \mathcal{S}(\mathcal{A}) \Leftrightarrow g \in \mathcal{S}(\mathcal{A})$$

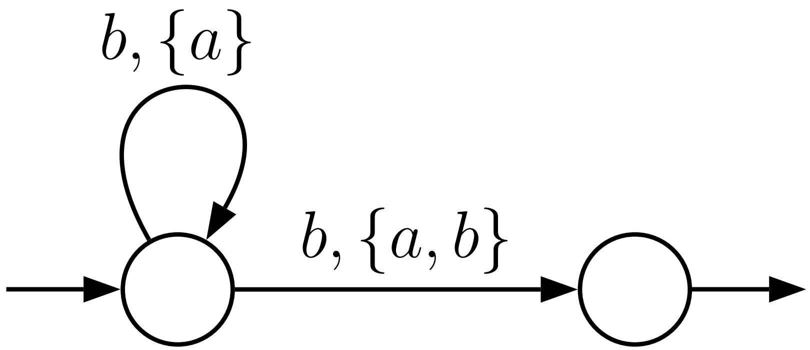
$[f]$ is a *signal*

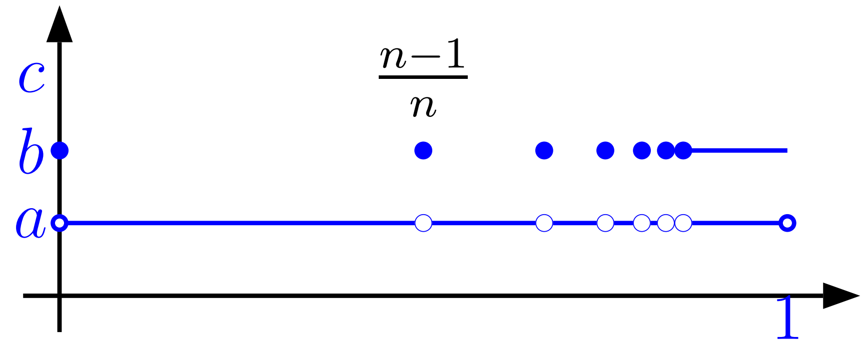
$\overset{\text{def}}{\Longleftrightarrow}$

$[f]$ is an equivalence class for $\approx$

Open question Characterize these classes (links to *scattered linear orders*)

Example of an acceptance

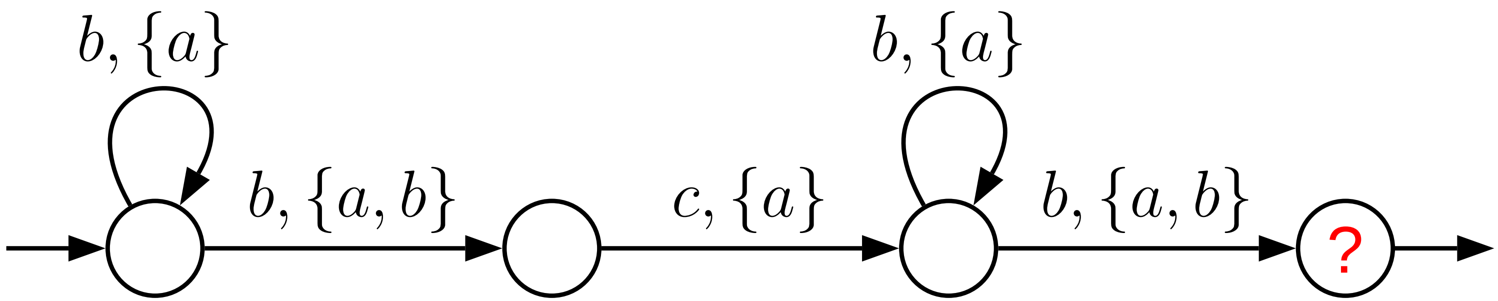


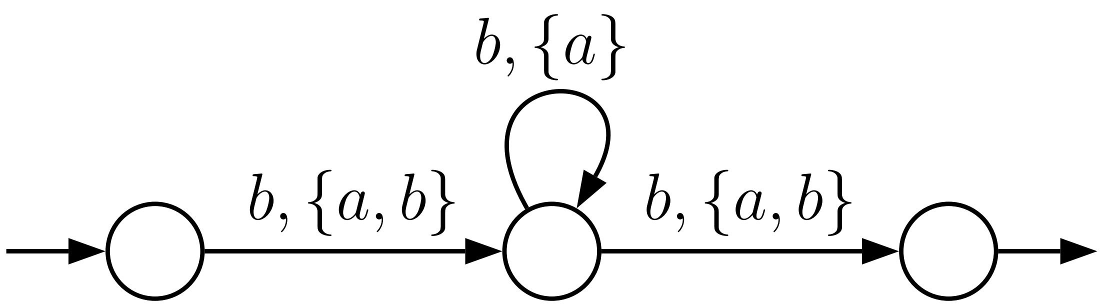


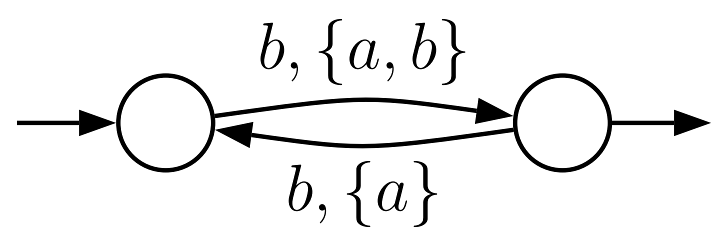
- $f \in \mathcal{S}(\mathcal{A})$
- $g \in \mathcal{S}(\mathcal{A}) \iff \exists z_1, z_2, \ldots \begin{cases} z_1 = 0 \\ z_i < z_{i+1} \\ \lim\limits_{i\to\infty} z_i = 1 \\ g(x) = b \Leftrightarrow \exists i, x = z_i \end{cases}$
- $[f] = \mathcal{S}(\mathcal{A})$

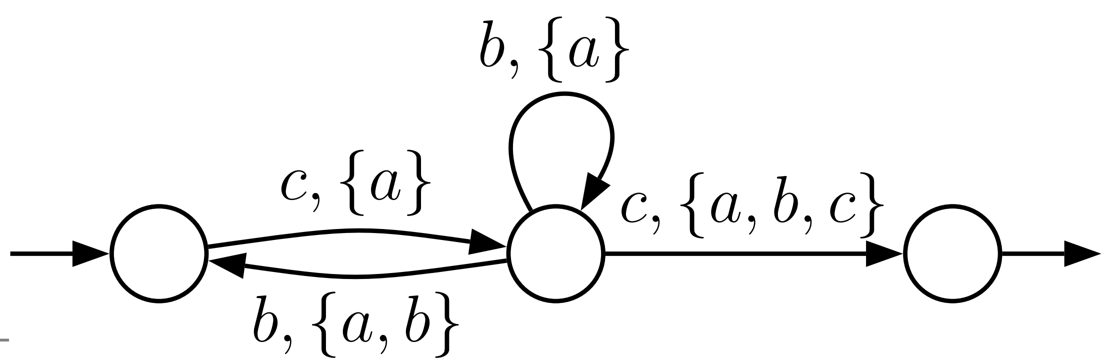
Exercises

Find the signal languages for:









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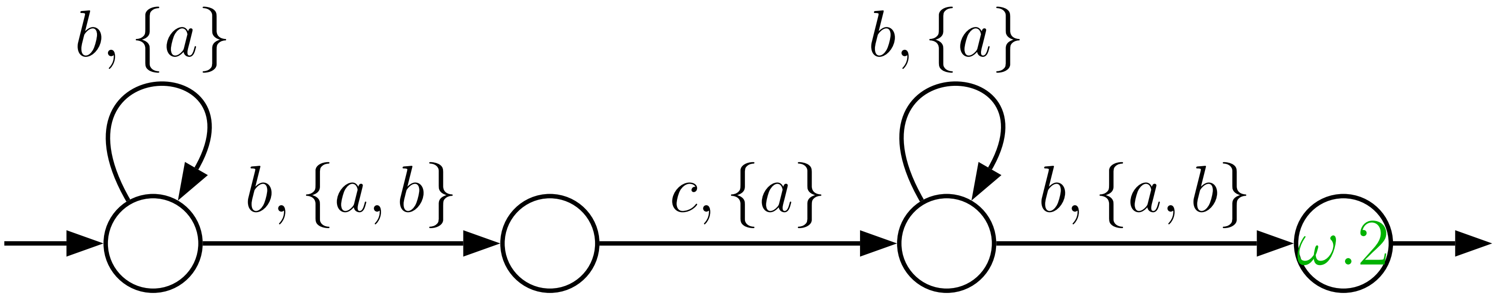

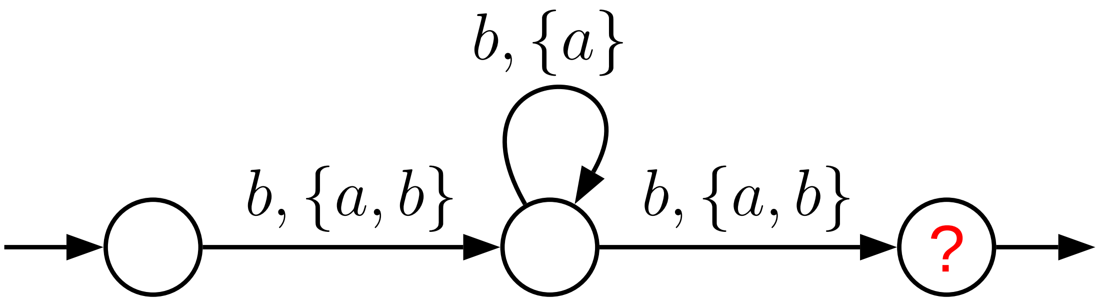

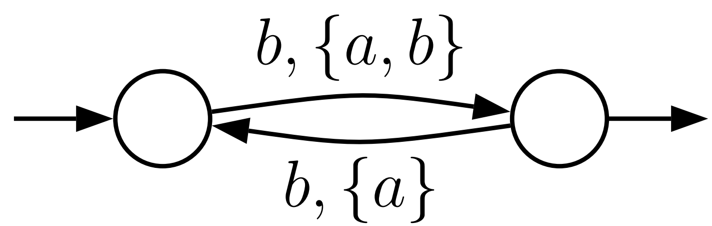

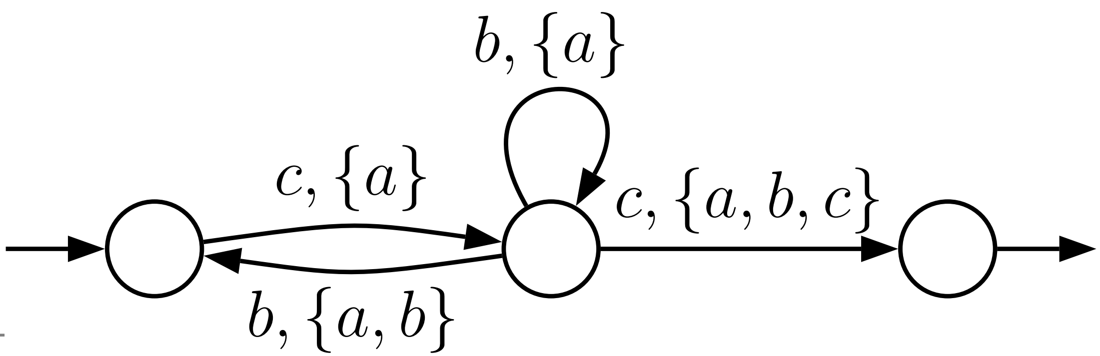

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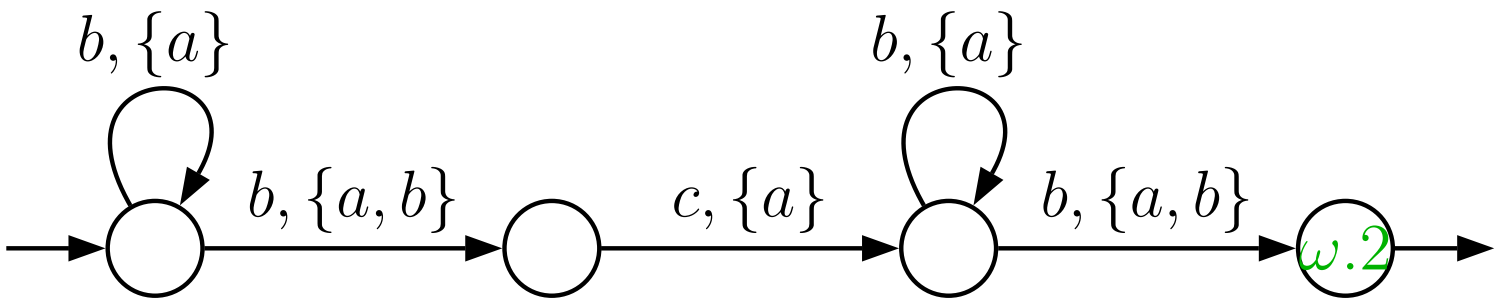

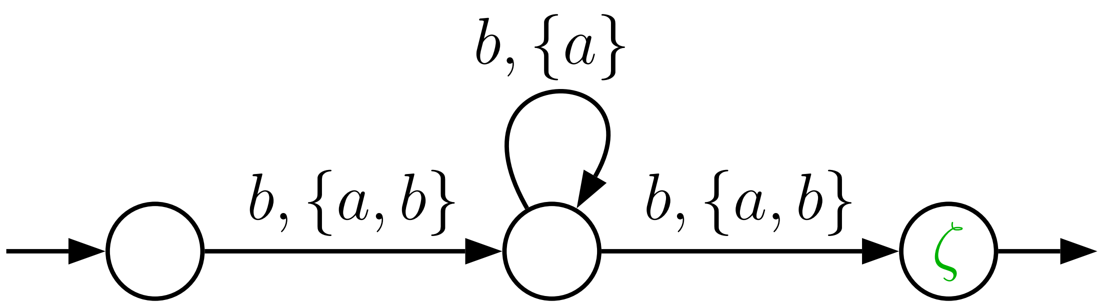

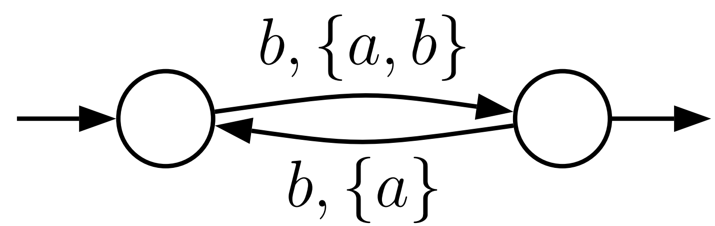

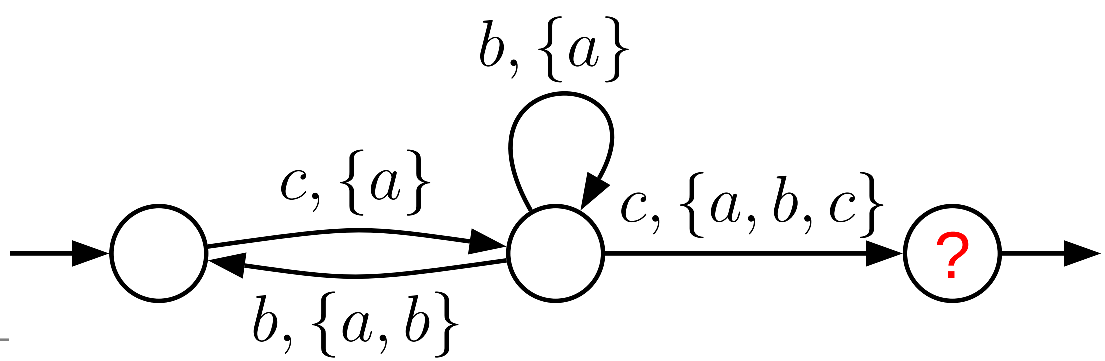

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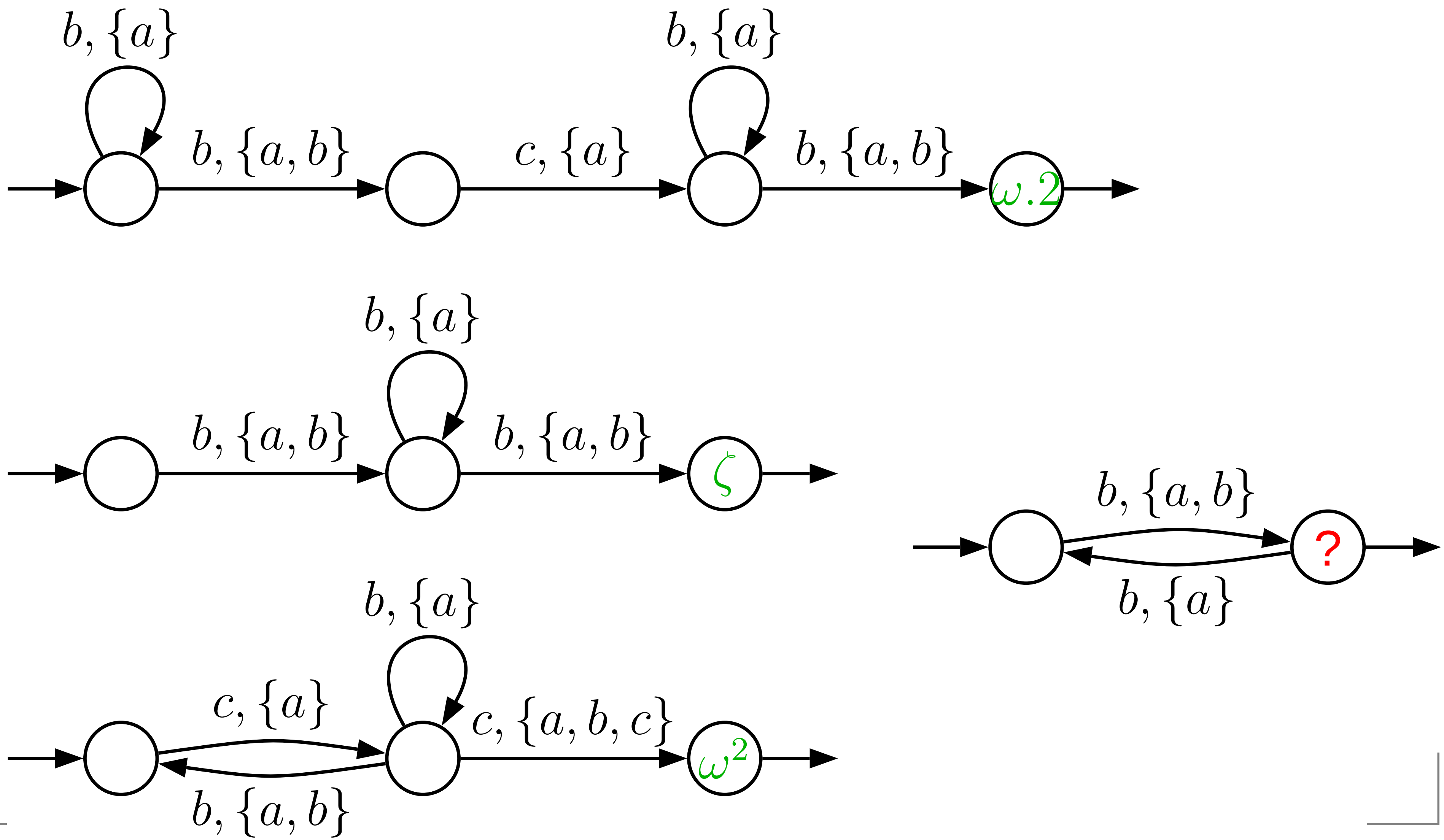

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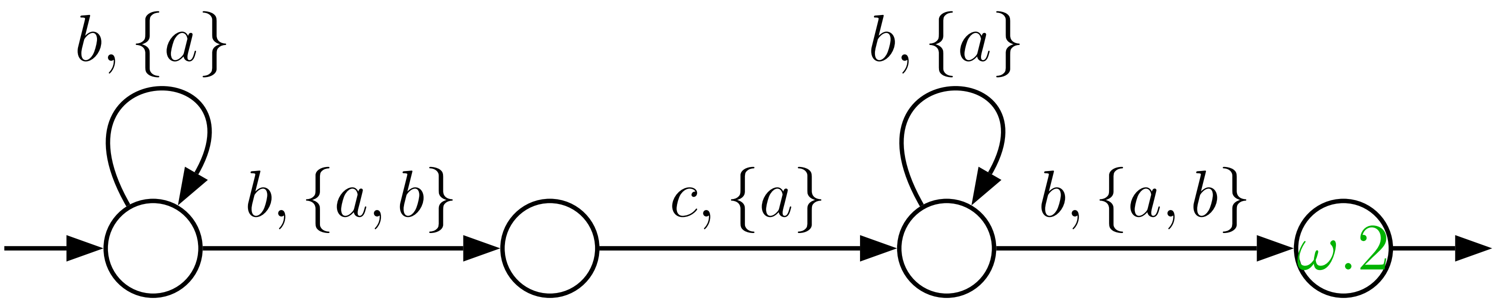

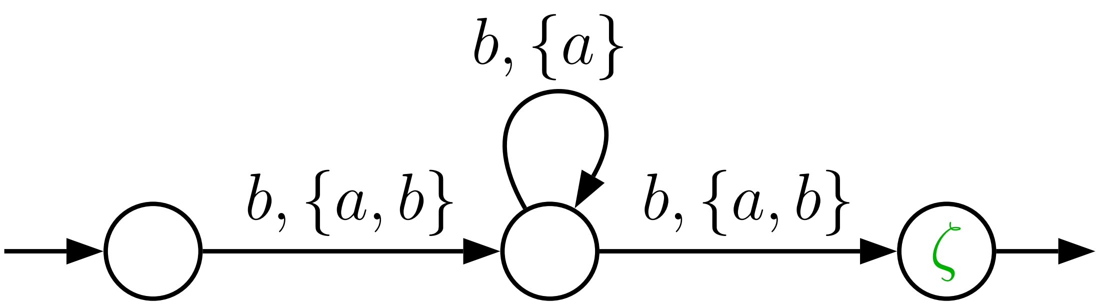

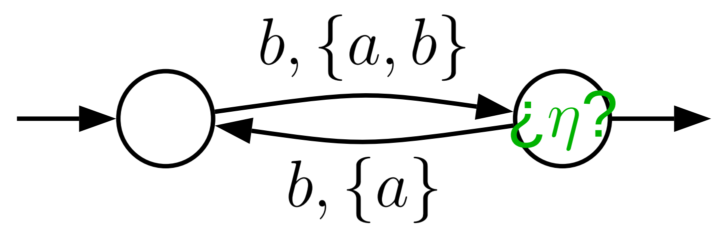

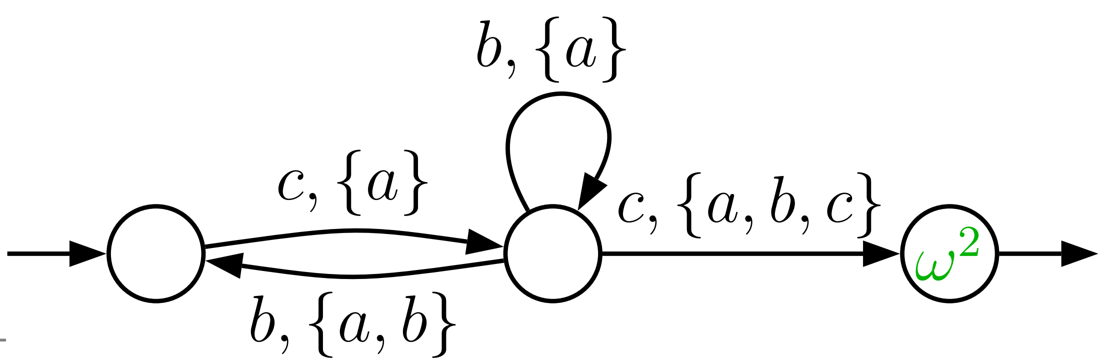

Cardinality of the set of signals

Notation: $\beth_0 = \omega$ et $\beth_{i+1} = 2^{\beth_i}$, (*e.g.* $|\mathbb{R}| = \beth_1$)

$\left|\{\mathcal{A} \text{ automate }\}\right| = \beth_0$

$\left|\{ \text{ signaux } \}\right| = ???$

$\left|\Sigma^{[0,1[}\right| = \beth_2$

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Almost no signal is characterized by an automaton

Open question Look at *companion signals*

$\left|\{\text{ signaux }\}\right| \leq \beth_1$

Let $\left\{\mathcal{A}_i\right\}_{i\in\mathbb{N}}$ be an enumeration of the automata

$$\begin{array}{ccl} & \varphi & \\ [f] & \longmapsto & w \in \{0,1\}^\omega \\ & & w_i{=}1 \Leftrightarrow [f] \subseteq \mathcal{S}(\mathcal{A}_i) \end{array}$$

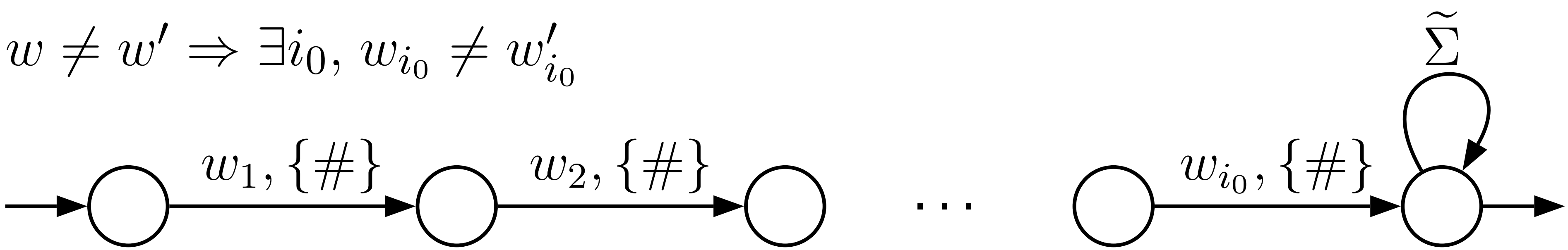
φ is one-to-one because

$$[f] = \bigcap_{x_i=1} \mathcal{S}(\mathcal{A}_i) \cap \bigcap_{x_i=0} \overline{\mathcal{S}(\mathcal{A}_i)}$$

$\beth_1 \leq \left|\{\ \textbf{signaux}\ \}\right|$

$$w \in \{0,1\}^\omega \quad \overset{\psi}{\longmapsto} \quad \begin{array}{l} f \in \{0,1,\#\}^{[0,1[} \\ f\left(\frac{n-1}{n}\right) = w_i \\ f\left(\left]\frac{n-1}{n}, \frac{n}{n+1}\right[\right) = \{\#\} \end{array}$$

$$w \neq w' \Rightarrow \exists i_0,\ w_{i_0} \neq w'_{i_0}$$



Accepts $\psi(w)$ but not $\psi(w')$ thus $[\psi(w)] \neq [\psi(w')]$

$[\psi(.)]$ is one-to-one

Possible with two letters

Open questions

- Classical operations on automata
  - Union OK
  - Concatenation. . . (I am missing an inclusion or a counterexample)
  - Star. . .
- Closure
  - Complements. . . (conj. no)
  - Intersection. . .
- Extra operations
  - ω, $-\omega$ and ζ-iterations ($\diamond$ and $\#$)
- Identification of signal languages
  - Regular expressions
  - KLEENE like theorem