Geometric computation on the plane

– signal machines –

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Methodology

Cellular automata word

(Discrete) Space-time diagrams

Dynamics

Conceptio

Methodology

Cellular automata word

(Discrete) Space-time diagrams

Observation

Discrete lines

Interpretation

Lines on the plane

Dynamics

Conceptio

Methodology

Cellular automata word

(Discrete) Space-time diagrams

Observation ↓ ↑ Implementation

Discrete lines

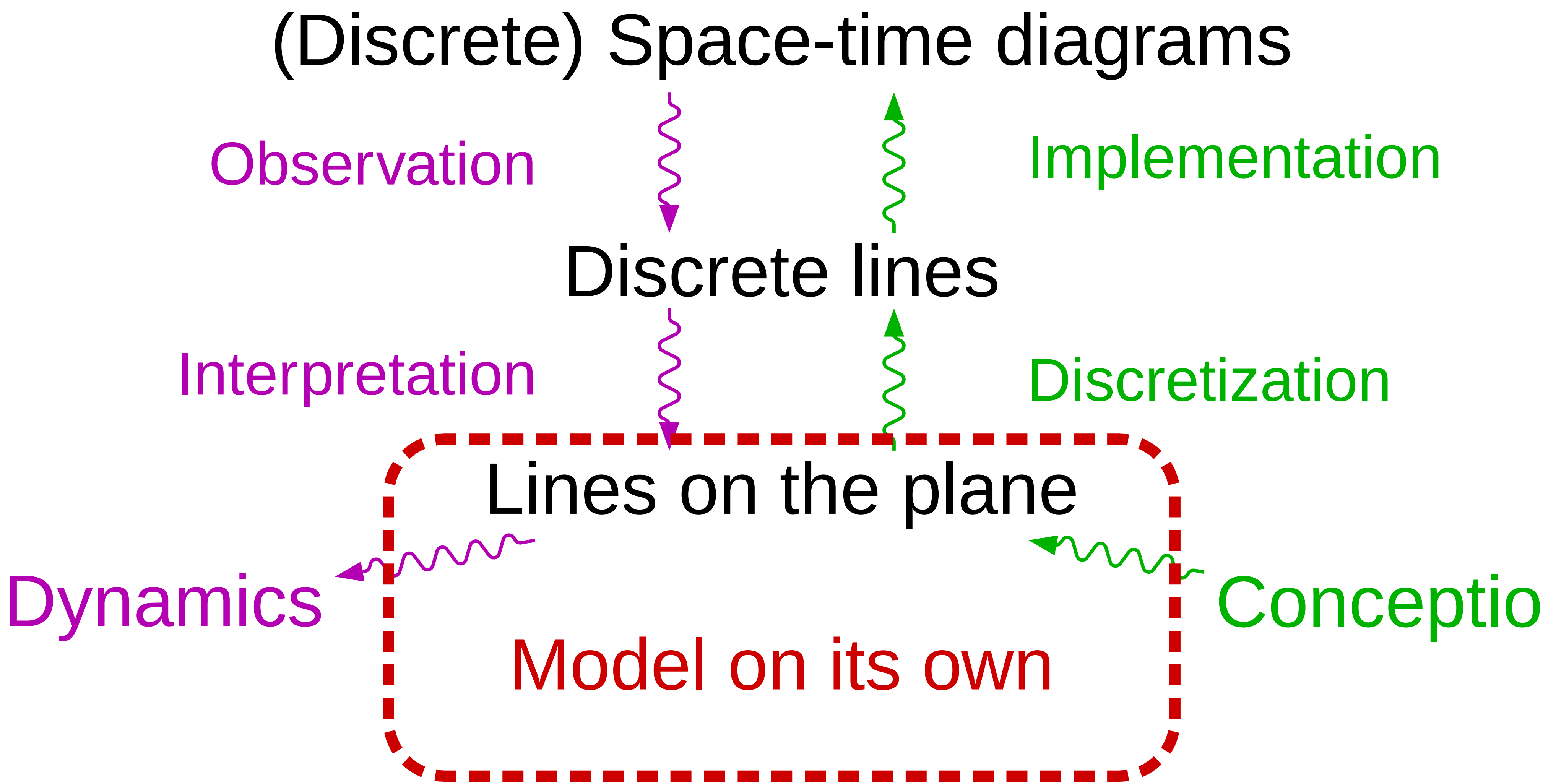
Interpretation ↓ ↑ Discretization

Lines on the plane

Dynamics ↙ ↖ Conceptio

Methodology

Cellular automata word



Outline

- Cellular automata, particles and signals
- Signal machines
  - Computational universality
  - Geometrical modifications
- Accumulation
  - Undecidability of appearance
  - Accumulation
- Conclusion and perspectives

Origins

— Cellular Automata —

Cellular Automata

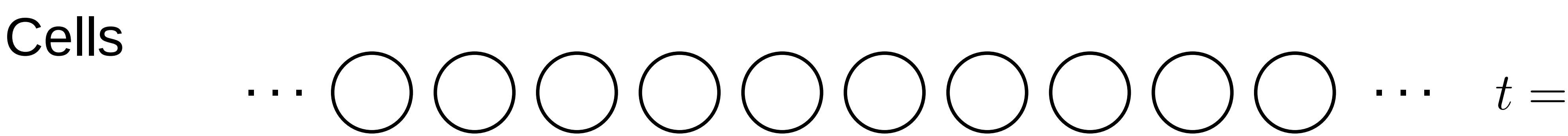
Modeling tools in biology, physics. . . parallelism

Dynamical systems

Cellular Automata

Modeling tools in biology, physics... parallelism

Dynamical systems



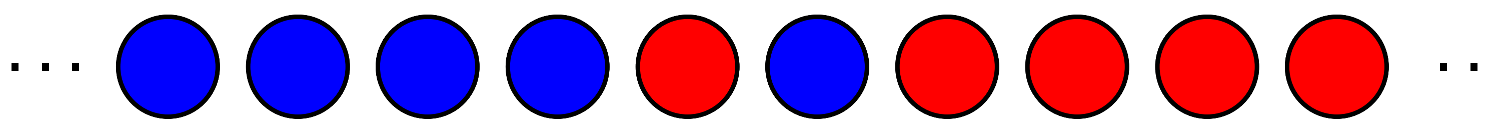
Cellular Automata

Modeling tools in biology, physics. . . parallelism

Dynamical systems

Cells

tates| $< \infty$

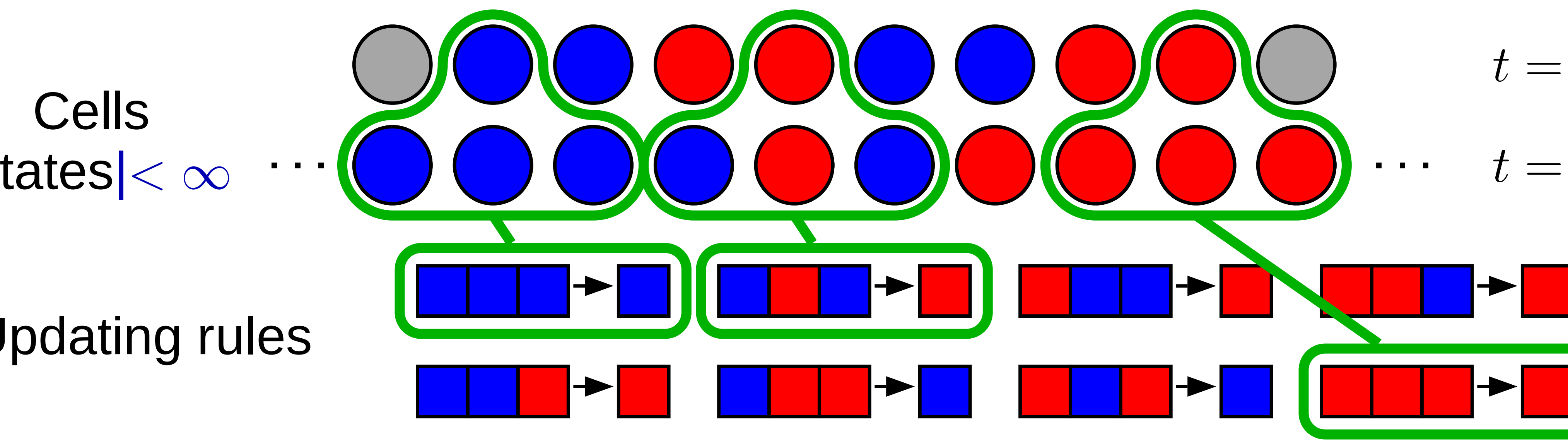


$t =$

Cellular Automata

Modeling tools in biology, physics... parallelism

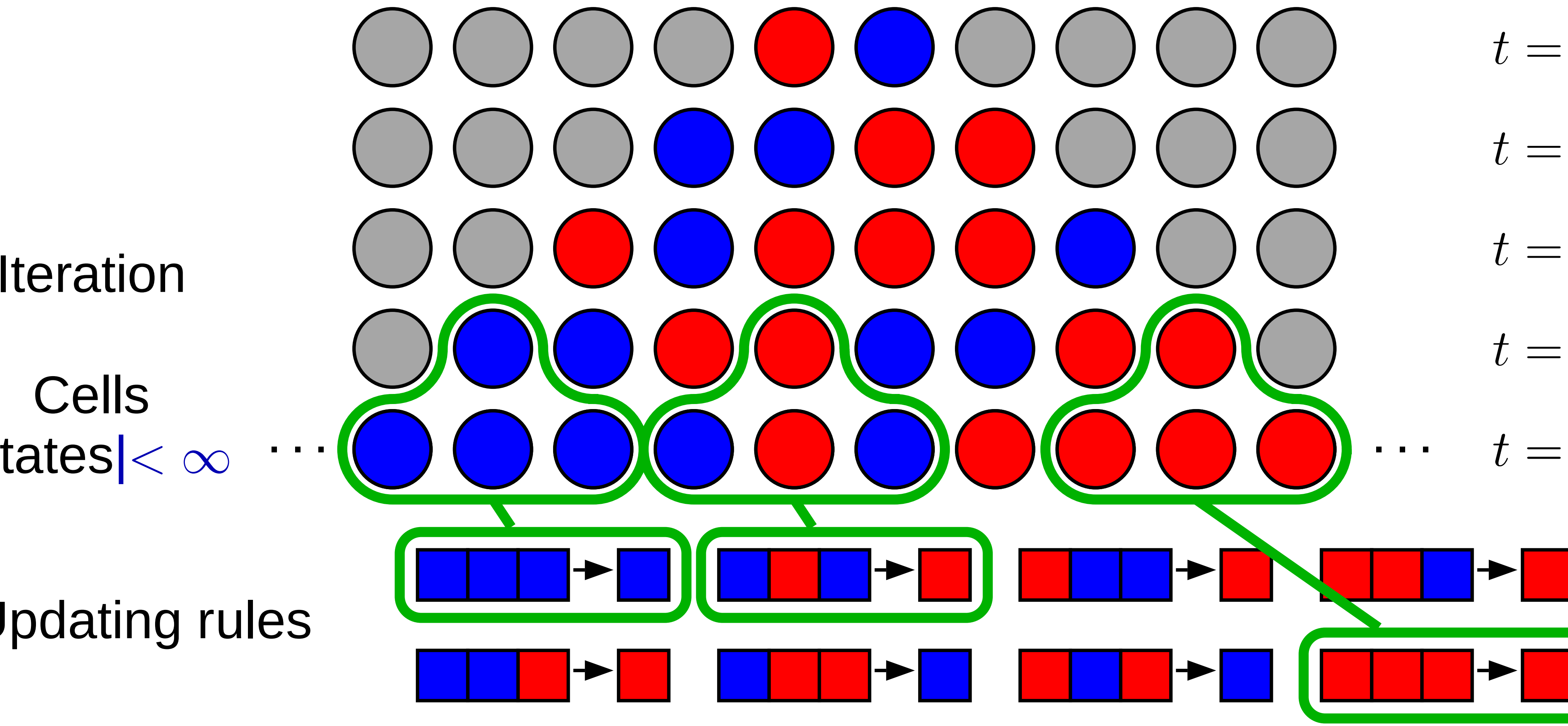
Dynamical systems



Cellular Automata

Modeling tools in biology, physics. . . parallelism

Dynamical systems



Cellular Automata

local

uniform

synchronous

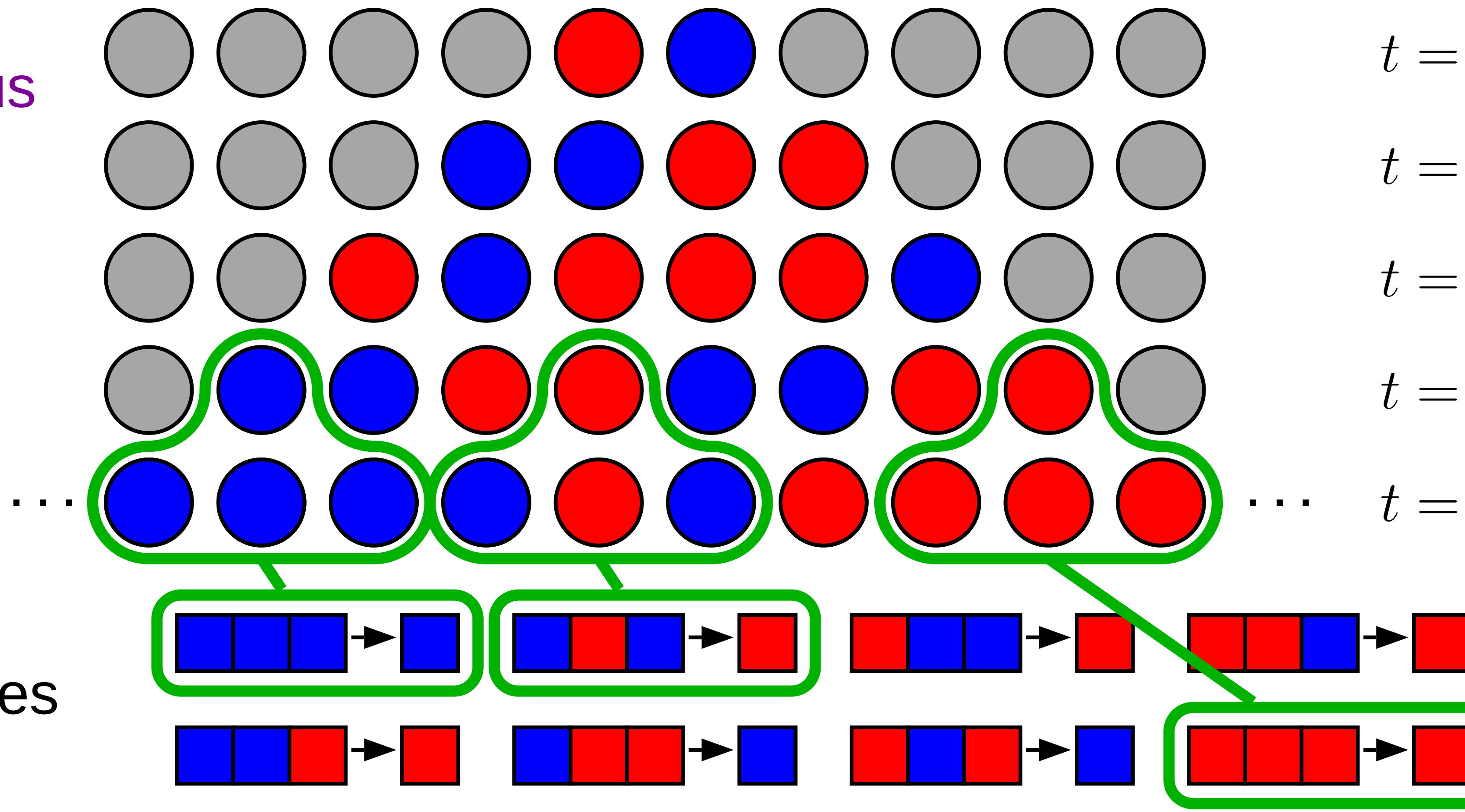
Space-time diagram

Iteration

Cells

tates| $< \infty$

Jpdating rules



Example of particles

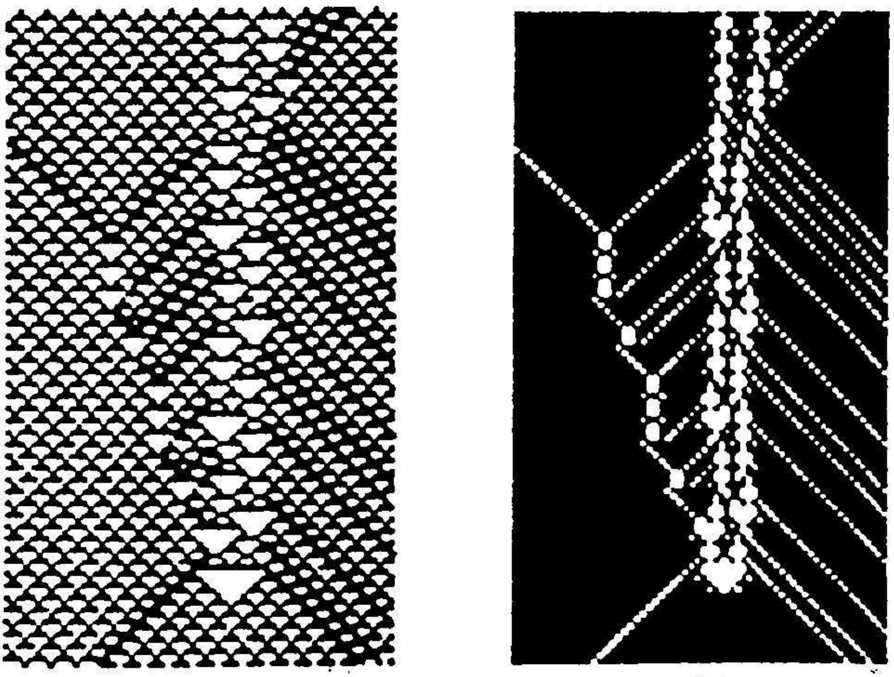


FIG. 7. Rule 54. (a) Annihilation of the radiating parti-
e. (b) The same as (a) with the mapping defined in Fig. 6.

[Boccara et al., 1991, Fig. 7]

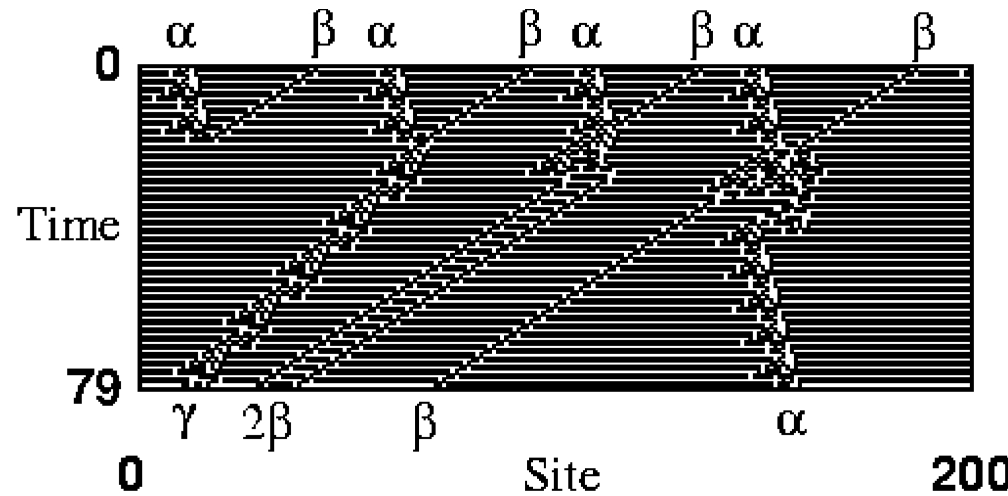


FIG. 7. The four different (out of 14 possible) interaction products for the $\alpha + \beta$ interactio

[Hordijk et al., 2001, Fig. 7]

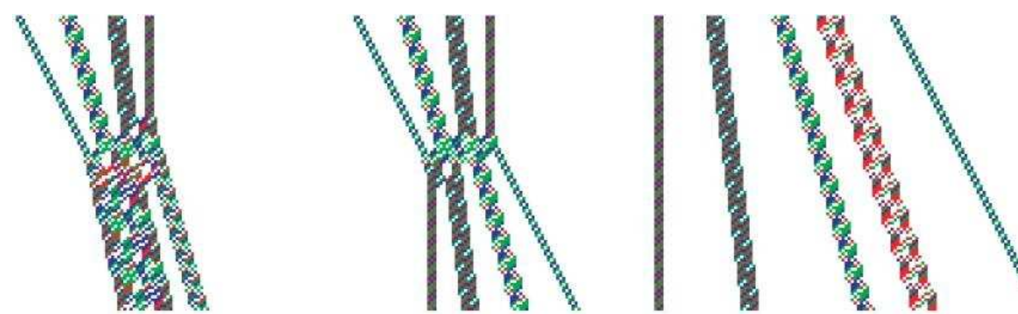


Figure 5. Two collisions of filtrons, and five free filtrons supported by the FPS model; ST diagra
applies $q = 1$.

[Siwak, 2001, Fig. 5]

To build an Turing-universal CA

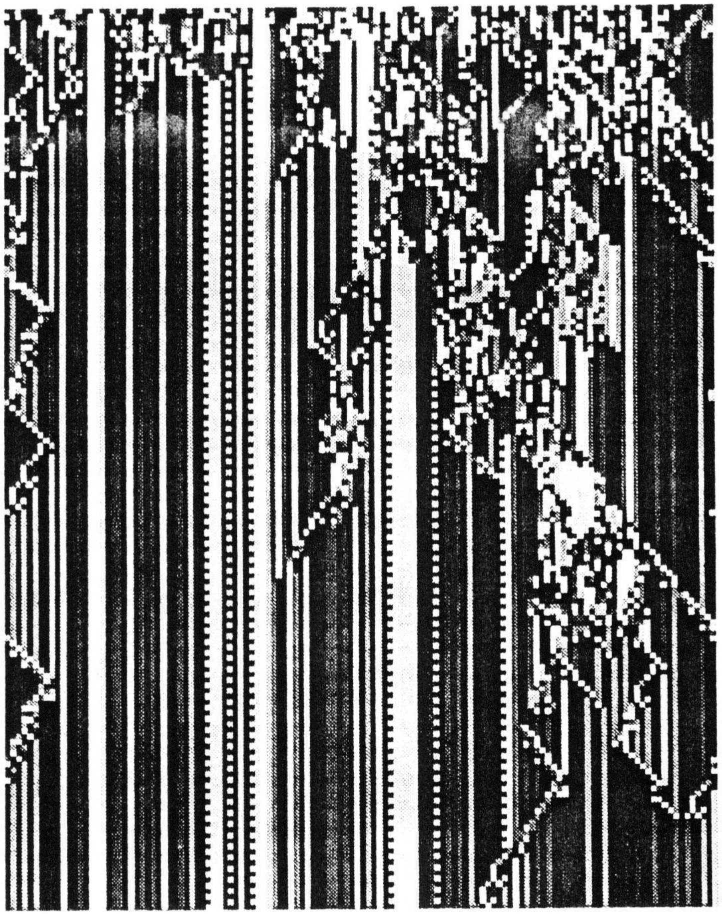


Figure 3: A simulation of the $k = 7$, $r = 1$ universal CA of table 3 for an uncorrelated initial state (with a density of blanks equal to 0.76). Symbols y, 0, 1, A, B, $\sqcup$, and T are represented by

[Lindgren and Nordahl, 1990, Fig. 4]

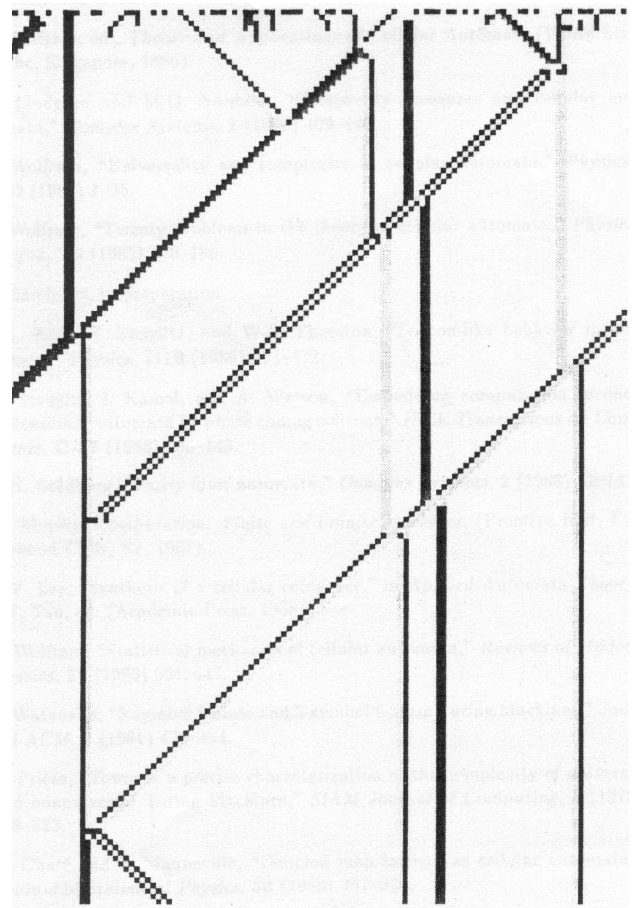


Figure 4: The $k = 4$, $r = 2$ universal cellular automaton of table 4 simulated starting from a random initial state. The symbols 0, 1, $\sqcup$, and + are represented by

[Lindgren and Nordahl, 1990, Fig. 3]

Signal algorithmic

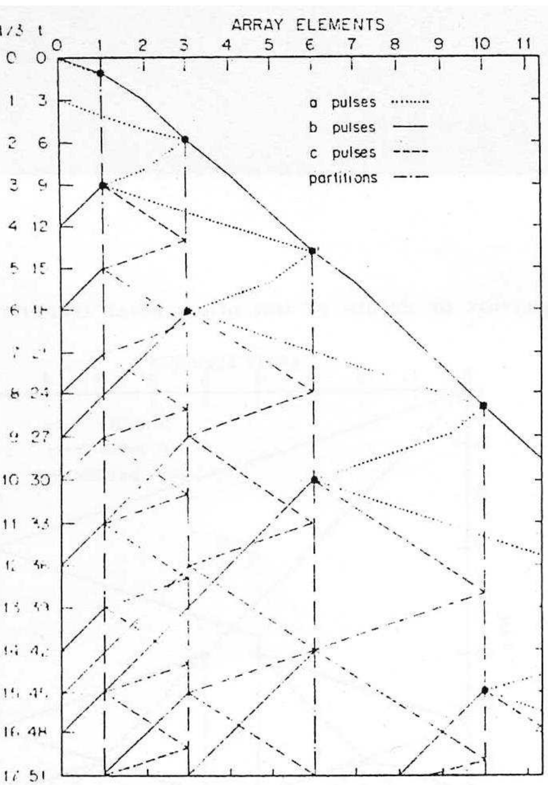


FIG. 2. Solution to the prime problem

[Fischer, 1965, Fig. 2]

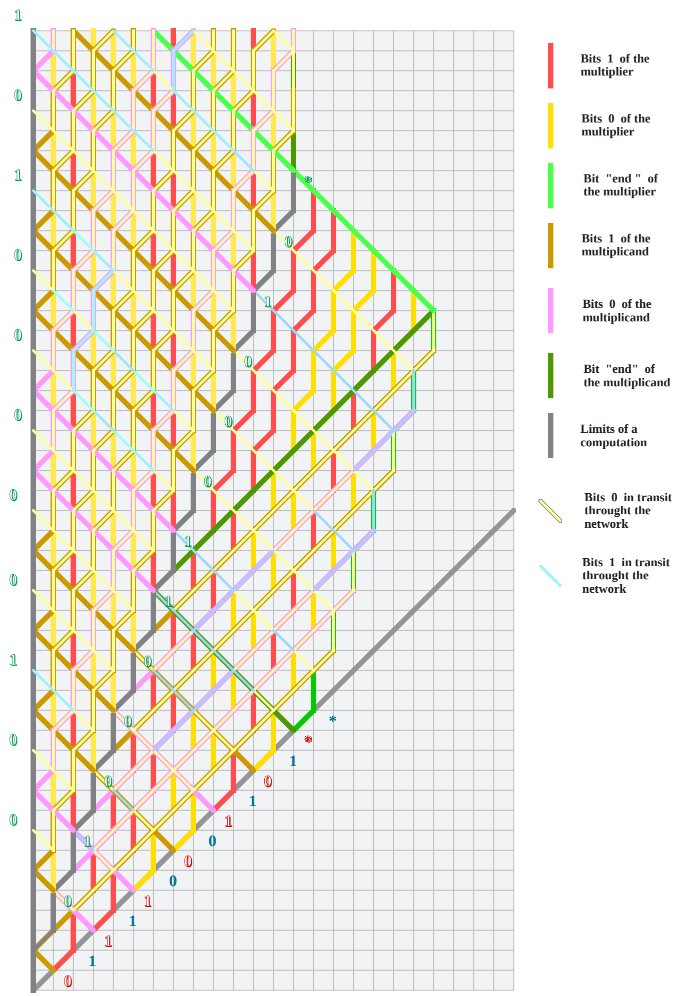


Figure 8: Computing $(ab)^2$.

[Mazoyer, 1996, Fig. 8]

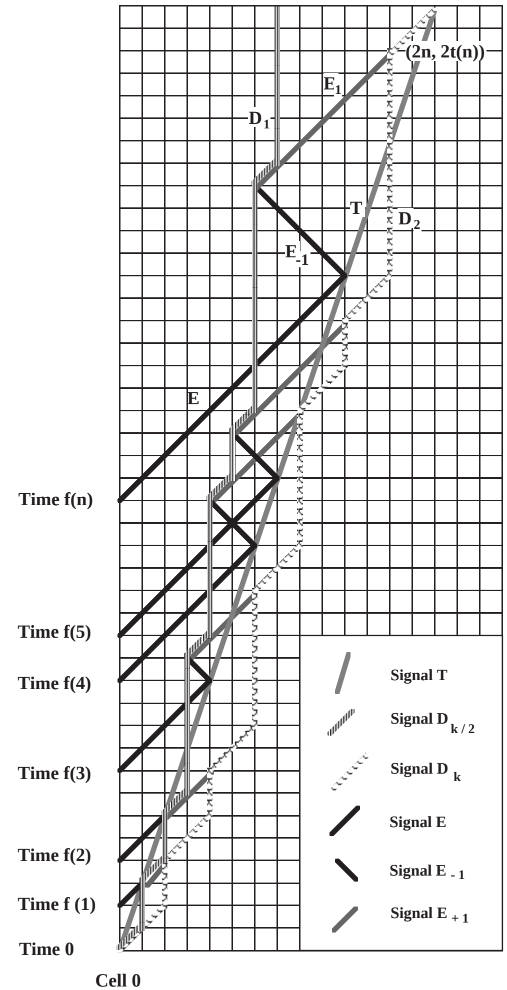


Figure 18: Characterization of the sites $(n, f(n))$.

[Mazoyer and Terrier, 1999, Fig. 1

Geometric algorithmic

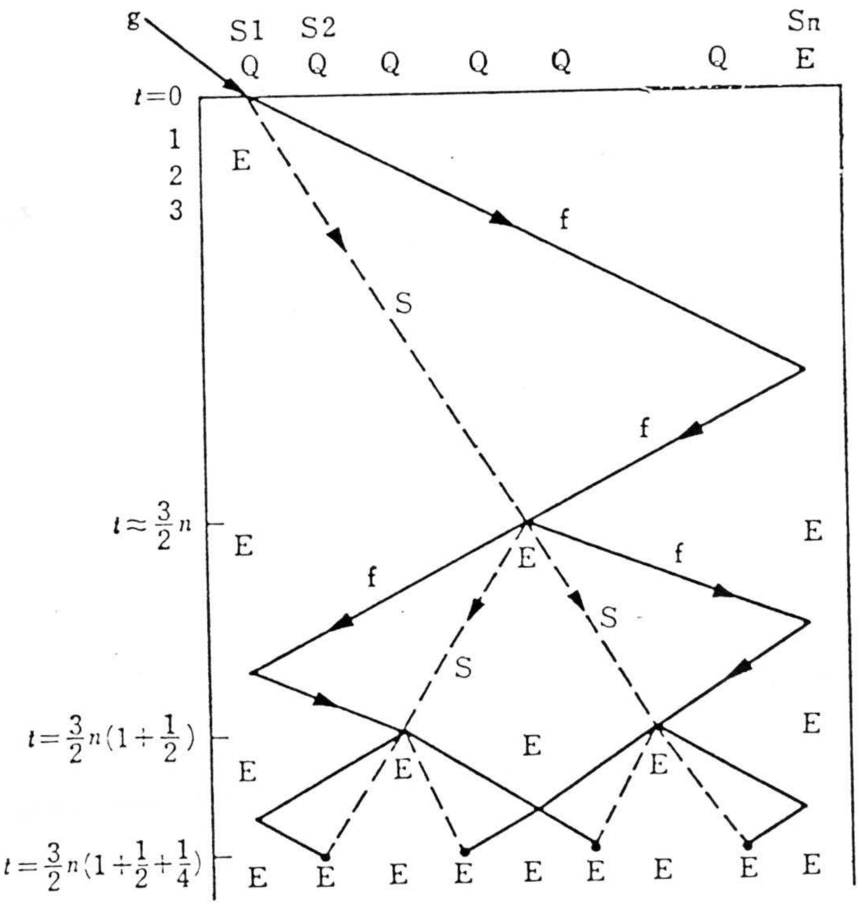
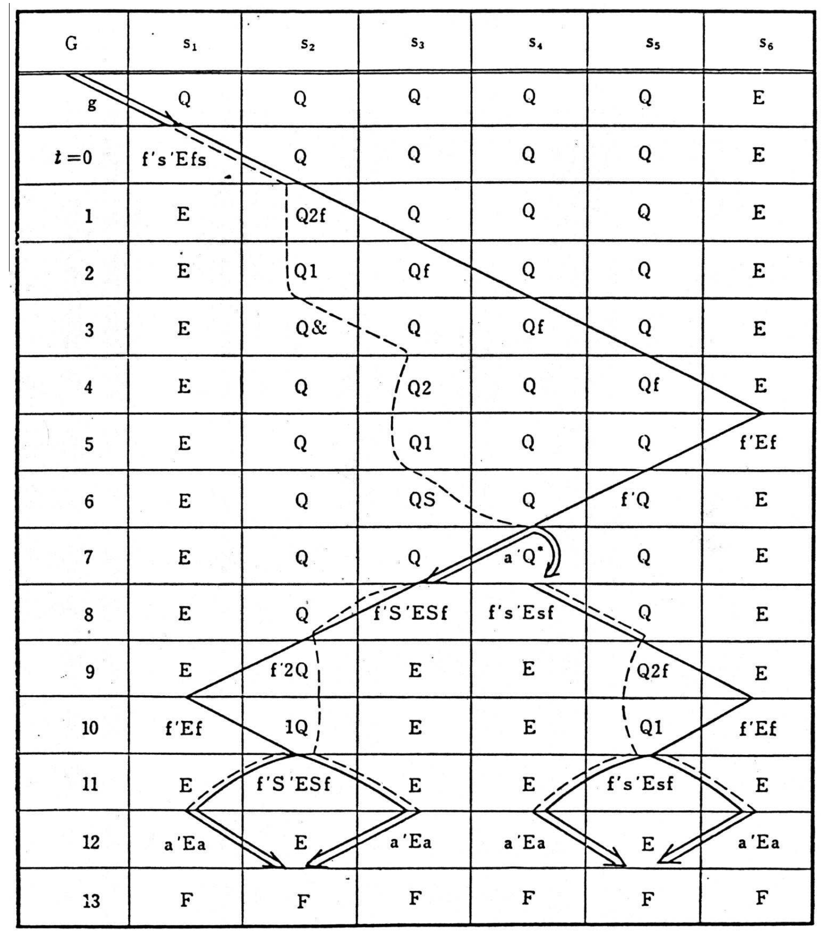


図 3·5 一斉射撃の問題（連続近似）

[Goto, 1966, Fig. 3]

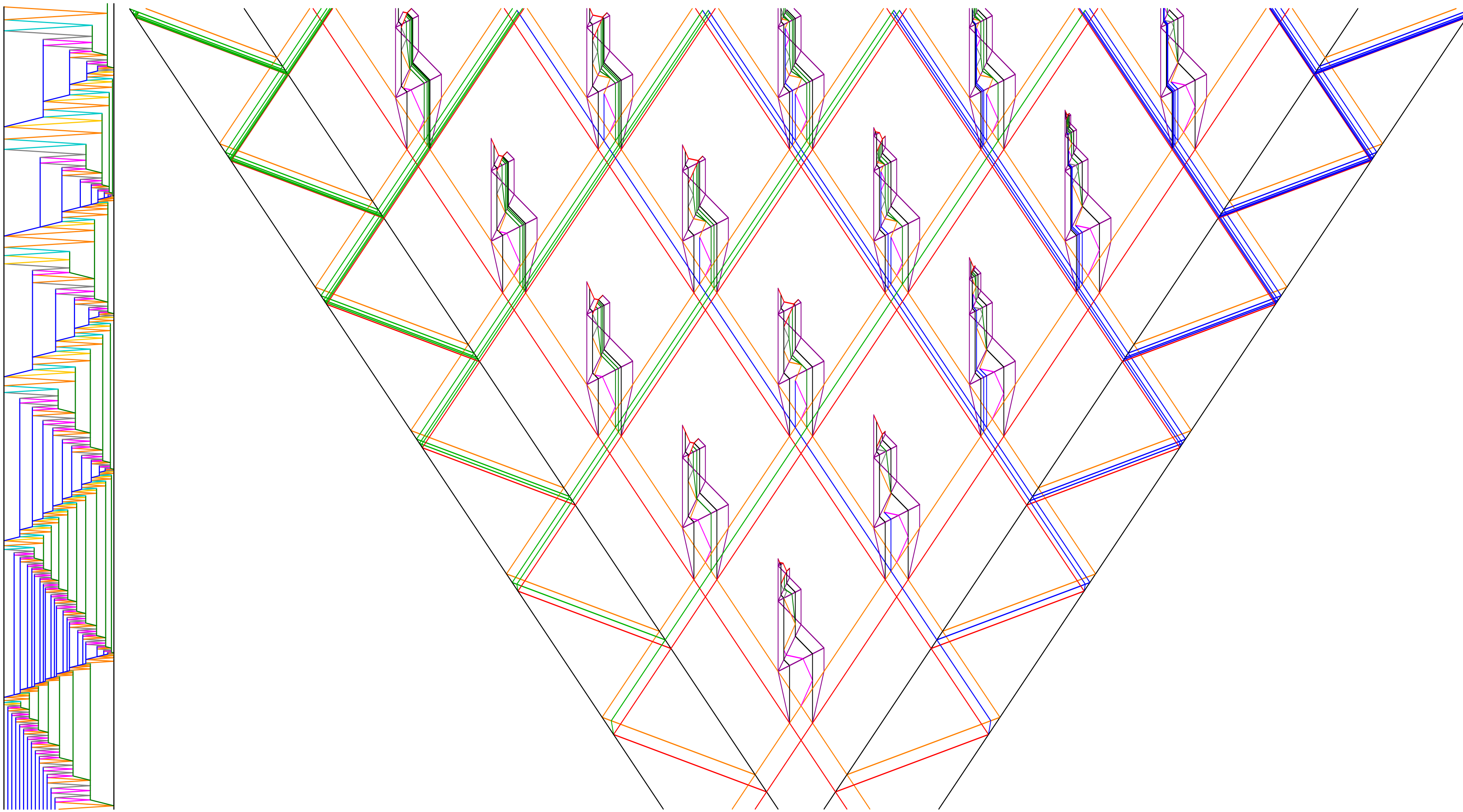


G	s_1	s_2	s_3	s_4	s_5	s_6
g	Q	Q	Q	Q	Q	E
$t=0$	f's'Efs	Q	Q	Q	Q	E
1	E	Q2f	Q	Q	Q	E
2	E	Q1	Qf	Q	Q	E
3	E	Q&	Q	Qf	Q	E
4	E	Q	Q2	Q	Qf	E
5	E	Q	Q1	Q	Q	f'Ef
6	E	Q	QS	Q	f'Q	E
7	E	Q	Q	a'Q*	Q	E
8	E	Q	f'S'ESf	f's'Esf	Q	E
9	E	f'2Q	E	E	Q2f	E
10	f'Ef	1Q	E	E	Q1	f'Ef
11	E	f'S'ESf	E	E	f's'Esf	E
12	a'Ea	E	a'Ea	a'Ea	E	a'Ea
13	F	F	F	F	F	F

図 3·6 一斉射撃解 ($n=6$)

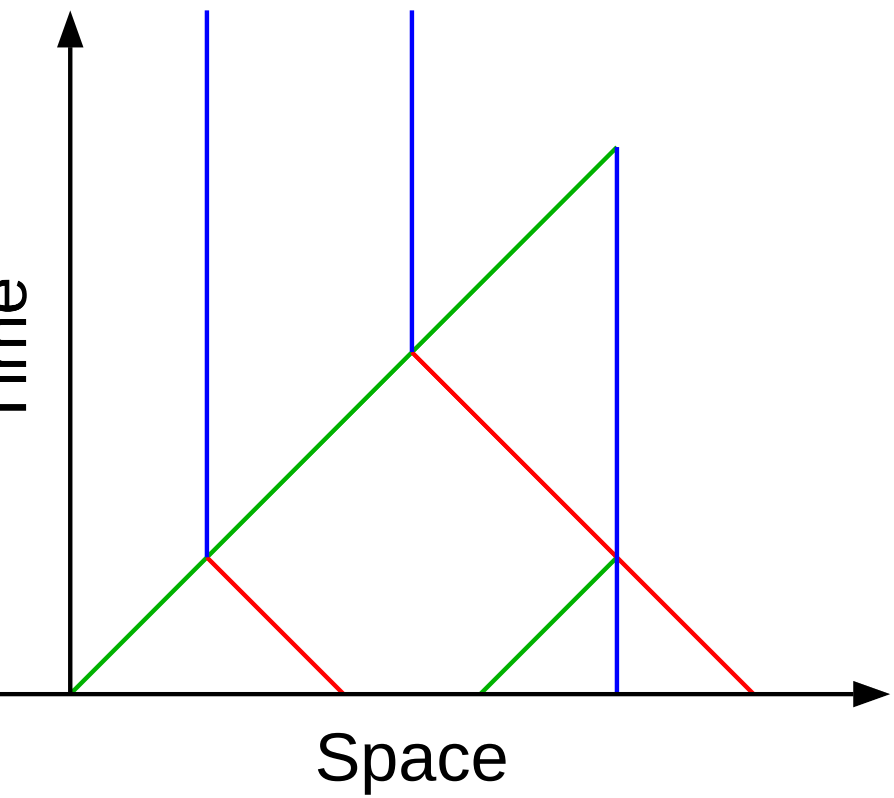
[Goto, 1966, Fig. 6]

Signal machines

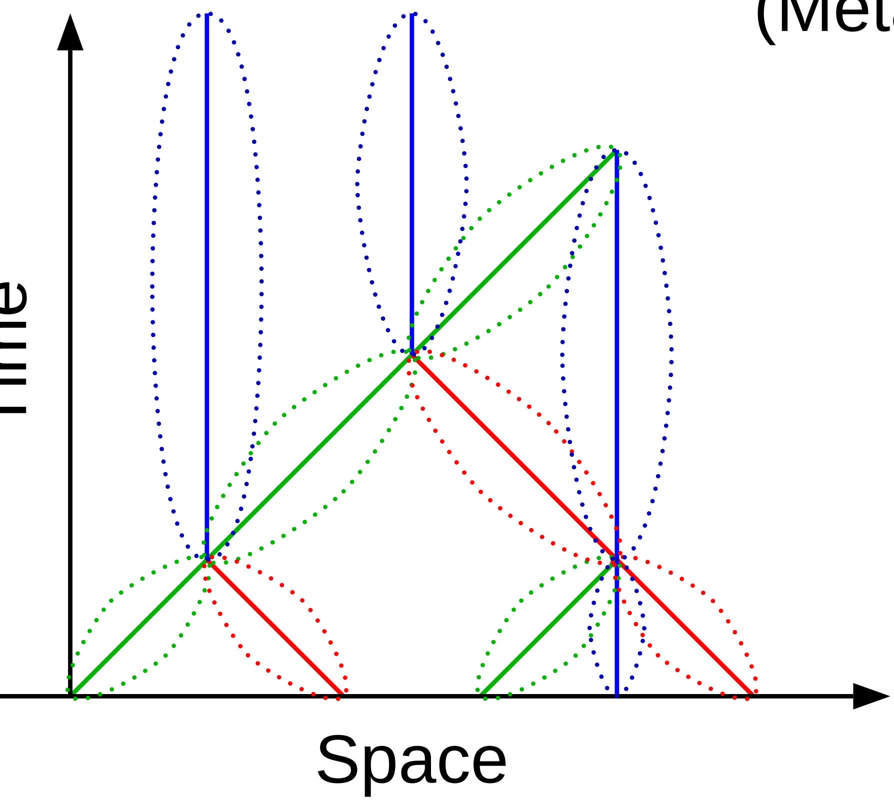


Signal Machines

Model analyze



Model analyze



Signal

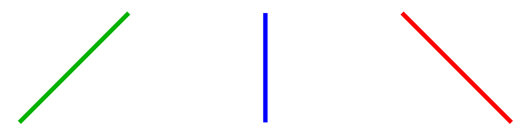
(Meta-signal, position)

Position

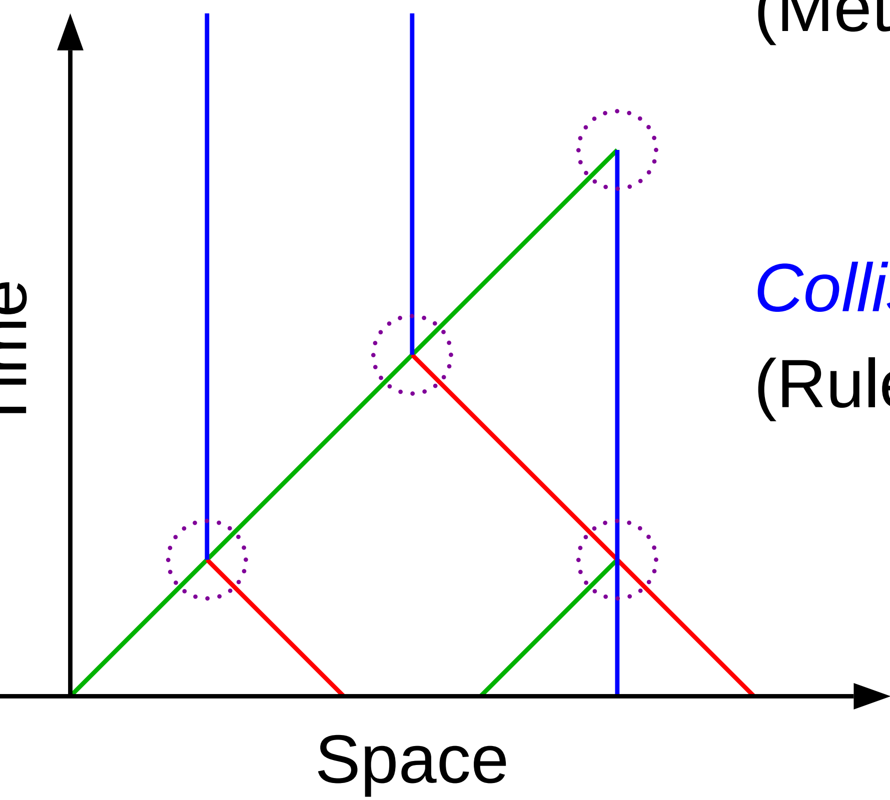
(x, t)

Meta-signal

$\mu = (\iota, \nu)$



Model analyze



Signal

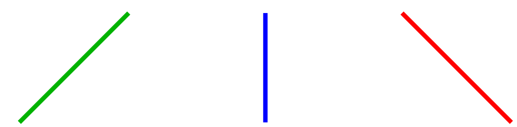
(Meta-signal, position)

Position

(x, t)

Meta-signal

$\mu = (\iota, \nu)$

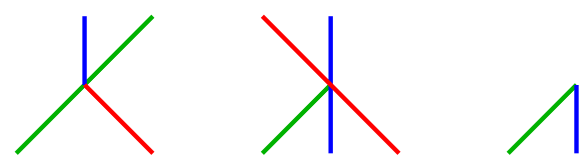


Collision

(Rule, position)

Rule

$\rho = \{\mu_i^-\}_i \rightarrow \{\mu_j^+\}_j$



Model definition

Machine

$\mathcal{M} = (\{\mu_i\}_i, \{\rho_j\}_j)$

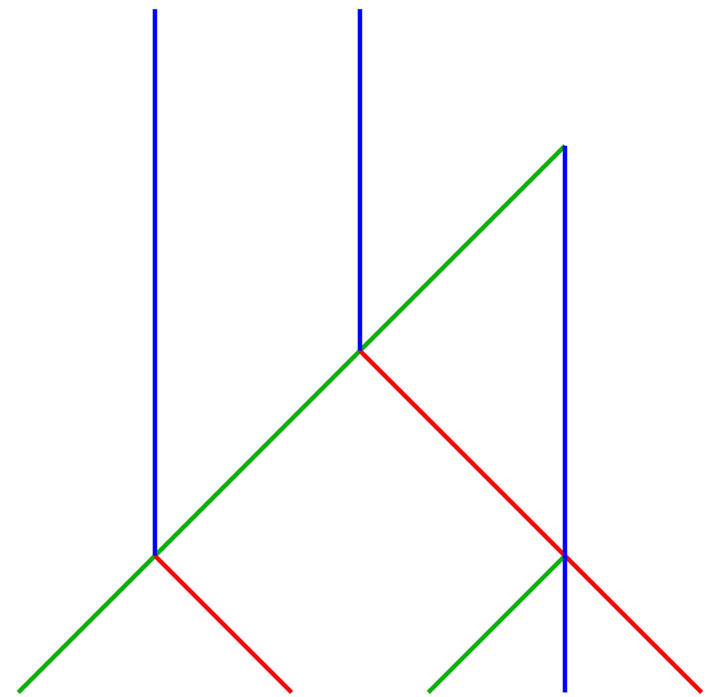
Finite description

Deterministic

Configuration (at t)

Positions of
signals and collisions

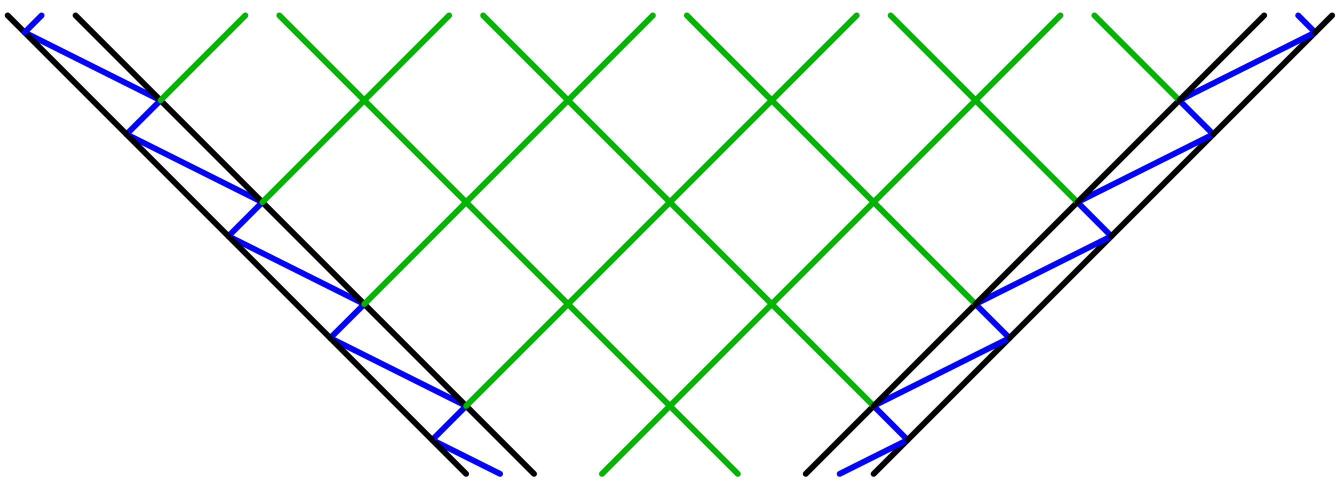
Space-time diagram

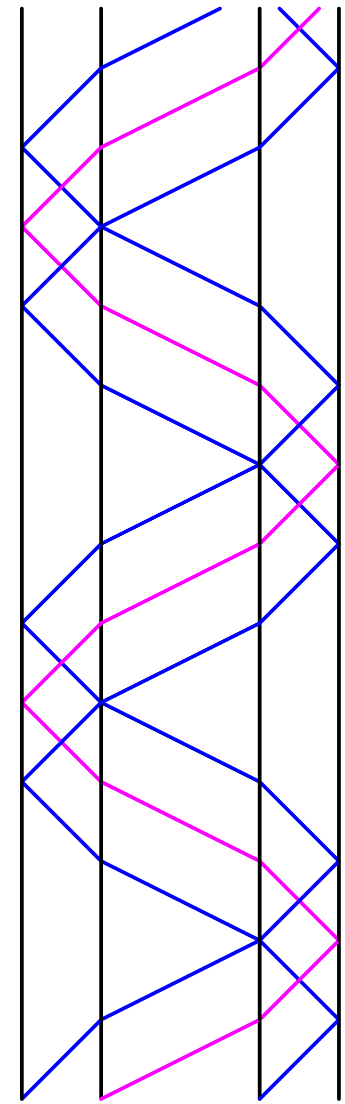


Computation
collisions +
dependence ordering

Properties

- Uniform in space and time
- Local
- Light cone
- Finite number of values & rules
- Continuous space & time





Turing-universal

2-counter automata simulation

```
beg:  B++
      A--
      A!=0  beg1
      B!=0  imp
beg1: A--
      A!=0  beg
pair: B--
      A++
      B!=0  pair
      A!=0  beg
imp:  B--
      A++
      A++
      B!=0  imp1
      A!=0  beg
imp1: B--
      A++
      A++
      A++
      B!=0  imp1
      A!=0  beg
```

A and B two non-negative integer counters

Operations

A++	B++
A--	A--
A != 0 <*label*>	B != 0 <*label*>

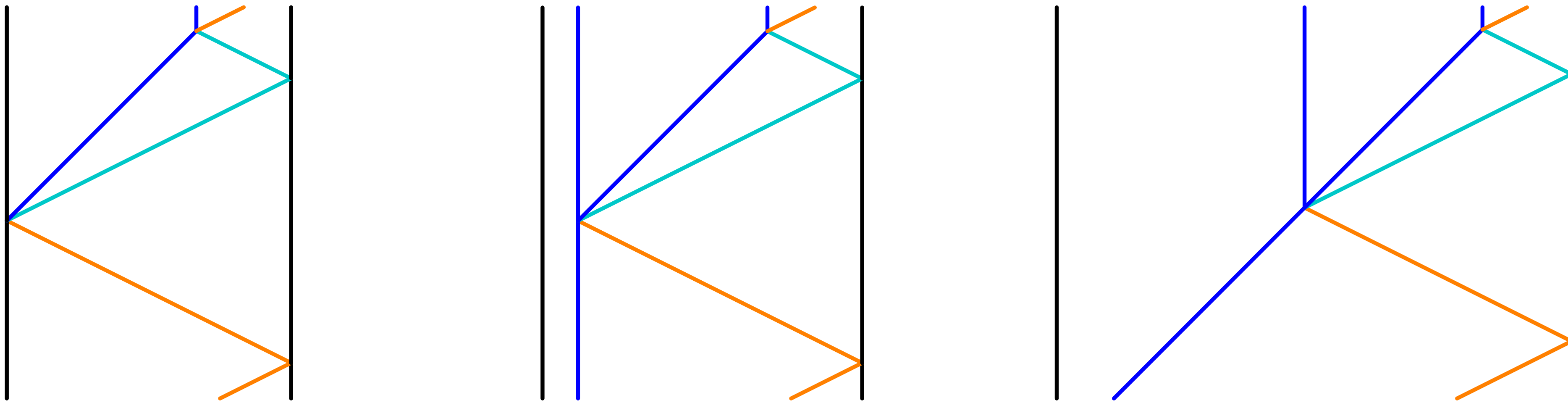
Encoding

bord $a_0 = 6$ instruction $b_0 = 2$ bord

Unbounded place for signals

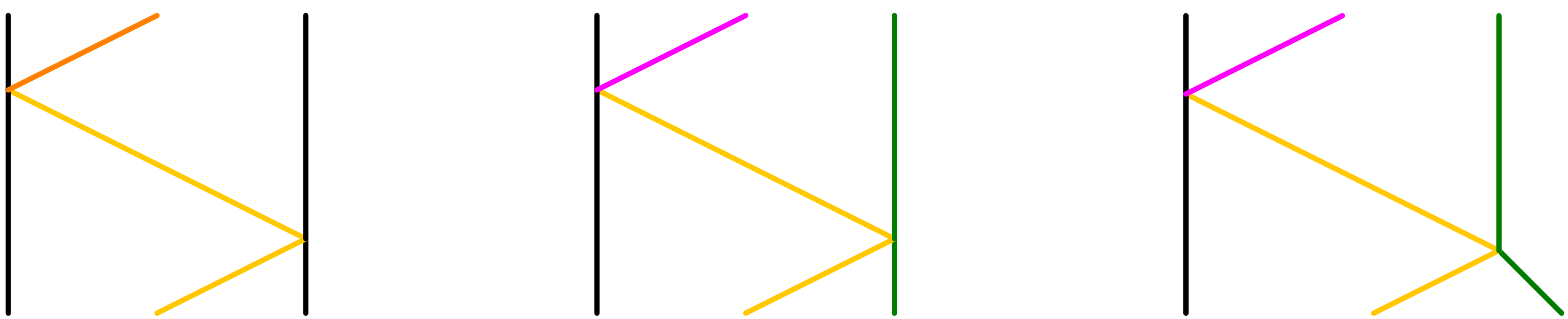
Transcription of the instructions

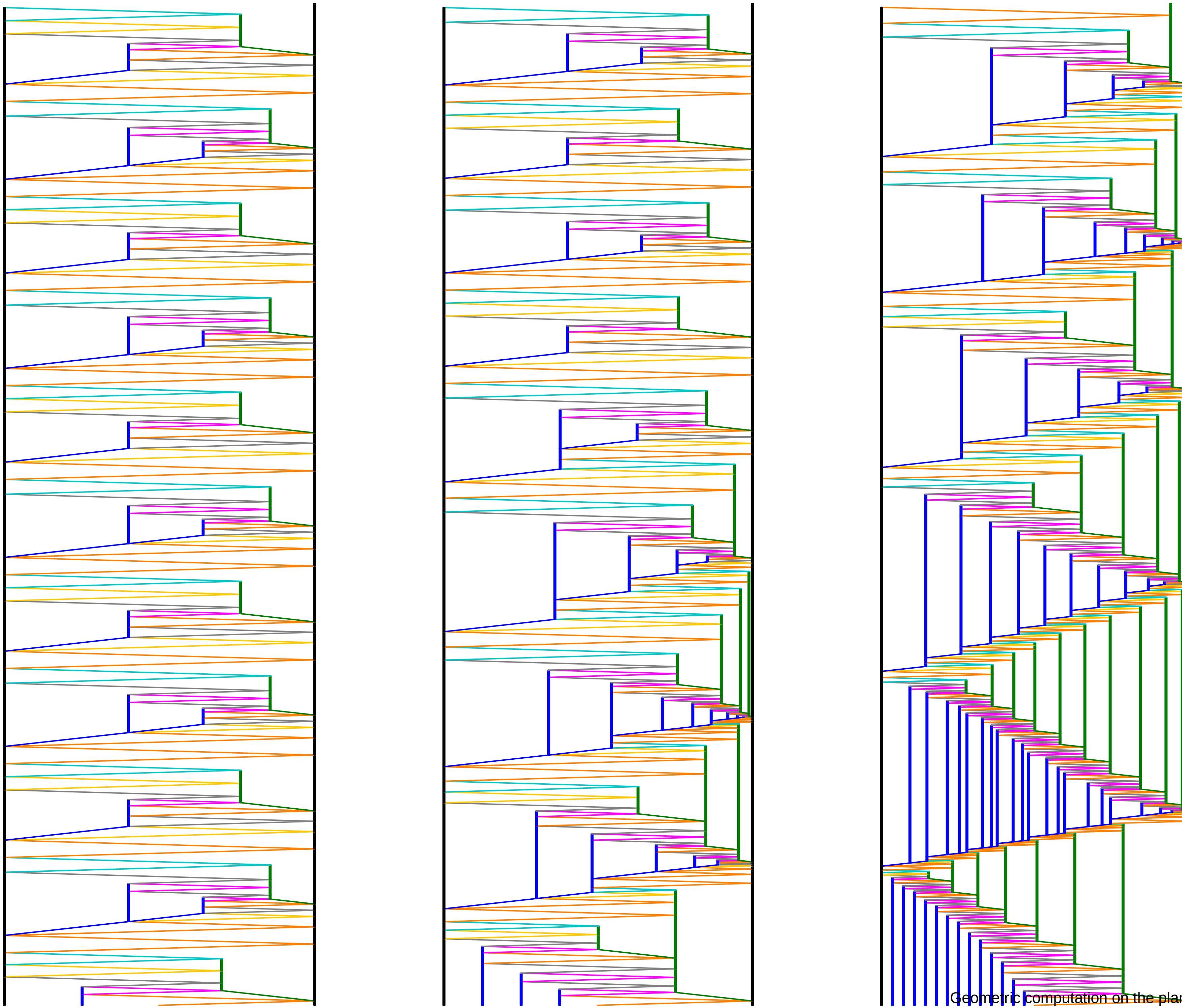
A++



$a = 0$ $a \neq 0$ After A++

B != 0 m





Turing-universal

Any recursive computation can be performed

$\rightsquigarrow$ *Highly unpredictable*

e.g. all these are undecidable

- Finite number of collisions
- Appearance of a meta-signal
- Collision with a signal

Geometrical constructions

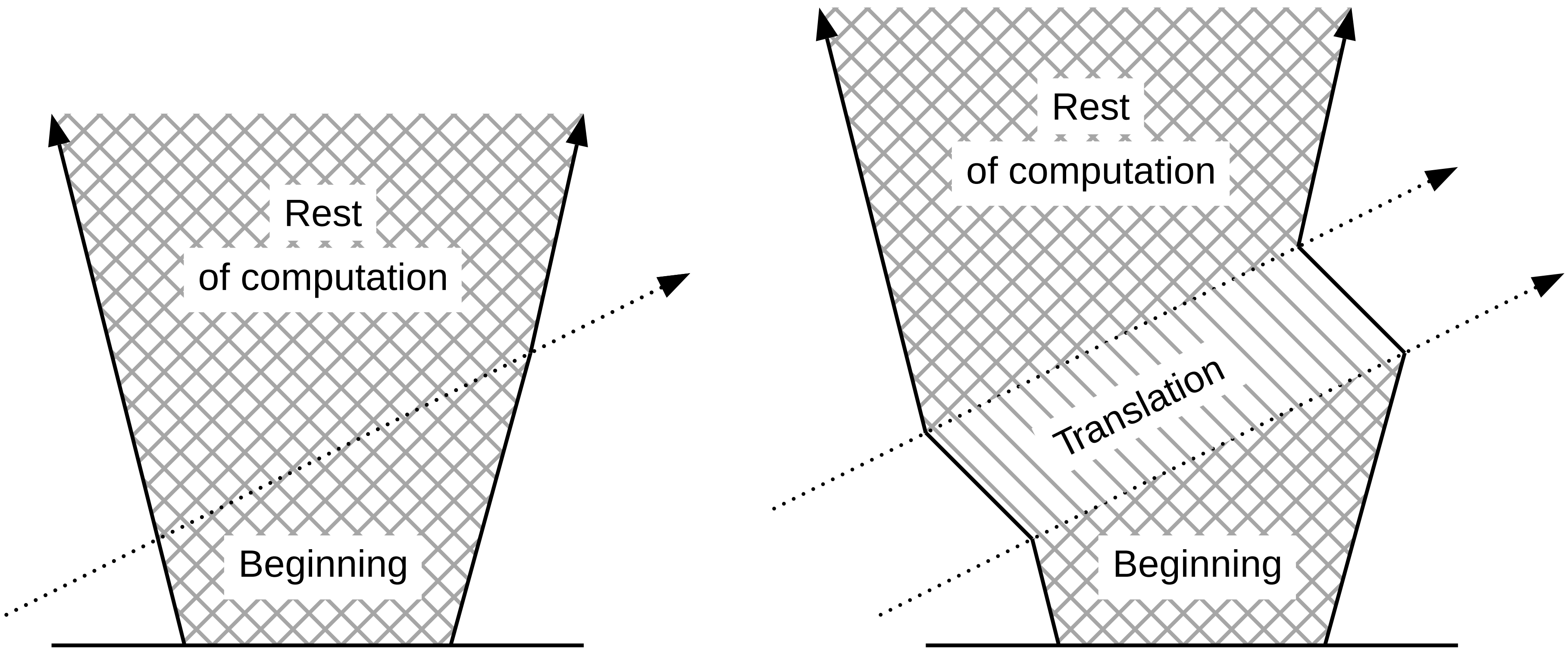
Modify the space-time diagram

but preserve the computation (*i.e.* ordering of collisions)

- Dynamic
  - Freezing
  - Translations
  - Homotecy
- Constructions with these “operators”
  - basic
  - iterated

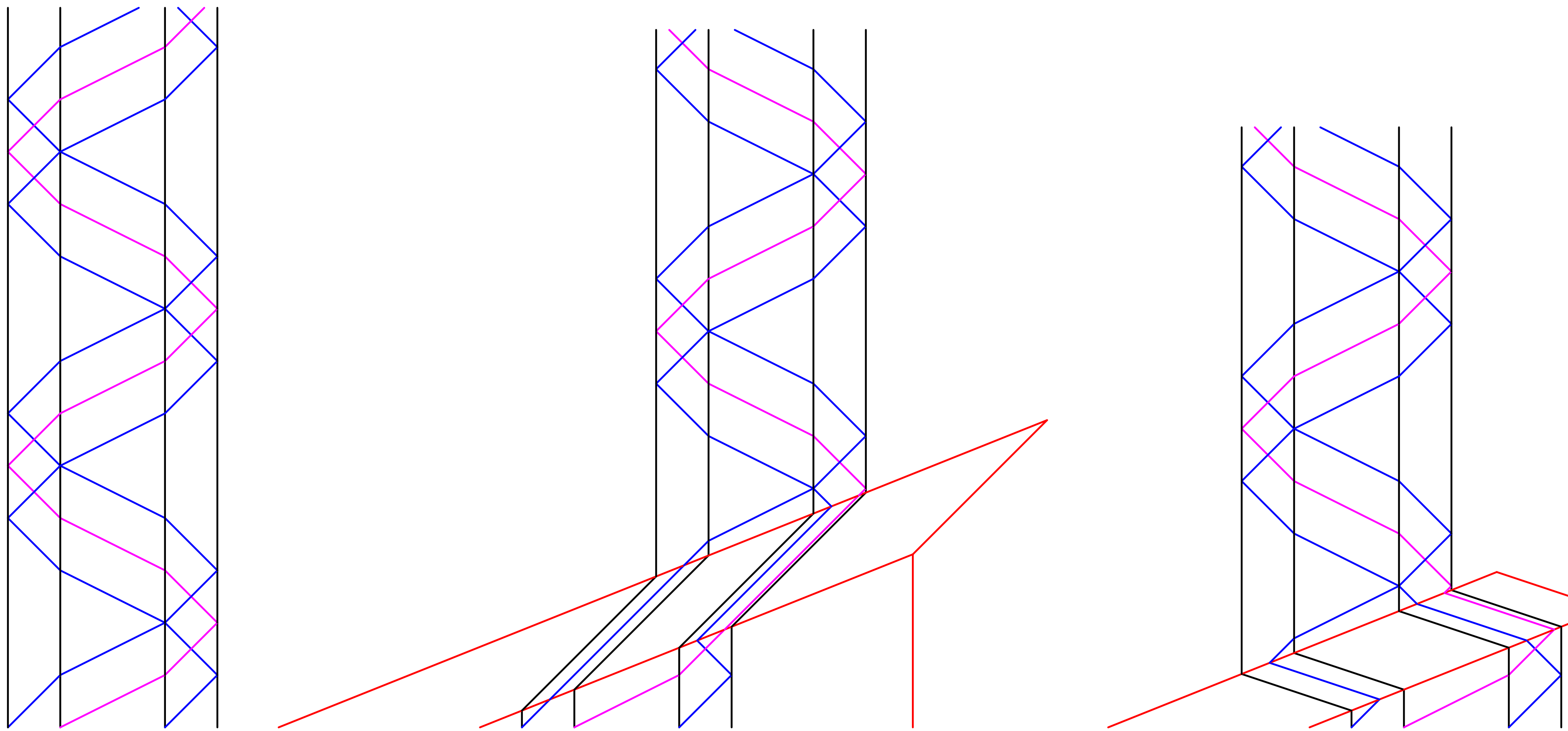
Freezing the computation

Freeze then restore



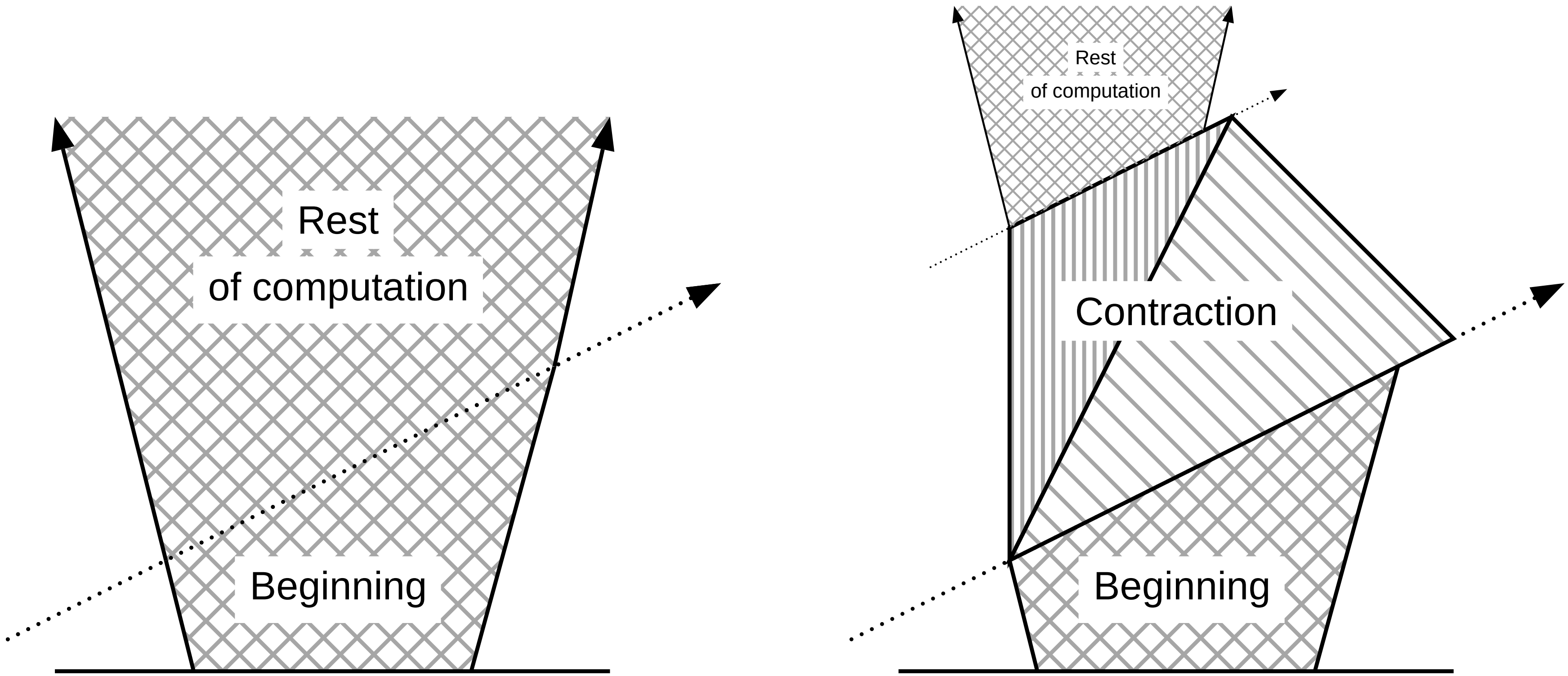
Freezing the computation

Freeze then restore

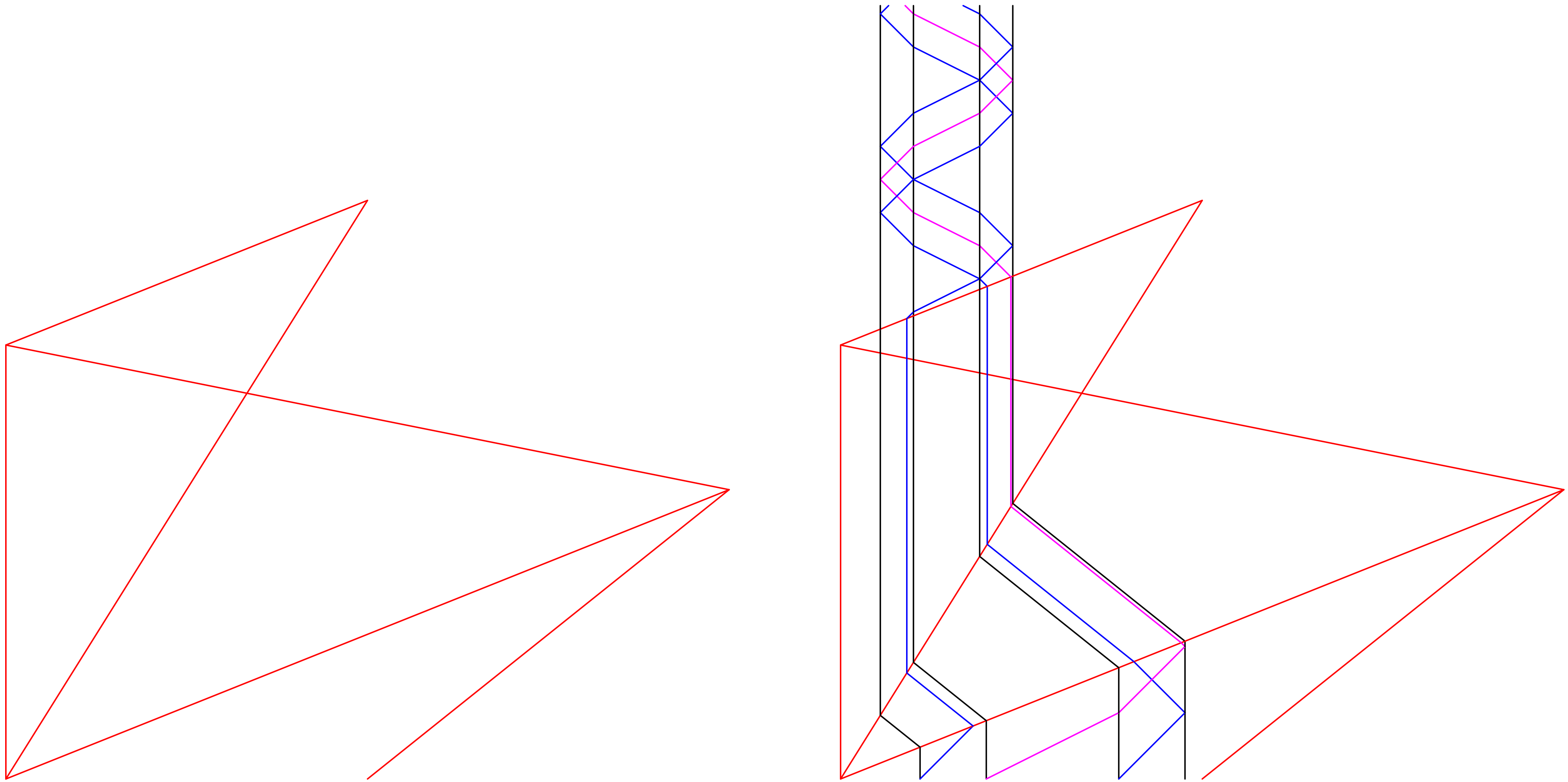


Only modifications: new meta-signals and rules

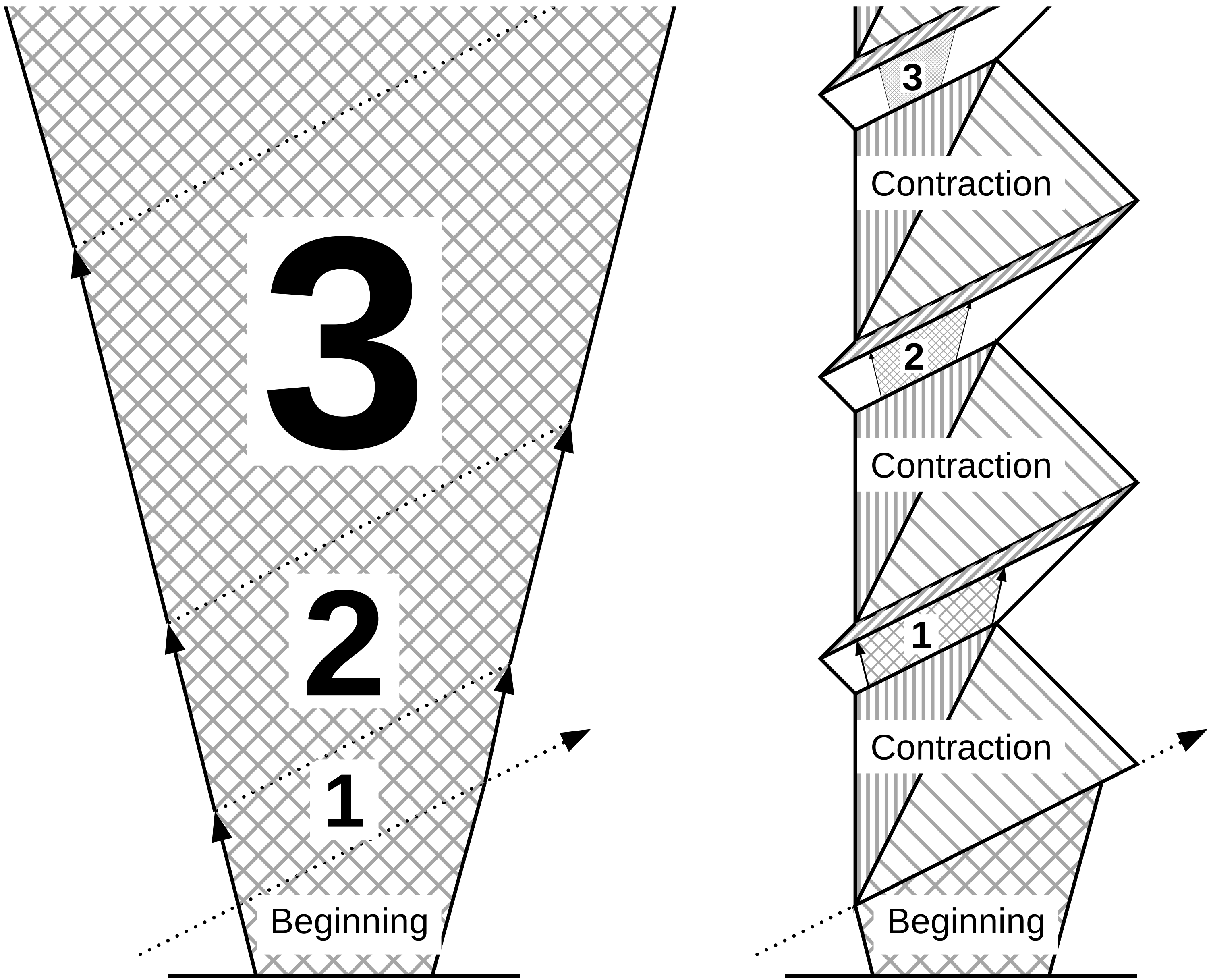
Contraction



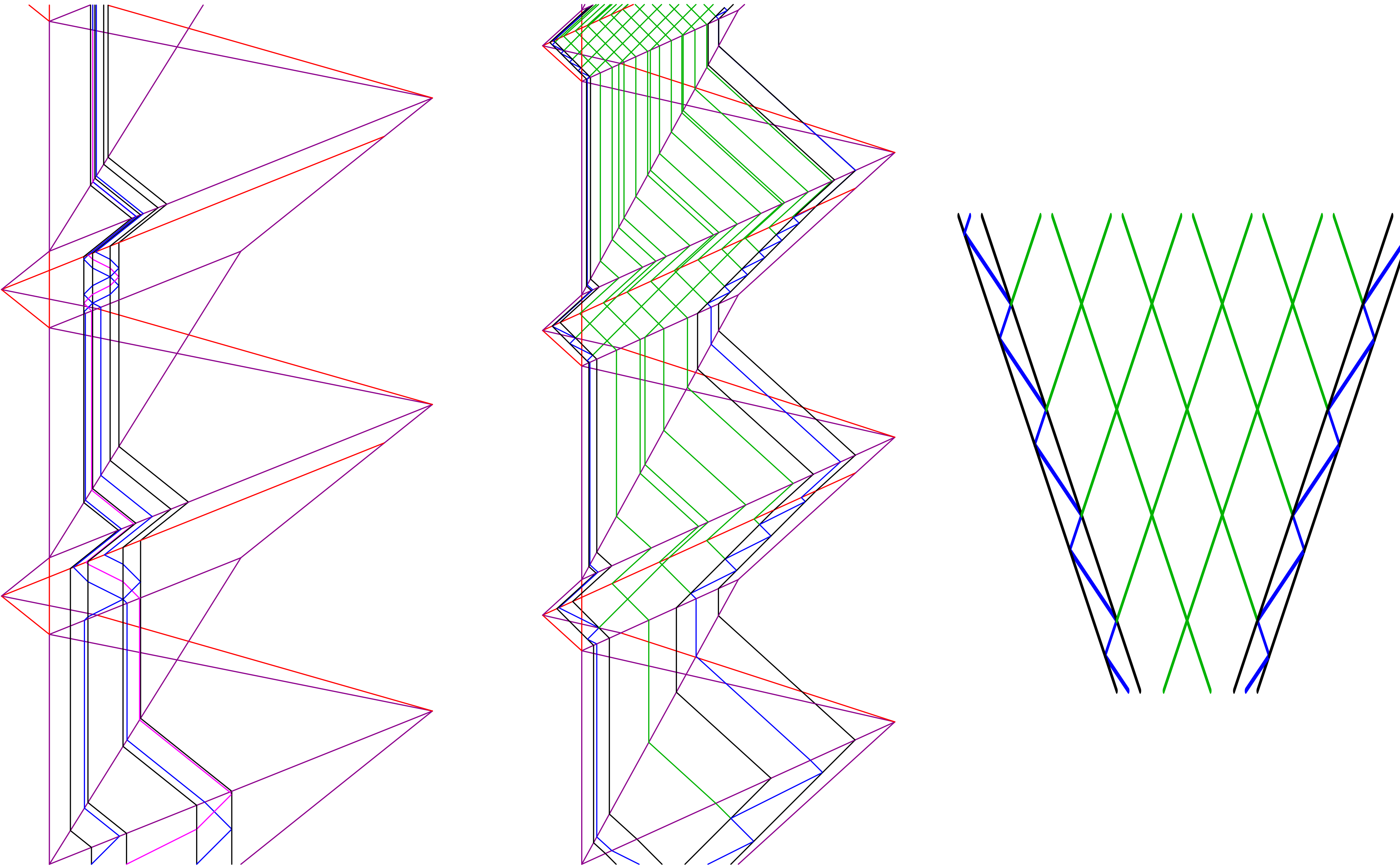
Contraction



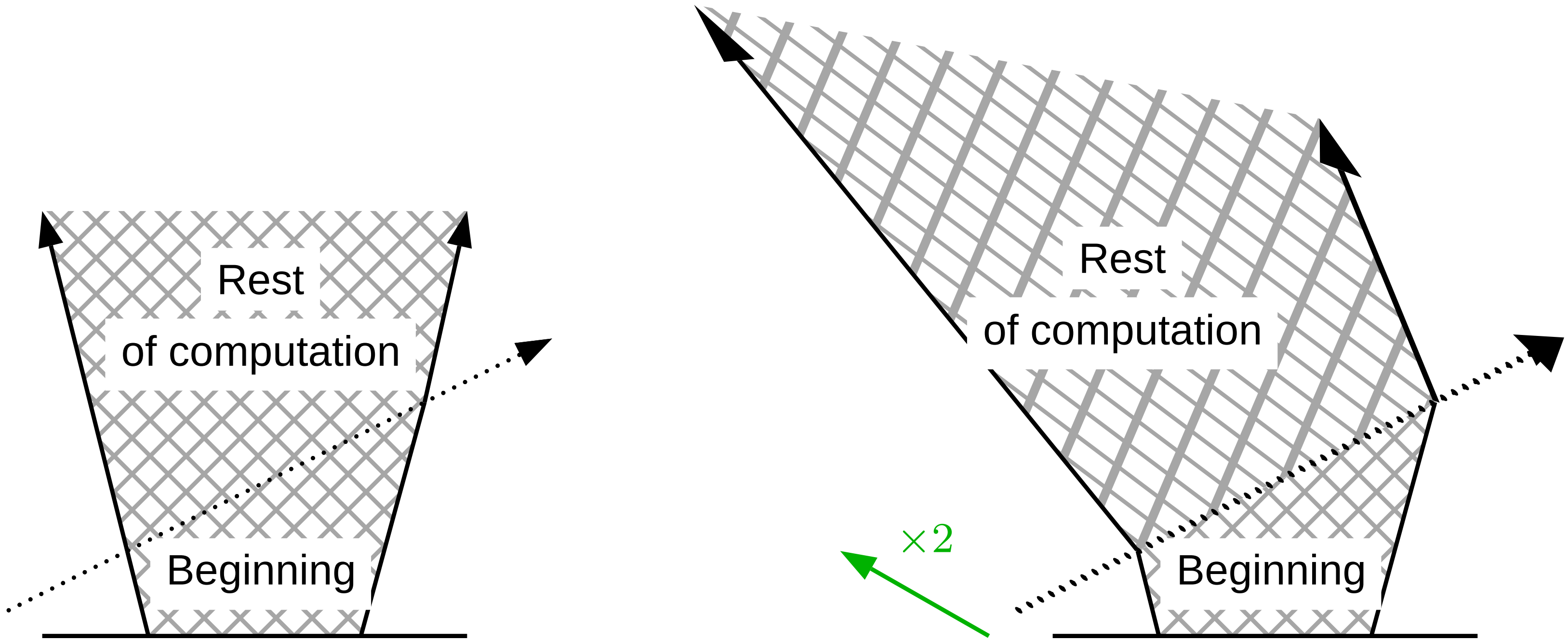
Contraction to a ribbon



Contraction to a ribbon

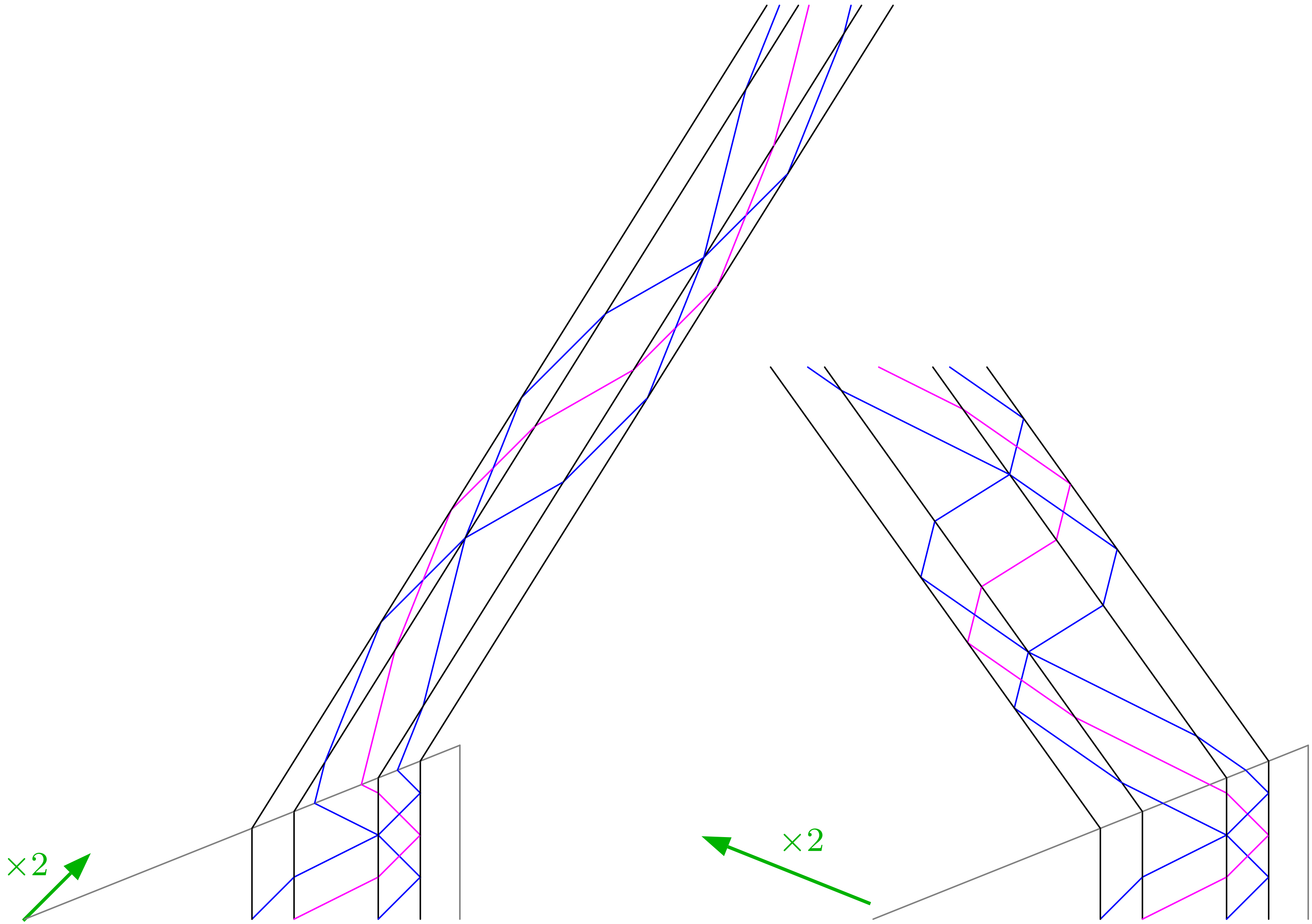


Distortion

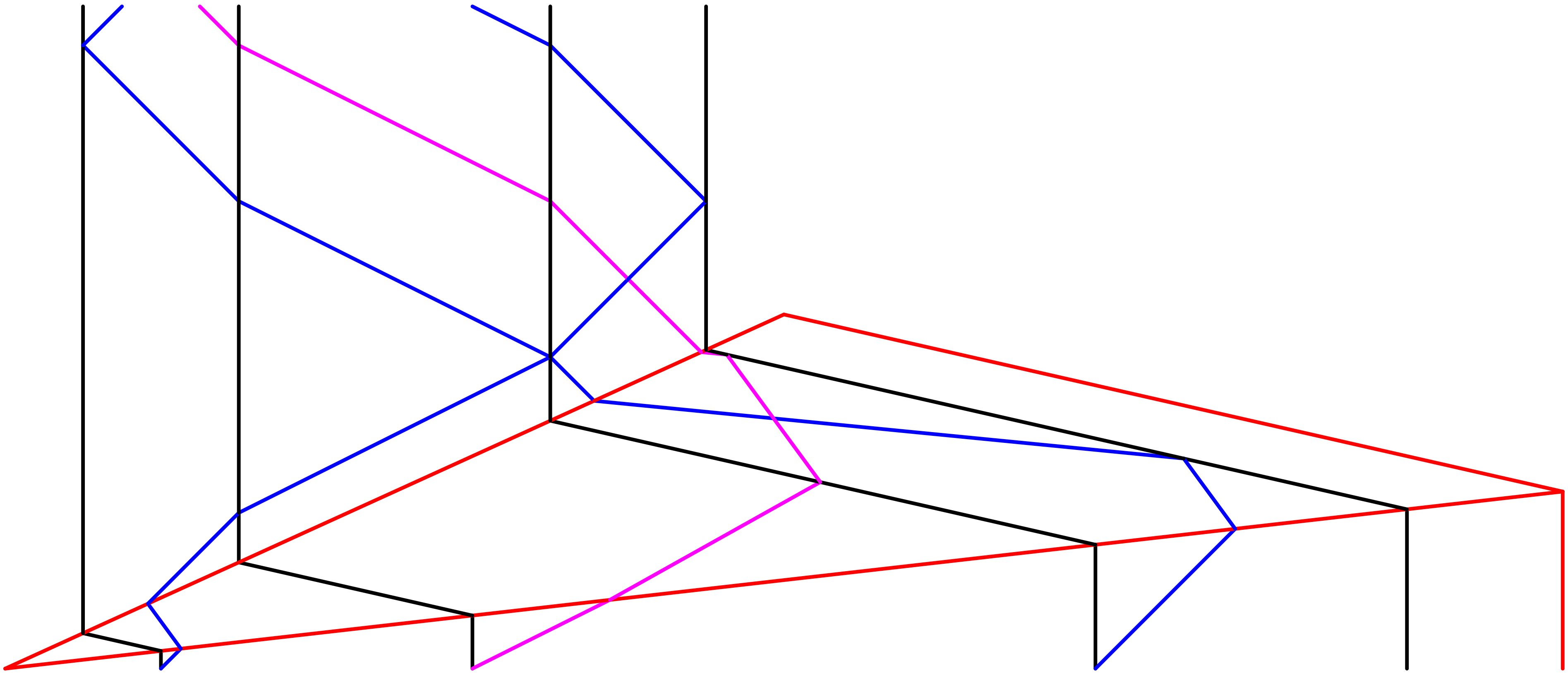


Rest
of computation
Beginning
×2
Rest
of computation
Beginning

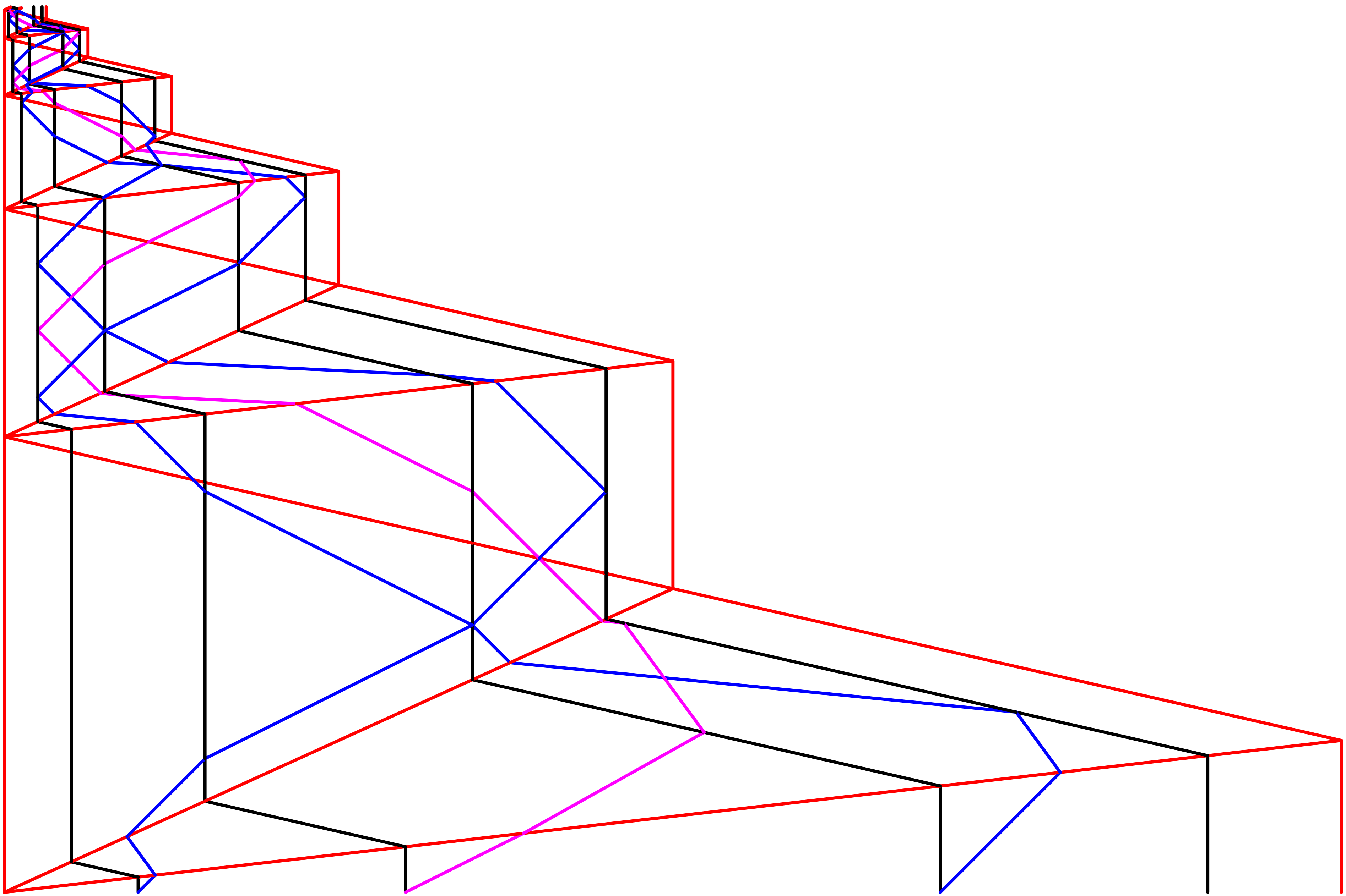
Distortion



Continuous contraction



Contraction to a triangle



Any computation can be embedded in a triangle

Great malleableness of space-time

Problem generation of an accumulation point

Accumulations

Undecidability of accumulation prevision

Instance

$\mathcal{M}$: signal machine,
c_0: finite initial configuration,
(all values in $\mathbb{Q}$)

Question

Will there be any accumulation?

$\exists (x,t) \in \mathbb{Z}\times\mathbb{N},\ \forall n \in \mathbb{N},$ There is at least n collisions in the light cone ending in (x,t)

Undecidability of accumulation prevision

Instance

$\mathcal{M}$: signal machine,
c_0: finite initial configuration,
(all values in $\mathbb{Q}$*)*

Question

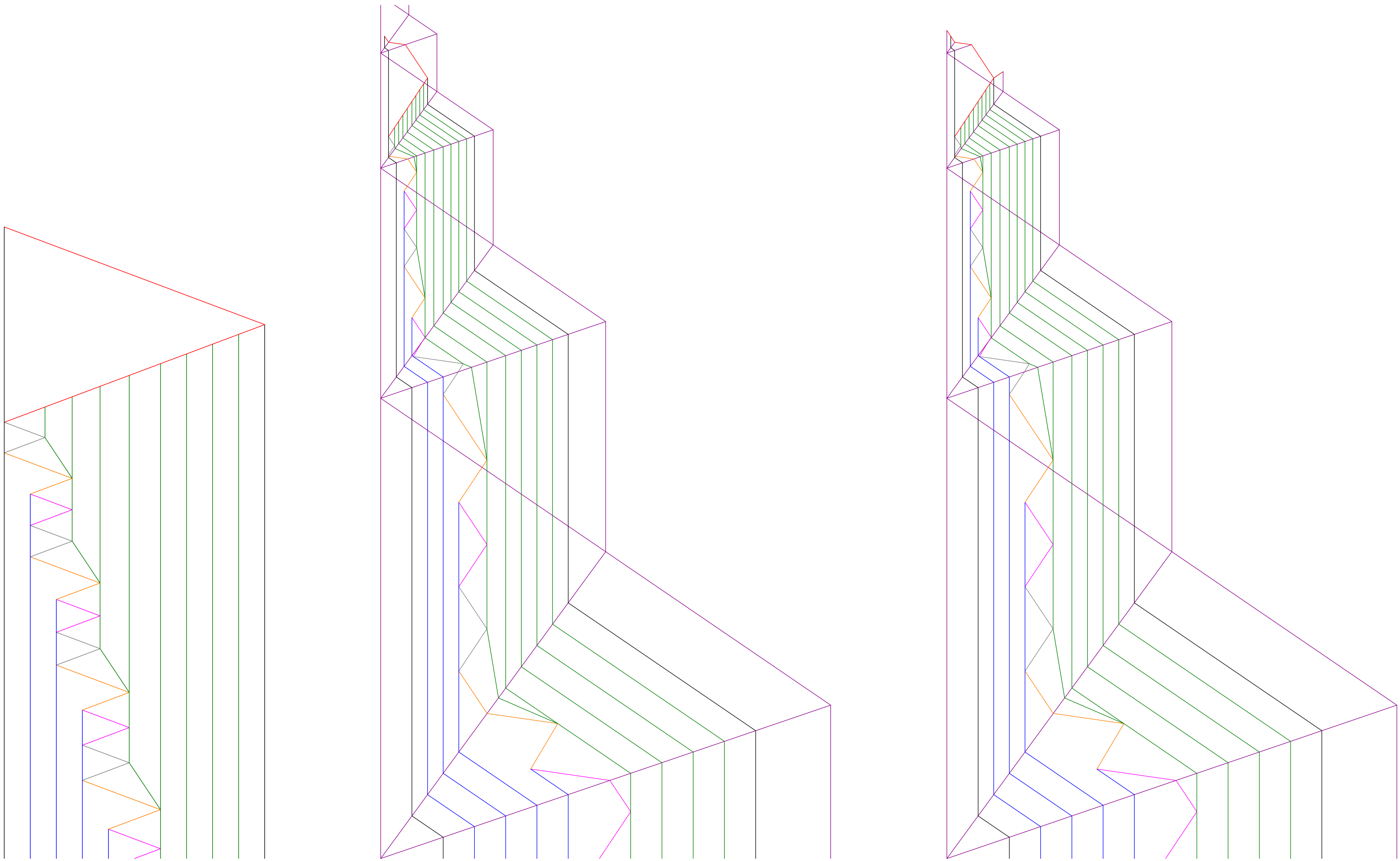
Will there be any accumulation?

recursive
total predicate

$\exists (x,t) \in \mathbb{Z} \times \mathbb{N},\ \forall n \in \mathbb{N},$ There is at least n collisions in the light cone ending in (x,t)

$\rightsquigarrow$ in Σ_2^0 (arithmetical hierarchy)

Reduction for Σ_1^0-hardness



Accumulation $\Leftrightarrow$ the 2 counter automaton does not stop

Placing all possible computations

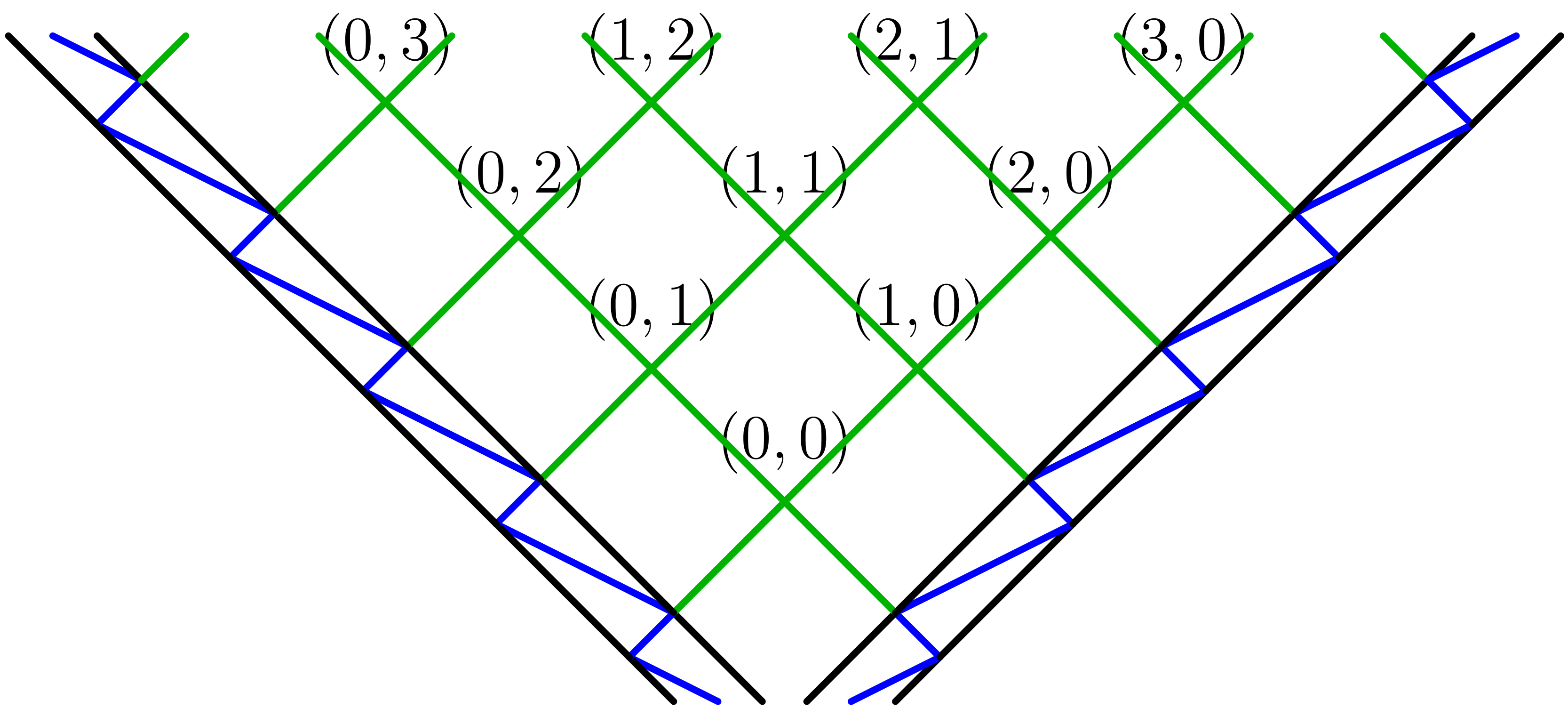
Σ_2^0-complete

Instance

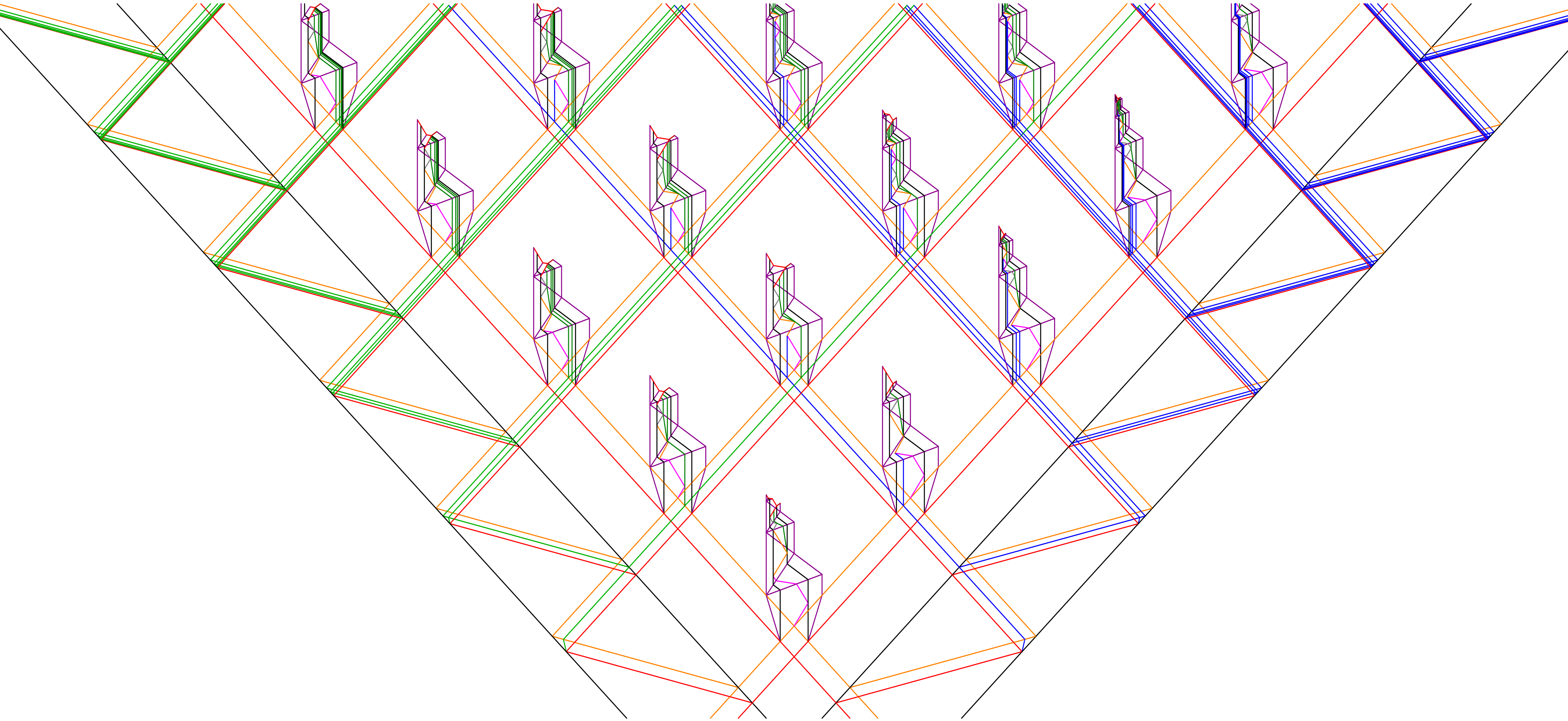
$\mathcal{A}$: 2-counter automata,

Question

Does the computation finished for all initials values?



Σ_2^0-hardness



$\rightsquigarrow$ Σ_2^0-complete

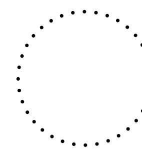
Topological reformulation

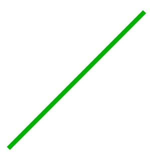
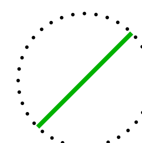
Space-time diagram

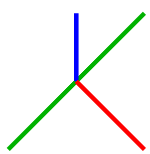
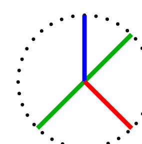
$$\mathbb{R} \times \mathbb{R}_{>0} \stackrel{\mathbb{D}}{\longmapsto} \{\oslash\} \cup \{\mu_i\}_i \cup \{\rho_j\}_j \cup \mathcal{S}$$

void → $\oslash$; singularities → $\mathcal{S}$

Conditions

$\mathbb{D}(p) = \oslash \iff \exists r > 0,\ \mathbb{D}(ball(p,r)) =$ 

$\mathbb{D}(p) =$  $\iff \exists r > 0,\ \mathbb{D}(ball(p,r)) =$ 

$\mathbb{D}(p) =$  $\iff \exists r > 0,\ \mathbb{D}(ball(p,r)) =$ 

$|\mathcal{S}| < \infty$ and $\lozenge \in \mathcal{S}$

Matches machine approach definition

Accumulated values

$\forall p \in \mathbb{R} \times \mathbb{R}_{>0}$

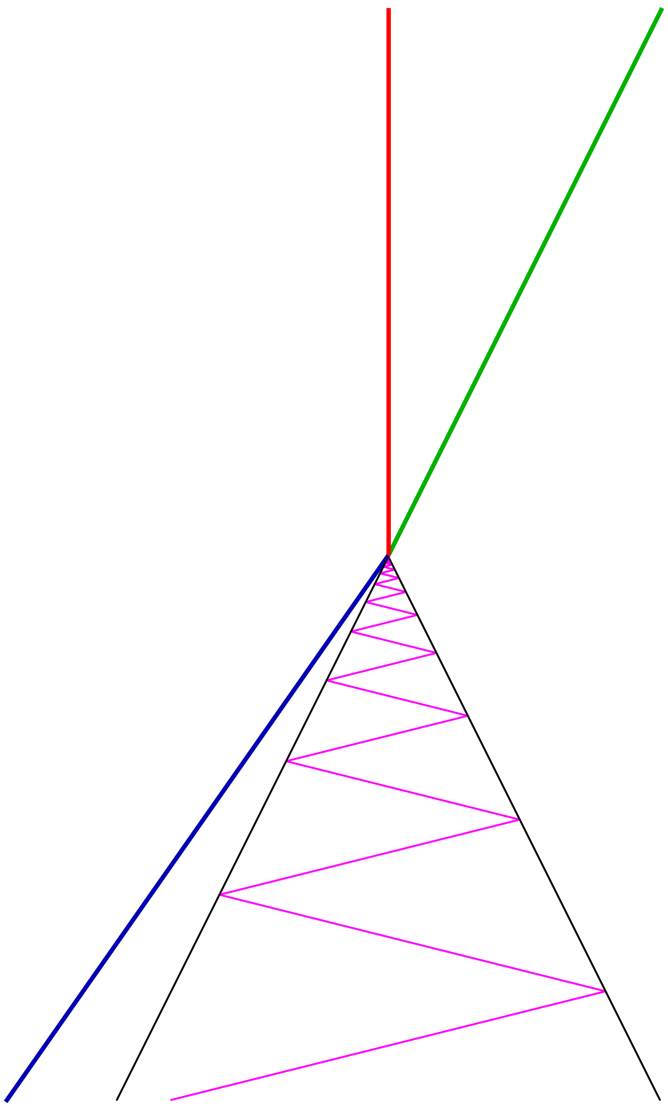
$\mathrm{AccVal}(p)$ set of values accumulating around p

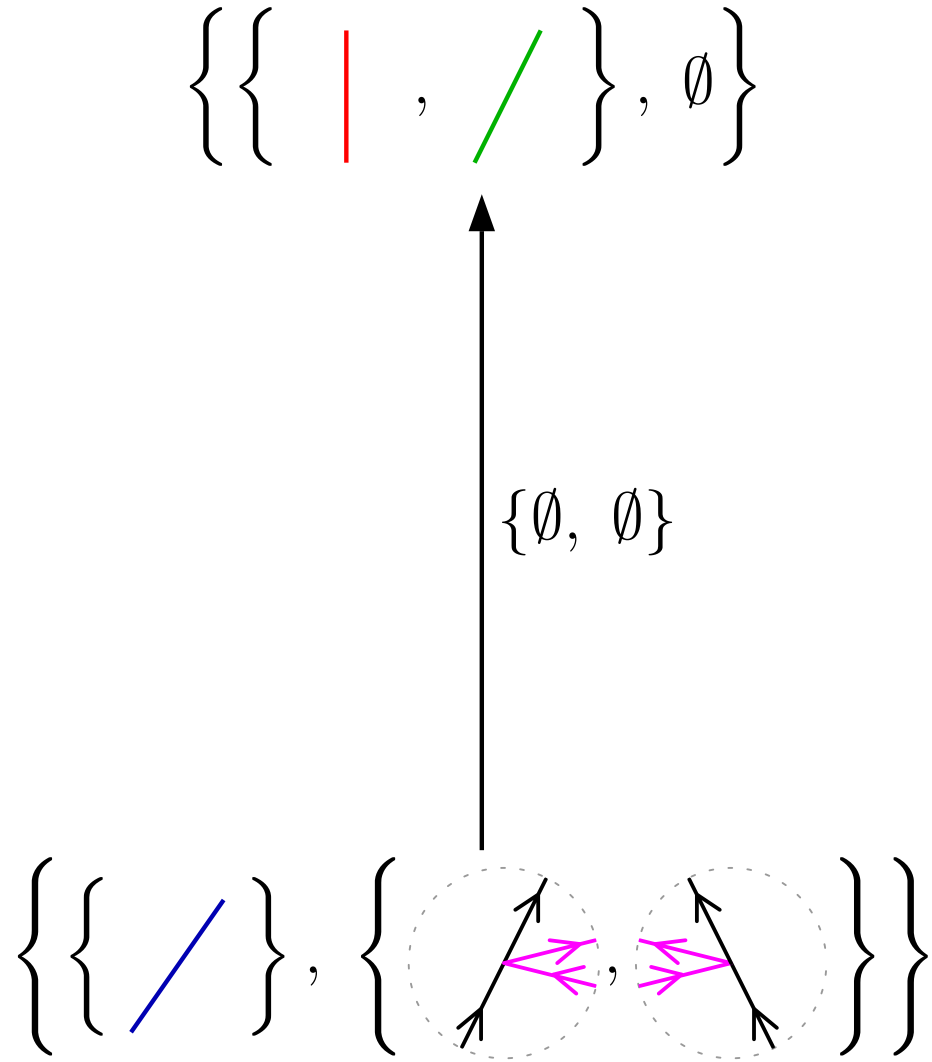
Singularity $\iff$ $\mathrm{AccVal}(p) \cap \{\{\rho_j\}_j \cup \mathcal{S}\} \neq \emptyset$

Isolated $\iff$ $\mathrm{AccVal}(p) \cap \mathcal{S} = \emptyset$

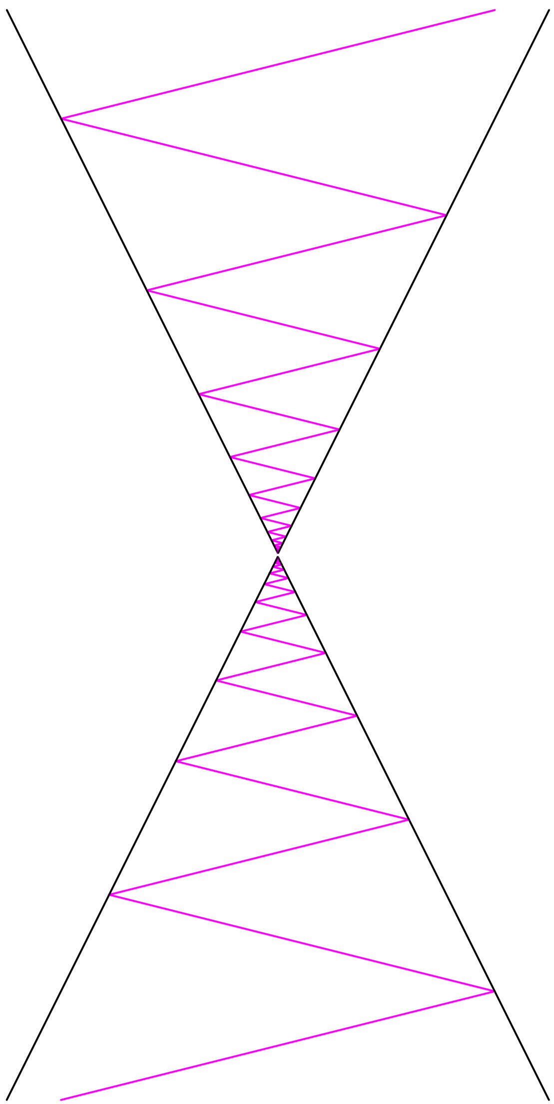
Closer look at isolated singularities

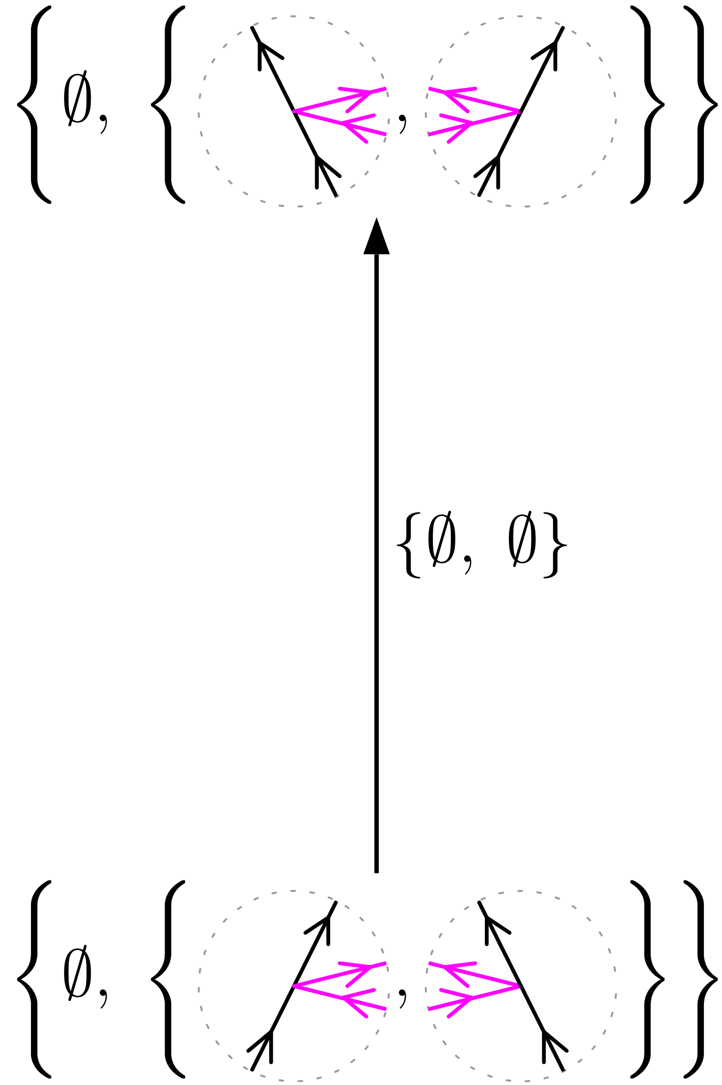
Meta-singularities



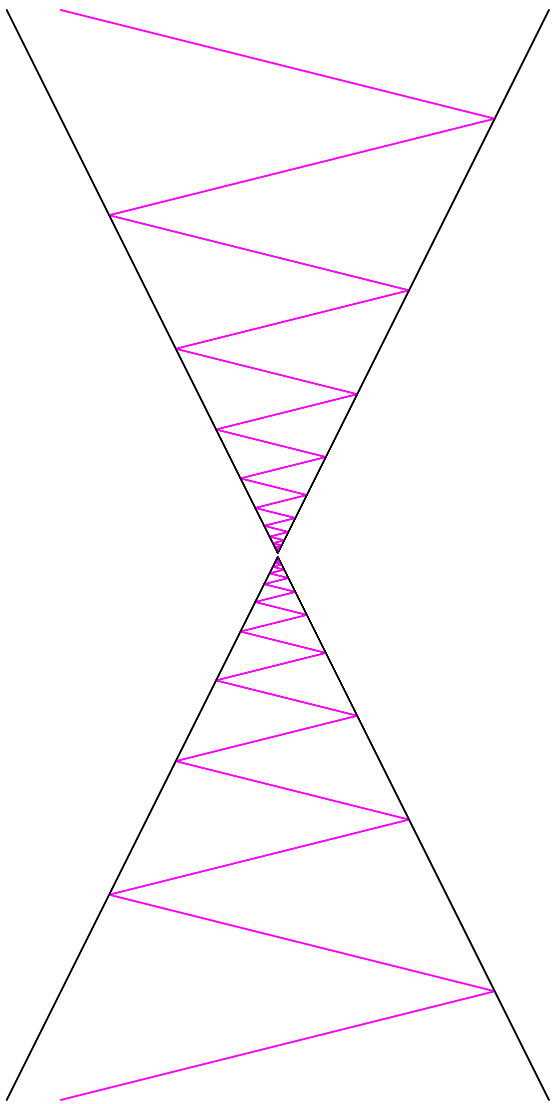


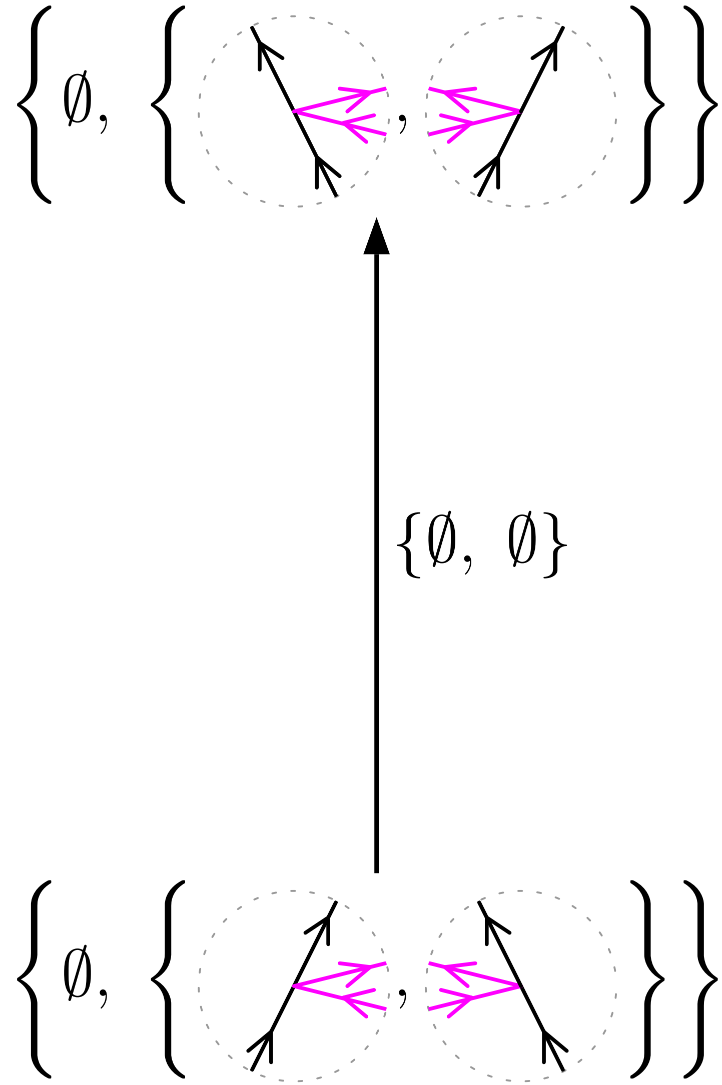
Non deterministic meta-singularities



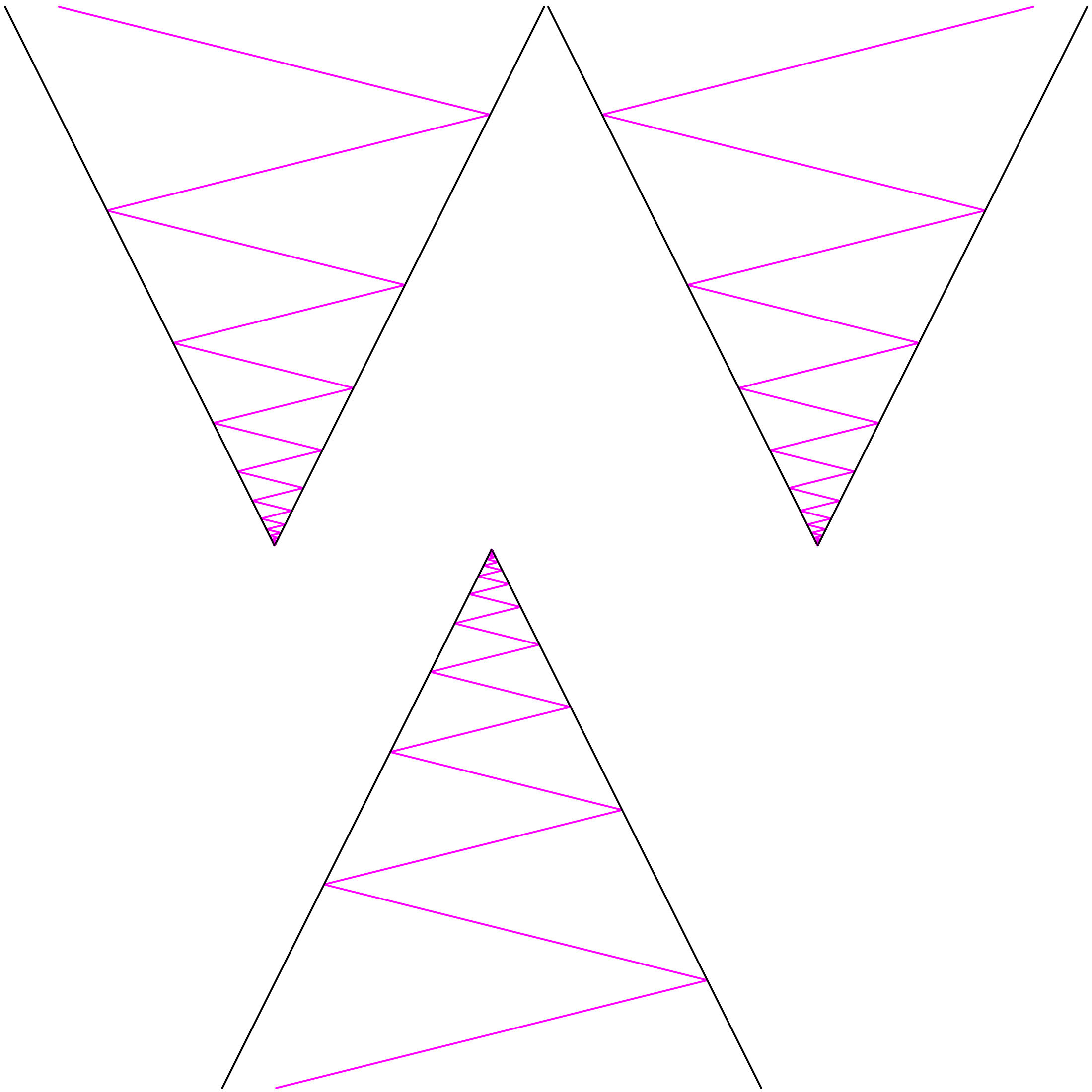


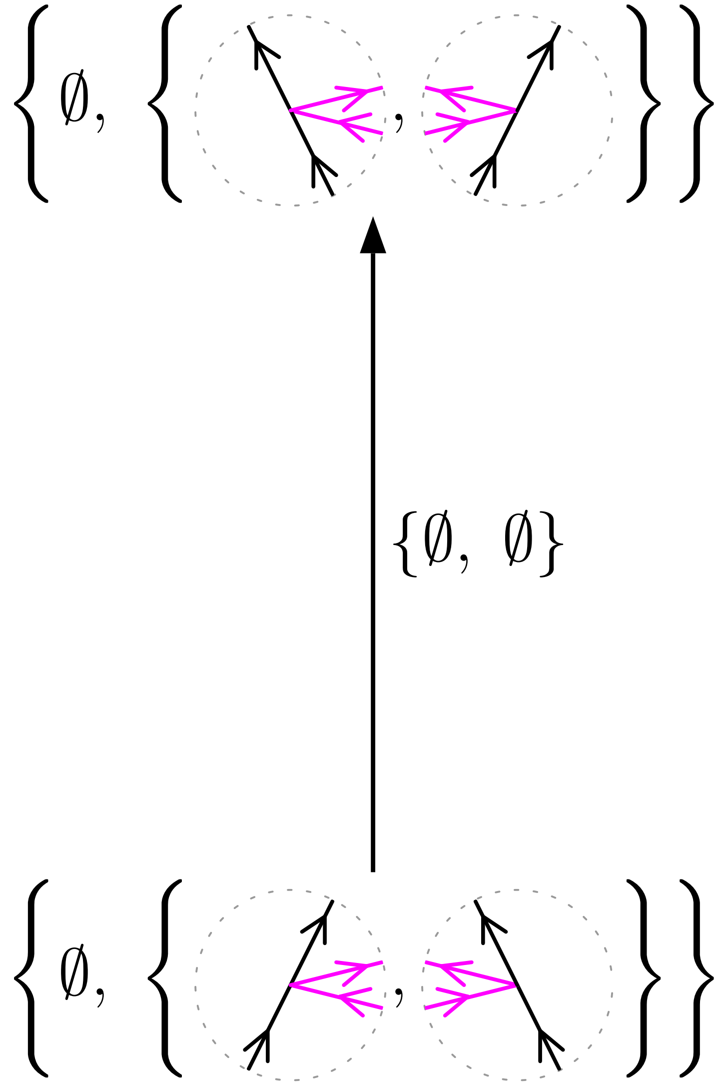
Non deterministic meta-singularities



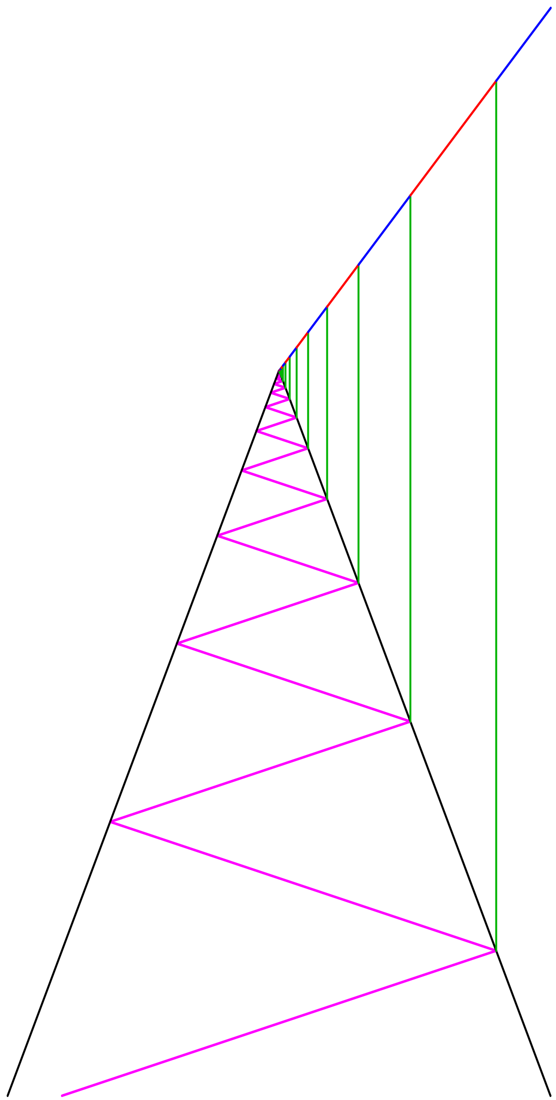


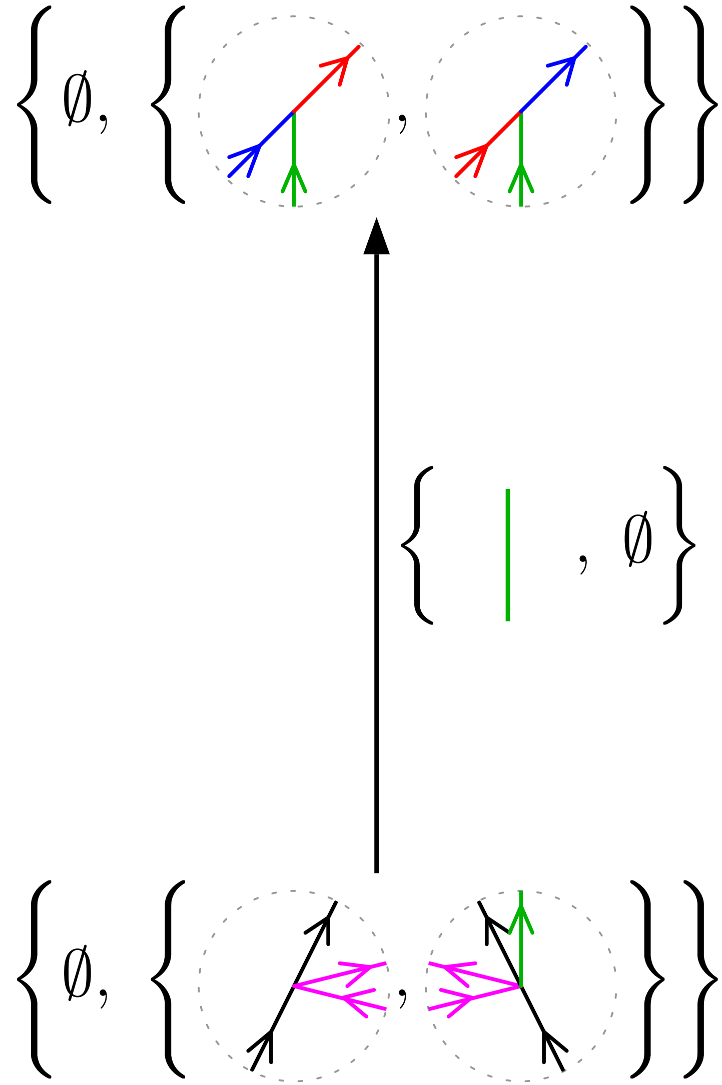
Non deterministic meta-singularities



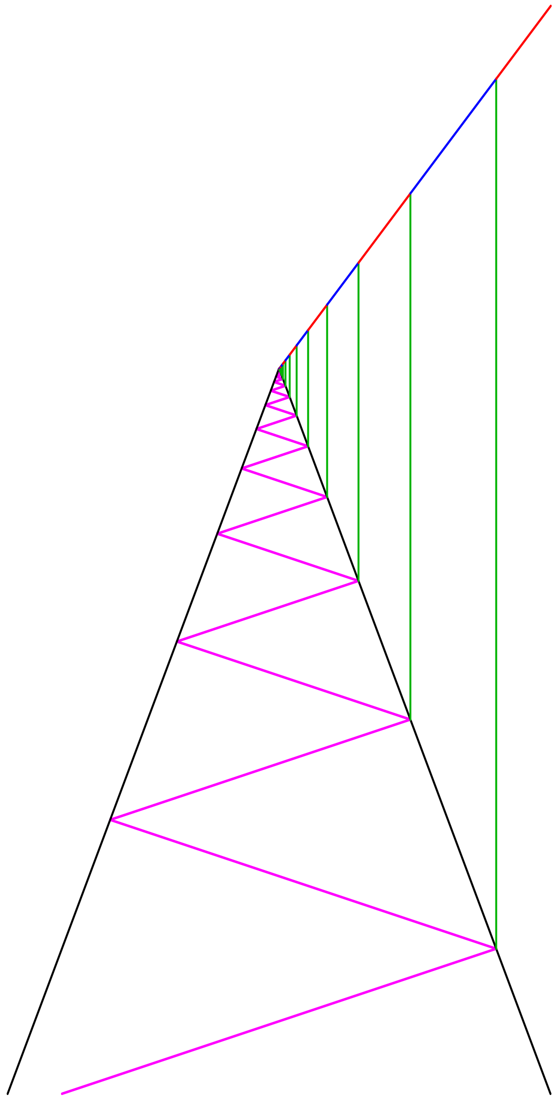


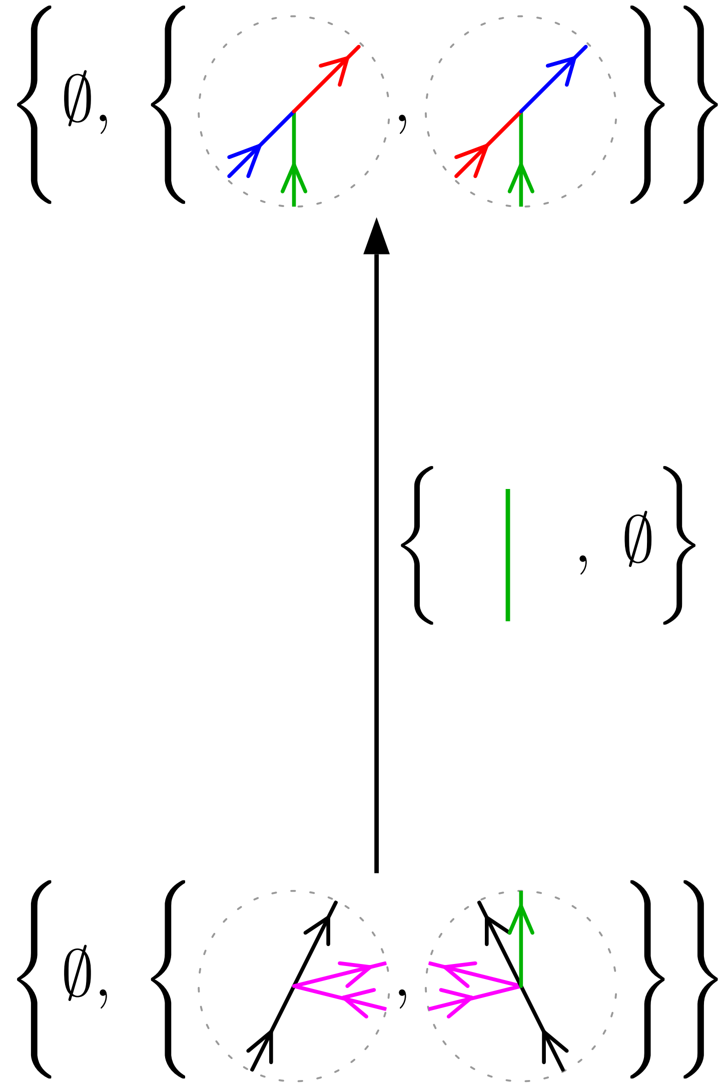
Non deterministic meta-singularities



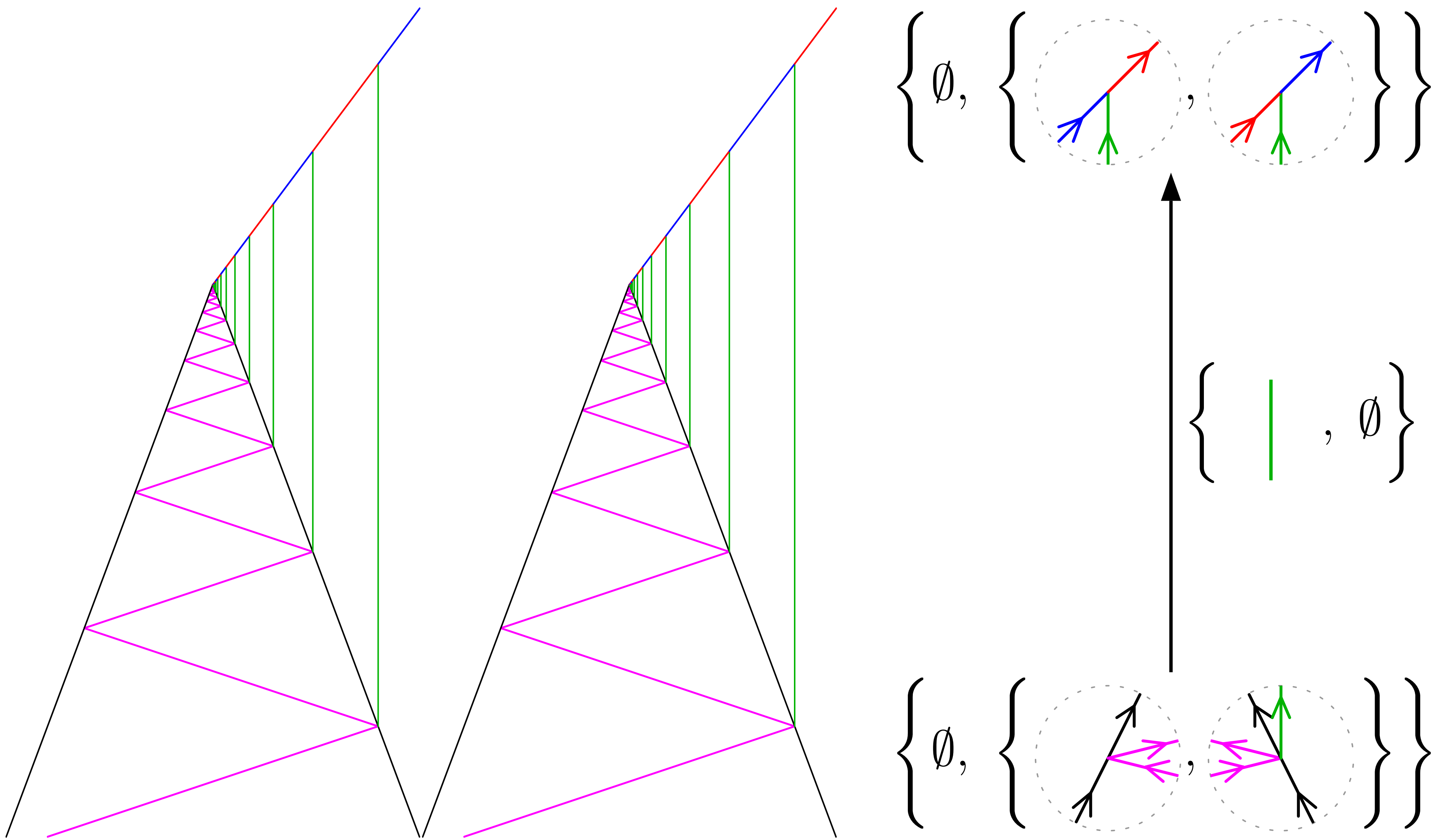


Non deterministic meta-singularities

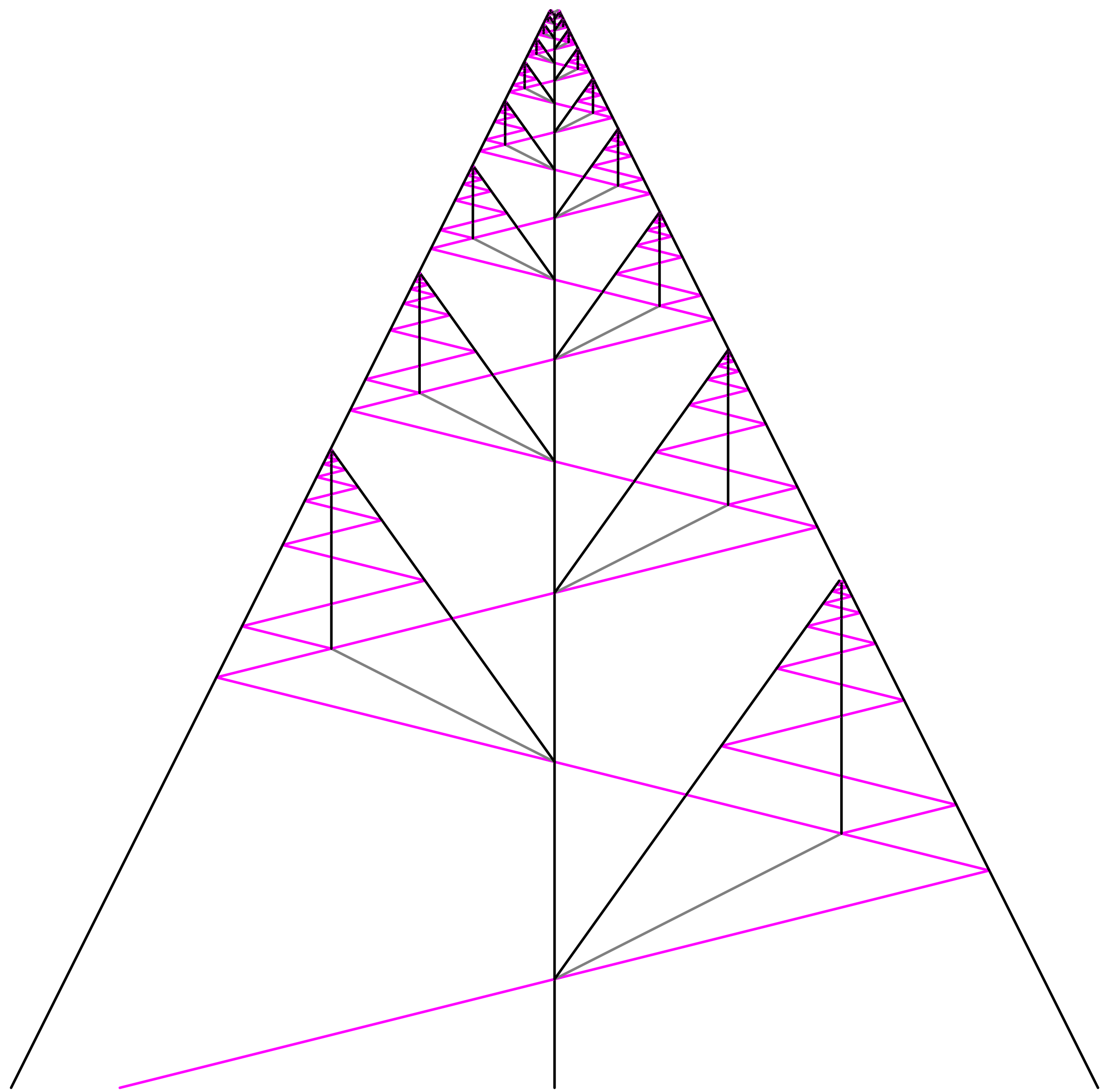




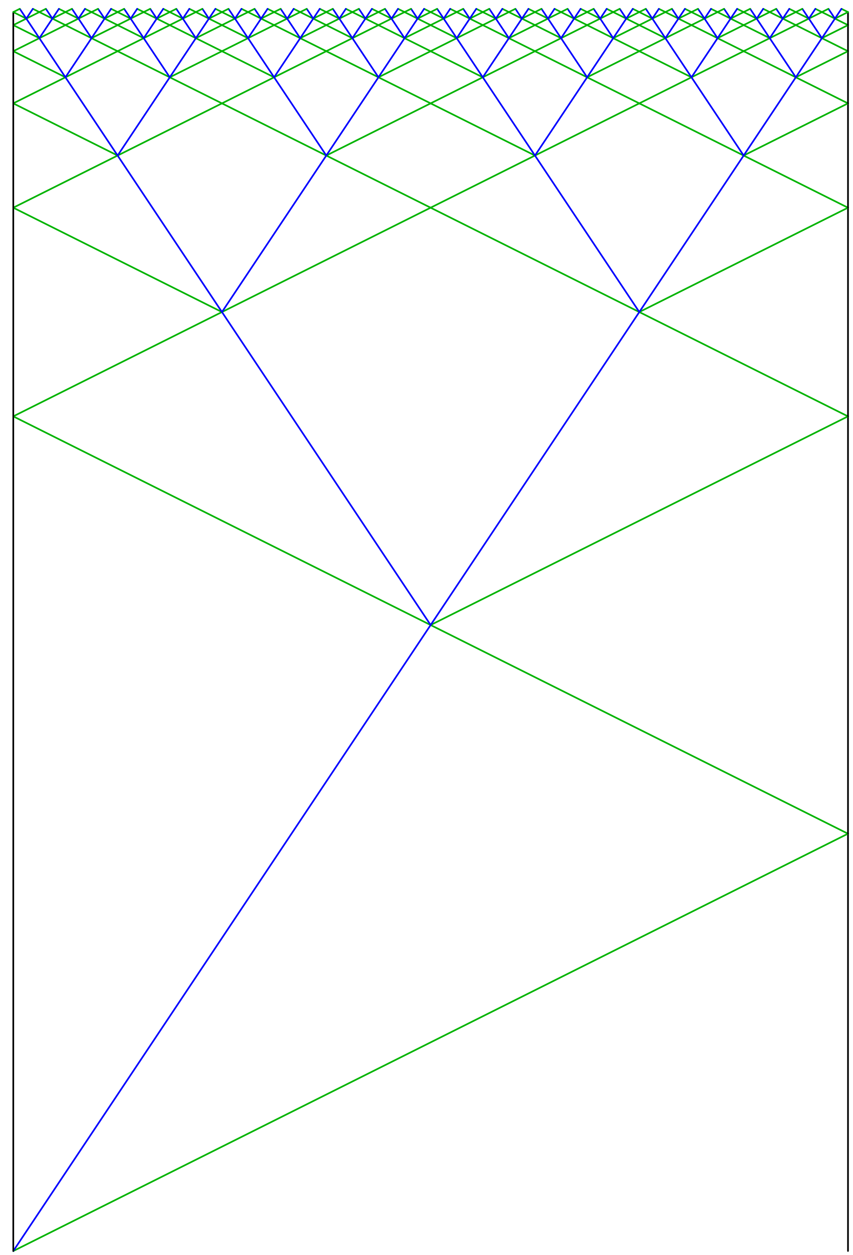
Non deterministic meta-singularities



Second order

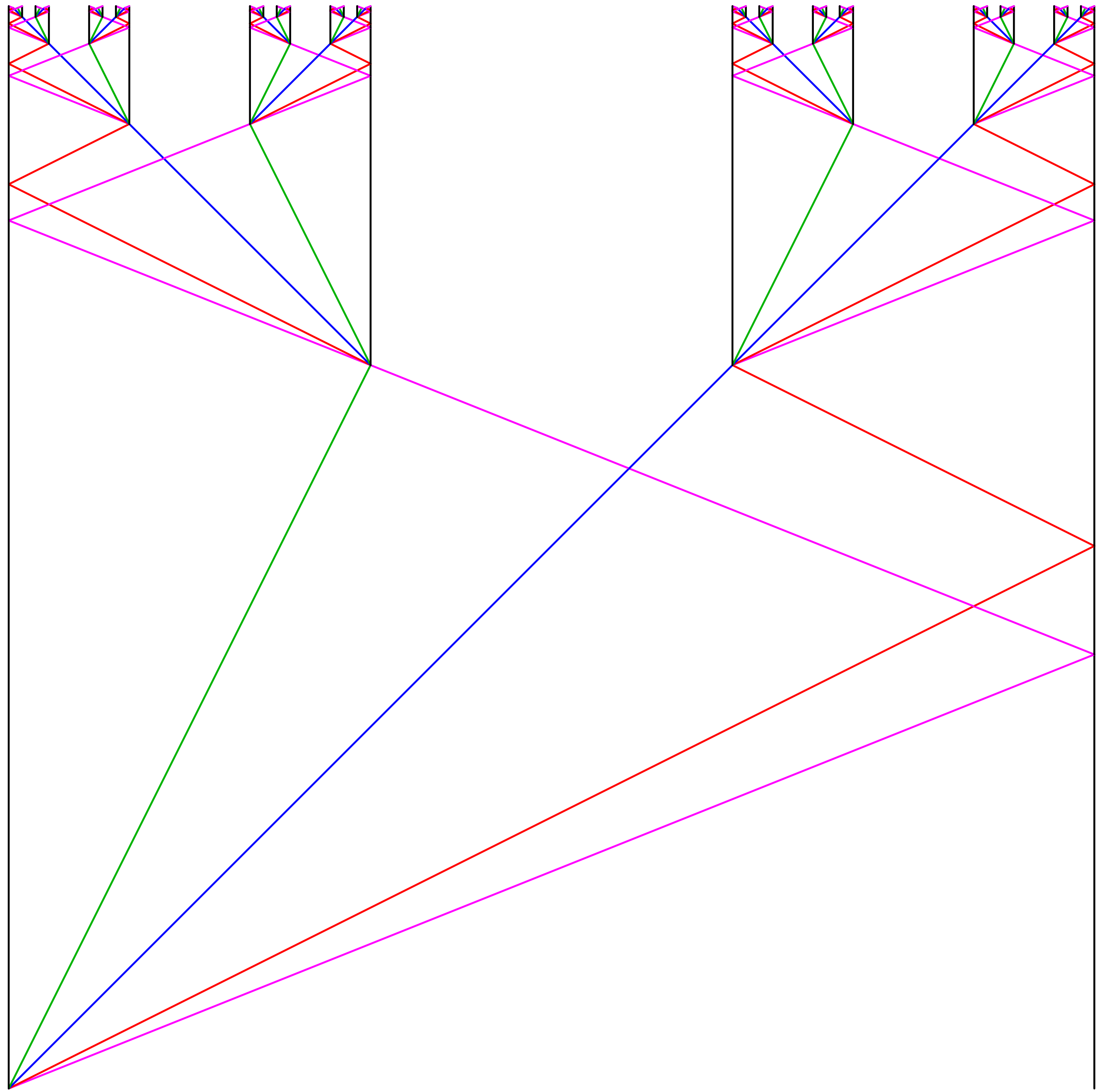


Non isolated singularities



Non isolated singularities

Cantor



Conclusion

- Geometrical computation model
- Turing-computability
- Continuous space and time aspects
- Great malleableness of space-time
- Accumulations
  - Decidability
  - Partial treatment

Perspectives – on the model

- Singularities
  - Comprehension
  - Utilization (super-Turing capability)

- Algorithmic principles
  - Complexity
  - Intrinsic universality

Perspectives – relating the model

- Link to other model
  - Understand continuous models
  - Continuous classes of complexity and decidability
- Cellular automata
  - Automatic discretization
  - Transfer theorem
    - Validate proofs

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