Computing with irrational 3-speed signal machines

MCU 2015 — Informal talk

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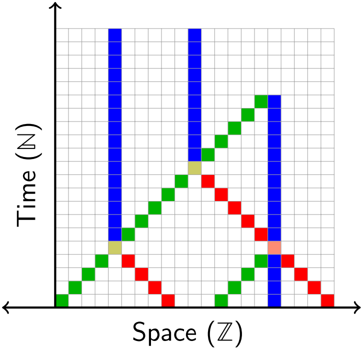
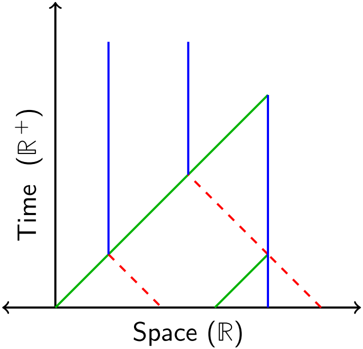
September 2015 — Famagusta, North Cyprus

1. Signal machines
2. Problematic: minimality for Turing capability
3. Known cases
4. Generalisation
   - Any irational ratio betwen distances at last position
   - Any irational ratio betwen distances
   - Any irrational ratio between speeds
5. Results and future work

Signals in cellular automata





- Signal (meta-signal)
- Collision (rule)

Vocabulary and example: finding the middle

Meta-signals (speed)

M (0)

Collision rules

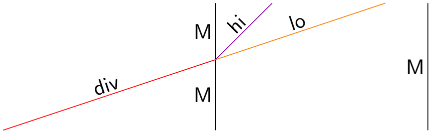
Vocabulary and example: finding the middle

Meta-signals (speed)

M	(0)
div	(3)

Collision rules

Vocabulary and example: finding the middle



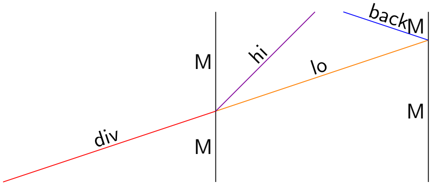
Meta-signals (speed)

M	(0)
div	(3)
hi	(1)
lo	(3)

Collision rules

{ div, M } → { M, hi, lo }

Vocabulary and example: finding the middle



Meta-signals (speed)

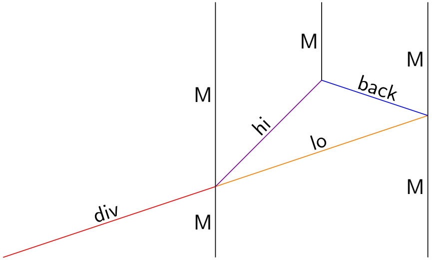
M	(0)
div	(3)
hi	(1)
lo	(3)
back	(-3)

Collision rules

$\{$ div, M $\} \rightarrow \{$ M, hi, lo $\}$

$\{$ lo, M $\} \rightarrow \{$ back, M $\}$

Vocabulary and example: finding the middle



Meta-signals (speed)

M	(0)
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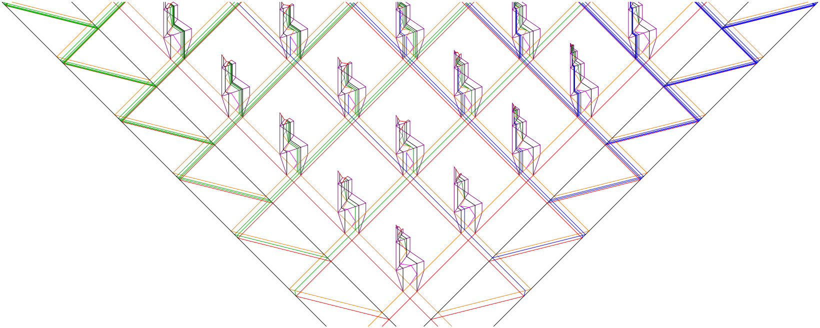
Collision rules

$\{$ div, M $\} \rightarrow \{$ M, hi, lo $\}$

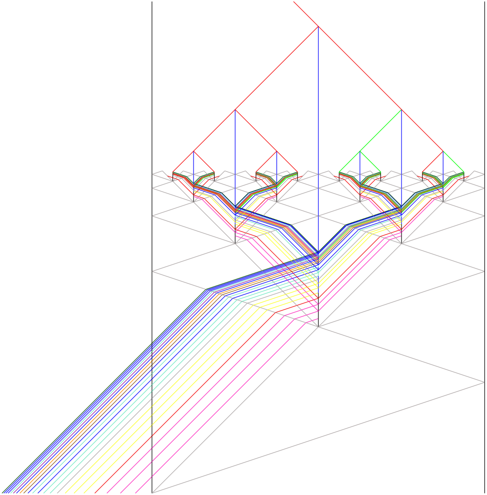
$\{$ lo, M $\} \rightarrow \{$ back, M $\}$

$\{$ hi, back $\} \rightarrow \{$ M $\}$

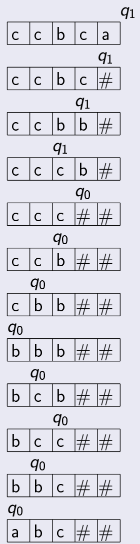
Complex behavior

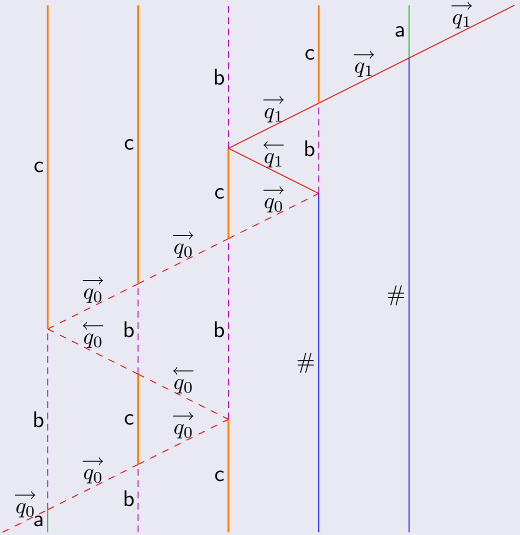


Complex behavior



Simulating a Turing machine on a finite tape





Minimality — bounding the number of...

Meta-signals and collision rules

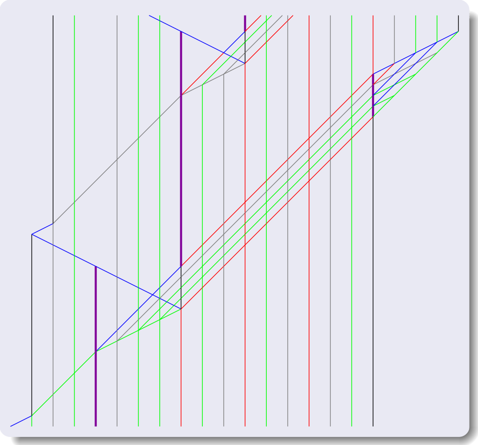
- 13 meta-signals
  (21 collision rules)
  Cyclic tag system
  [Durand-Lose, 2011]

3 Speeds rational

- Impossible
  [Durand-Lose, 2013]

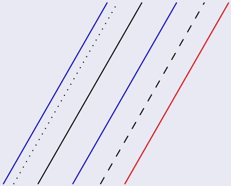
3 Speeds with irrationality

- Possible
  [Durand-Lose, 2013]
- Always possible
  (generic meta-signal, rules and initial configuration)
  this talk



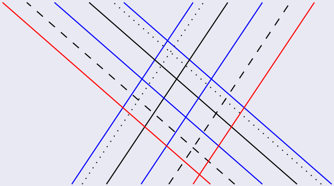
1 or 2 speeds

1 speed



No collision at all

2 speeds



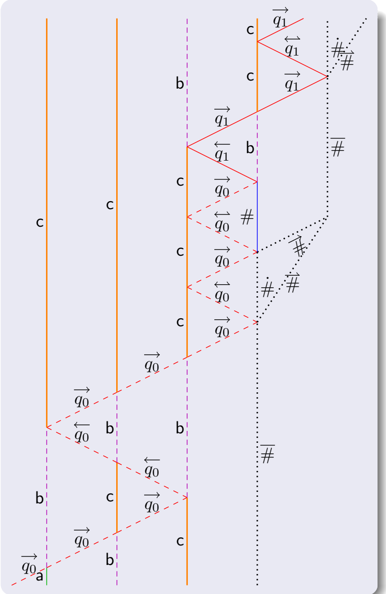
Bounded number of collisions

- Not Turing-universal

4 speeds

Turing-universal

- TM simulation
- Fourth speed to enlarge the tape



3 speed rational case

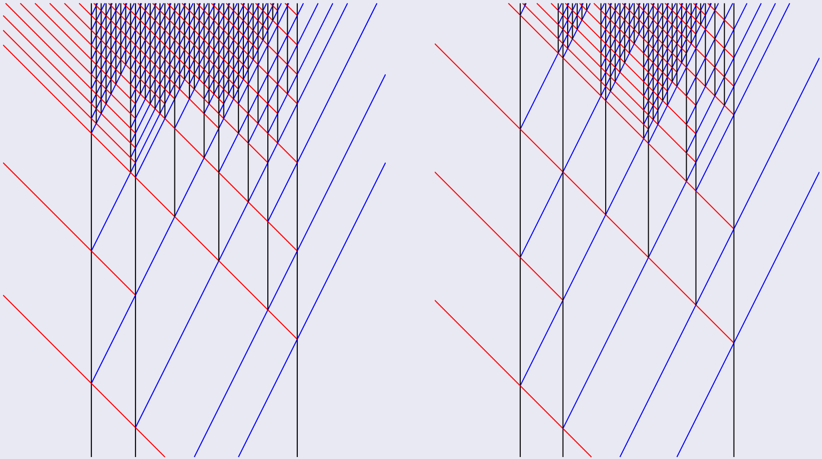
- Rational speeds ($\in \mathbb{Q}$)
- Rational initial positions

$\rightsquigarrow$ Collisions at rational positions
as the solution of systems of two rational linear equations

Implemented in Java

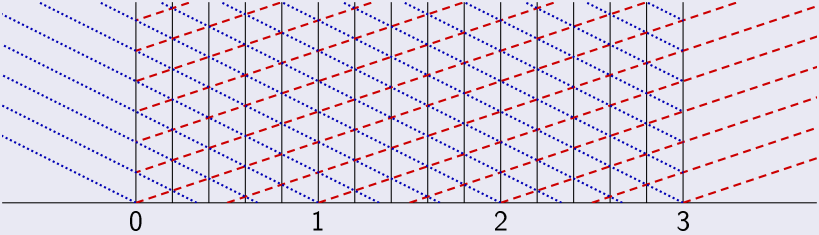
- Exact precision (on $\mathbb{Q}$)
- Tons of space-time diagrams

Rational space-time diagrams



Embedded in a mesh [Becker et al., 2013]

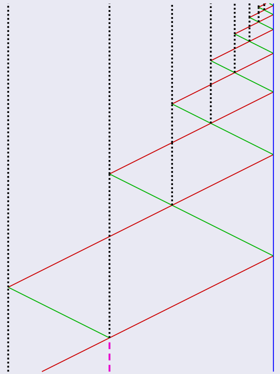
Mesh



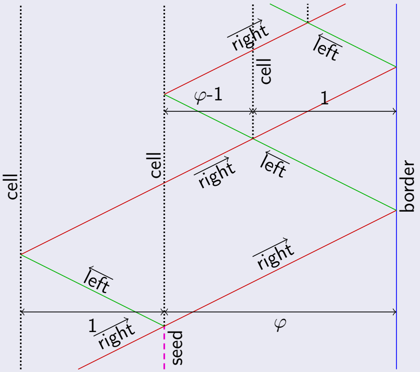
- No computation

Simple fractal construction [Becker et al., 2013]

Fractal



Fractal construction

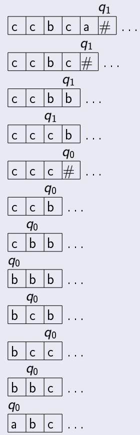


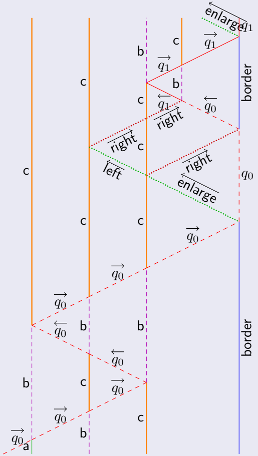
Irrational initial positions $(-1, -0.6, 0, \varphi)$, *rational* speeds $(-2, 0, 2)$

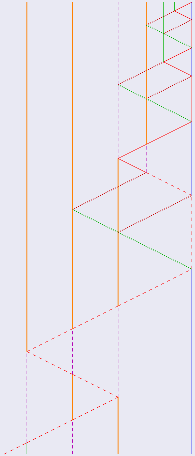
φ must satisfy $\frac{\varphi}{1} = \frac{1}{\varphi - 1}$ φ is the *Golden ratio*

How to enlarge the tape?

- Use the fractal... without generating it!







Final situation

$q_0 \qquad \overleftarrow{\mathsf{enl}_{ok}} \qquad \overline{\#} \xleftrightarrow{\alpha'} \mathsf{bord}_1^R \xleftrightarrow{\beta'} \mathsf{bord}_2^R \qquad \text{with } \frac{\alpha'}{\beta'} \notin \mathbb{Q}$

Initial situation

$\overrightarrow{q_0} \qquad \overline{\#} \xleftrightarrow{\alpha} \mathsf{bord}_1^R \xleftrightarrow{\beta} \mathsf{bord}_2^R \qquad \text{with } \frac{\alpha}{\beta} \notin \mathbb{Q}$

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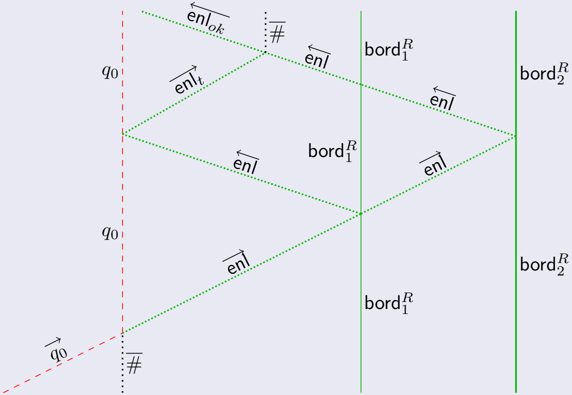

Initial situation

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Final situation

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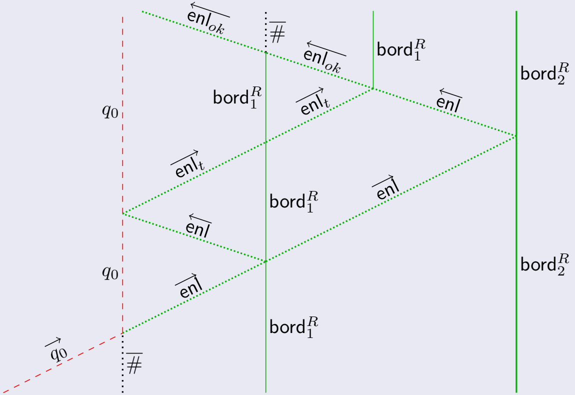

Initial situation

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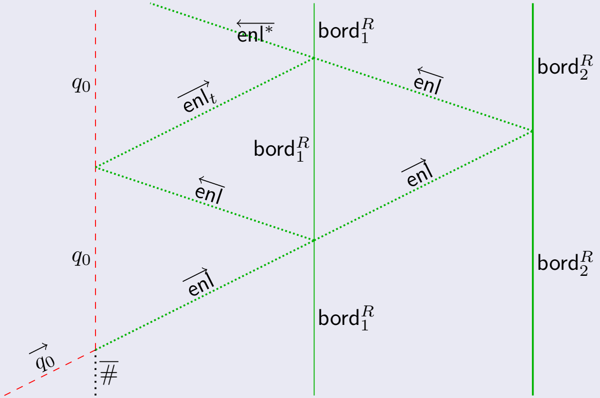
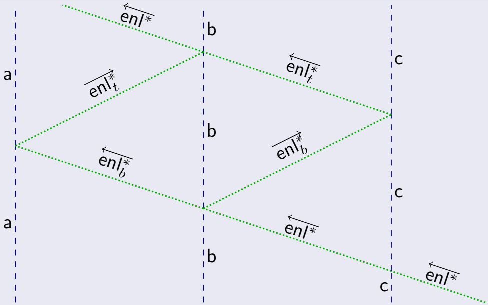
Previous stategy works unless



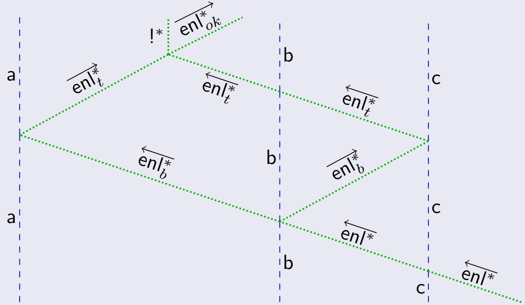
3-signal collision identifies the case

$\rightsquigarrow$ try to find a ratio different from 1 on the left

Not found yet

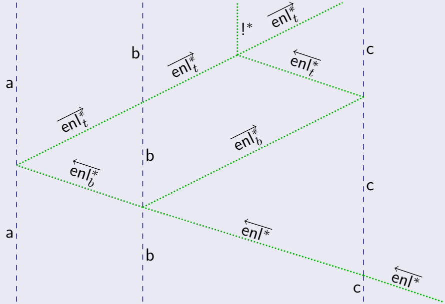


Found



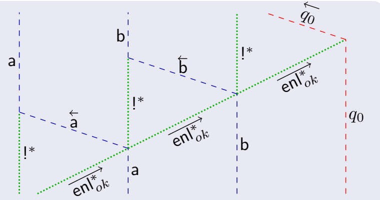
A new vertical signal (!*) is generated

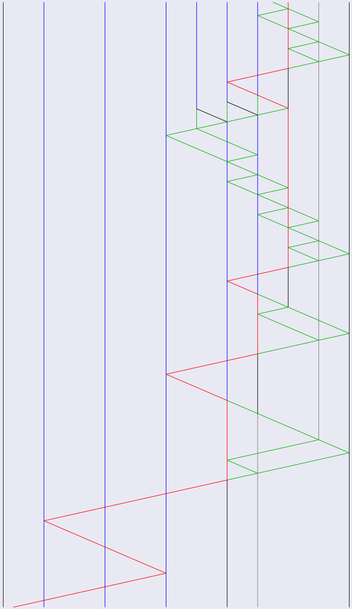
Found



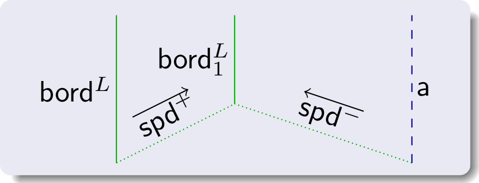
A new vertical signal (!*) is generated

Cells
have to be
shifted





Transformed into irrational ratio between distances



$$\text{bord}^L \;@\; 0$$
$$\text{a} \;@\; 1$$
$$S\left(\overrightarrow{\text{spd}^+}\right) = s^+$$
$$S\left(\overleftarrow{\text{spd}^-}\right) = \text{-}s^-$$

$$\text{bord}_1^L \;@\; \frac{1}{1+\frac{s^+}{s^-}}$$

Results

Rational signal machines (Up to *normalization*)

- 4 speeds are needed and enough to compute

3 speeds: possible to compute with either

- any 2 speeds with irrational ratio, or
- initial positions with irrational ratio between distances

Unique machinery

- collision rules and
- ordered signals in the initial configuration

Future work

Speed order unknown

- superposition
- identifying the middle speed and form barriers

- Use irrational values as oracle

- Black hole (hyper-)computation

- Analog computation?

Becker, F., Chapelle, M., Durand-Lose, J., Levorato, V., and Senot, M. (2013).

Abstract geometrical computation 8: Small machines, accumulations & rationality.

Submitted.

Durand-Lose, J. (2011).

Abstract geometrical computation 4: small Turing universal signal machines.

Theoret. Comp. Sci., 412:57–67.

Durand-Lose, J. (2013).

Irrationality is needed to compute with signal machines with only three speeds.

In Bonizzoni, P., Brattka, V., and Löwe, B., editors, *CiE '13, The Nature of Computation*, number 7921 in LNCS, pages 108–119. Springer.

Invited talk for special session *Computation in nature*.