The signal point of view: from cellular automata to signal machines

Jérôme Durand-Lose

Laboratoire d'Informatique Fondamentale d'Orléans, Orléans, FRANCE

Journées Automates Cellulaires 2008 — 21 au 25 avril — Uzès
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2 Implicit use of signals

3 Discrete signals

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Cellular Automata

**Definition**

(do you really need one?)

**Dynamical system**

*Global function, $\mathcal{G} : Q^\mathbb{Z} \rightarrow Q^\mathbb{Z}$*

**Orbit and space-time diagram**

*Value in $Q^{\mathbb{Z} \times \mathbb{N}}$*

*Image with big pixels*
Background and Signals

**Background**

(2-d) Pattern that may form a valid space-time diagram by bi-periodic repetition.

**Signal**

- Pattern that (legally) repeats 1-periodically on a background
- Pattern repeating 1-periodically and separating two backgrounds

Illustration by examples
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Implicit use of signals

Understanding the dynamics

FIG. 7. Rule 54. (a) Annihilation of the radiating particle. (b) The same as (a) with the mapping defined in Fig. 6. [Boccara et al., 1991, Fig. 7]

FIG. 7. The four different (out of 14 possible) interaction products for the $\alpha + \beta$ interaction. [Hordijk et al., 2001, Fig. 7]

Figure 5. Two collisions of filtons, and five free filtons supported by the FPS model; ST diagram applies $q = 1$. [Siwak, 2001, Fig. 5]
Implicit use of signals

Computing by simulating a Turing machine

Figure 4: The $k = 4$, $r = 2$ universal cellular automaton of table 4 simulated starting from a random initial state. The symbols 0, 1, $\omega$, and + are represented by

[Lindgren and Nordahl, 1990, Fig. 4]
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Generating primes

[Fischer, 1965, Fig. 2]
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Firing Squad Synchronization

[Goto, 1966, Fig. 3+6]
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Discrete signals

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Firing Squad Synchronization (again)

[Varshavsky et al., 1970, Fig 1 and 3]
Multiplication

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Figure 1: A human multiplier

One cell out of two computes one time out of two:

\[ C = (\alpha \land \beta) \oplus (A \lor B) \]
\[ D = (\alpha \land \beta \land A) \lor (\alpha \land \beta \land B) \lor (A \land B). \]

Figure 3: Computations done on one cell out of two, one unit of time out of two

[Jack Mazoyer, 1996, Fig. 1, 3 and 4]
A whole programming system

[Figure 8: Computing $|ab|^2$.]

[Figure 9: Setting up an infinite family of regular safe grids (the darkness of the grid indicates its rank).]

[Figure 18: Characterization of the sites $(n, f(n))$.]

[Mazoyer, 1996, Fig. 8 and 19] and [Mazoyer and Terrier, 1999, Fig. 18]
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Moving to the continuum

Forget about discreteness

⇝ continuous
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Signal Machines

Vocabulary

- Signal (meta-signal)
- Collision (rule)
New kinds of *monsters*
Computability and undecidability [Durand-Lose, 2005]

Two-counter simulation
Turing-machine can also be simulated directly

Undecidable
- total erasing
- finite number of signal
- signal/collision apparition
Scaling down and bounding the duration
Computing inside bounded room
Accumulation forecasting is $\Sigma_0^2$-complete

[Durand-Lose, 2006b]
**Principe**

Two different timelike half-curves such that
- they have a point in common (used to set things and start)
- one is upward-infinite and fully contained in the casual past of a point of the other

**Solving recursively enumerable problems**

- **Accept**
  - calcul

- **Refuse**
  - calcul

- **Does not stop**
  - calcul

Link with the Black hole model [Durand-Lose, 2006a]
Links with the Blum, Shub and Smale model

**Classical BSS model**

Variables hold real numbers in exact precision
- input / output
- test 0 <
- shift (to access other variables)
- compute a polynomial function

**Linear BSS [Durand-Lose, 2007]**

Restriction
- only linear function
- *i.e.* no inner multiplication
Encoding real numbers

- Common scale for all variables
- Sign test trivial
Encoding real numbers

**Scale + distance**

1
scale

2.71
ba

**Common scale for all variables**

**Sign test trivial**
Encoding real numbers

Scale + distance

- Common scale for all variables
- Sign test trivial
Addition
External multiplication

\[ \text{line}_{n+1} \]

\[ \text{accum} \] \[ \text{val} \]

\[ \text{mul} \] \[ \text{mul}_a \] \[ \text{mul}_b \] \[ \text{mul}_c \]

\[ \text{val} \] \[ \text{line}_{n+1} \]
Internal multiplication [Durand-Lose, 2008]

Computation
- Pre-treatment to ensure \(0 < y < 1\)
- Binary extension of \(y\):
  \[y = y_0 \cdot y_1 y_2 y_3 \cdots\]
- Computation
  \[xy = \sum_{0 \leq i} y_i \left( \frac{x}{2^i} \right)\]

Principe
- Computation on the margin
  - the margin is scaling down geometrically
- Square rooting is also possible!
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Conclusion

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Conclusion

- Natural filiation with CA
- Continuous time
  - Zeno effect
  - Unpredictability

Links with other models
- Black hole model
- Blum, Shub and Smale model

Future work
- Relate with CA
- Characterize the analog computing power
Particle-like structures and interactions in spatio-temporal patterns generated by one-dimensional deterministic cellular automaton rules.

Abstract geometrical computation: Turing computing ability and undecidability.

Abstract geometrical computation 1: Embedding black hole computations with rational numbers.

Conclusion

Forcasting black holes in abstract geometrical computation is highly unpredictable.


**Durand-Lose, J. (2007).**

Abstract geometrical computation and the linear Blum, Shub and Smale model.


**Durand-Lose, J. (2008).**

Abstract geometrical computation with accumulations: Beyond the Blum, Shub and Smale model.

Generation of primes by a one-dimensional real-time iterative array.

Ōtomaton ni kansuru pazuru [Puzzles on automata].

An upper bound on the products of particle interactions in cellular automata.

Universal computation in simple one-dimensional cellular automata.
Computations on one dimensional cellular automata.

Signals in one-dimensional cellular automata.

Soliton-like dynamics of filtrons of cycle automata.

Synchronization of interacting automata.