Finite Automata and Regular Expressions

SITE : http://www.info.univ-tours.fr/˜mirian/

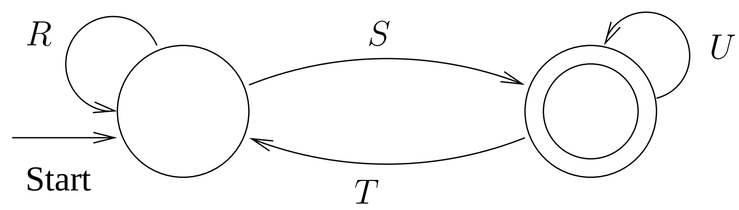
Theorem

If $L = L(A)$ for some DFA, then there is a regular expression R such that $L = L(R)$ $\square$

- We are going to construct regular expressions from a DFA by eliminating states.
- When we eliminate a state s, all the paths that went through s no longer exist in the automaton.
- If the language of the automaton is not to change, we must include, on an arc that goes directly from q to p, the labels of paths that went from some state q to state p, through s.
- The label of this arc can now involve strings, rather than single symbols (may be an infinite number of strings).
- We use a regular expression to represent all such strings.
- Thus, we consider automata that have regular expressions as labels.

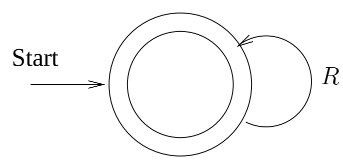
Constructing a regular expression from a finite automaton

1. For each accepting state q, apply the reduction process to produce an equivalent automaton with regular expression labels on the arcs. Eliminate all states except q and the start state q_0.
2. If $q \neq q_0$, then we shall be left with a two-state automata:



One regular expression that describes the accepted strings: $(R + SU^*T)^*SU^*$

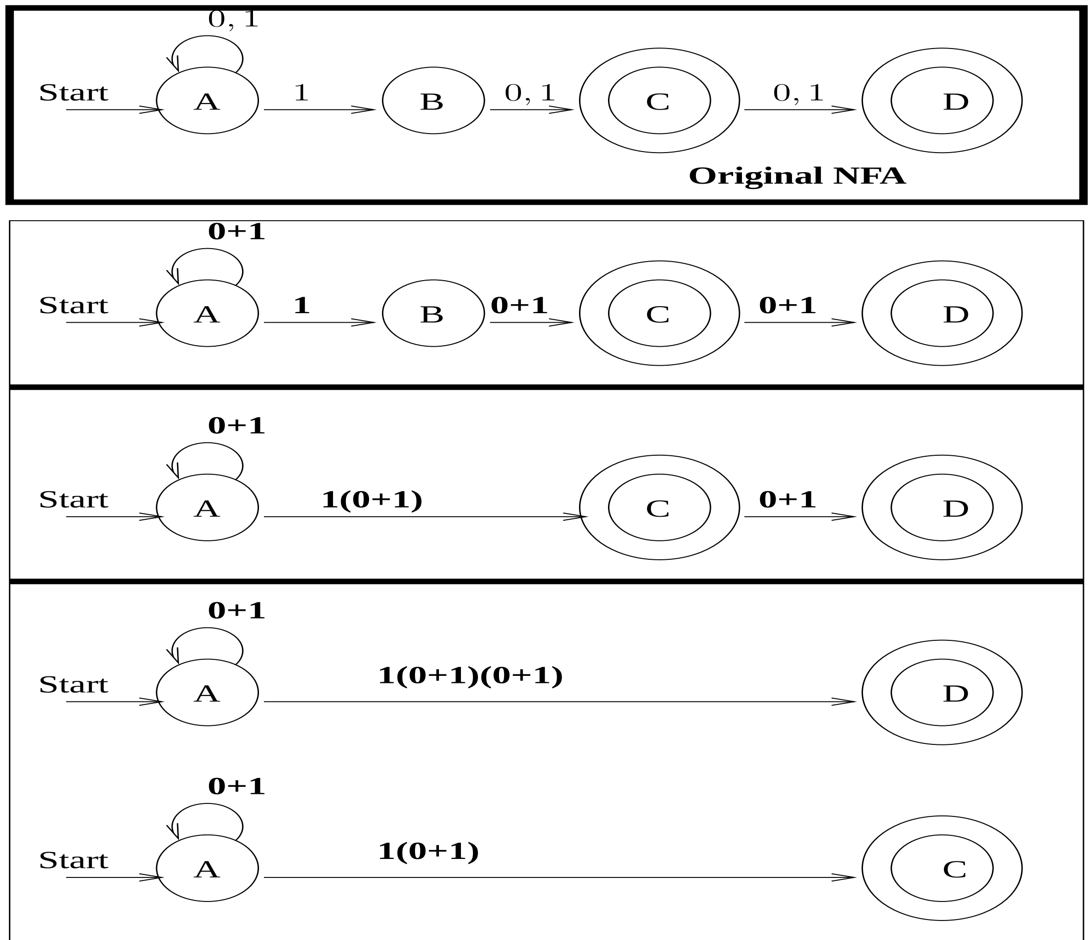
3. If the start state is also a final state, then we are left with a one-state automaton and the regular expression denoting strings that it accepts is R^*



4. The desired regular expression is the union of all the expressions derived from the reduced automata for each accepting states.

Example

Example: NFA accepts strings of 0 and 1 such that either the second or the third position from the end has a 1. Represented by the regular expression $(\mathbf{0}+\mathbf{1})^*\mathbf{1}(\mathbf{0}+\mathbf{1}) + (\mathbf{0}+\mathbf{1})^*\mathbf{1}(\mathbf{0}+\mathbf{1})(\mathbf{0}+\mathbf{1})$



Theorem

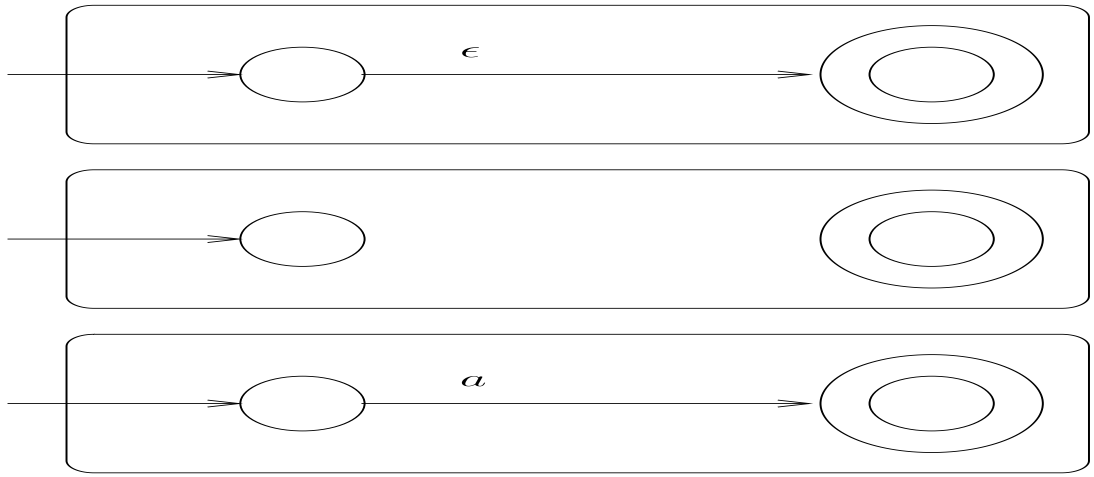
Every language defined by a regular expression is also defined by a finite automaton.

- Suppose $L = L(R)$ for a regular expression R. We show that $L = L(E)$ for some $\epsilon - NFA\ E$ with
  1. Exactly one accepting state
  2. No arcs into the initial state
  3. No arcs out of the accepting state
- The proof is by structural induction on R, following the recursive definition of regular expressions.

Proof

Basis

The basis of the construction of fsa from regular expressions:



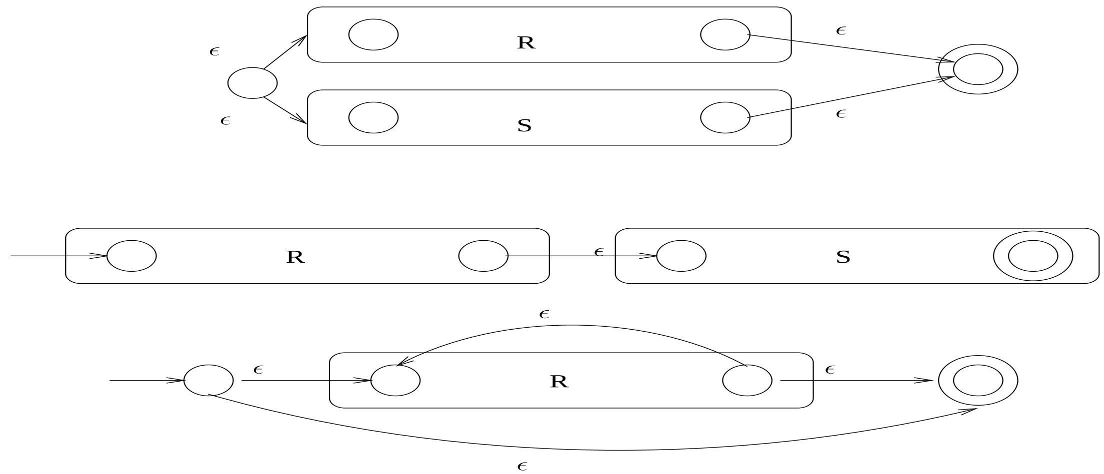
1. Expression ϵ: the language of the FSA is $\{\epsilon\}$.
2. Expression $\emptyset$: $\emptyset$ is the language of FSA.
3. Expression **a**: the language of the FSA is $\{a\}$.

All these automata satisfies the three initial conditions

Proof

Induction

The inductive step of the construction of fsa from regular expressions



1. The expression is $R + S$ for some smaller expressions R and S.
2. The expression is RS for some smaller expressions R and S.
3. The expression is R^* for some smaller expression R.
4. The expression is (R) for some smaller expression R.