PDA - Final part

SITE : http://www.sir.blois.univ-tours.fr/˜mirian/

PDA- Acceptance by final state

Let

$$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

be a PDA. The **language accepted by P by final state is**:

$$L(P) = \{w \mid (q_0, w, Z_0) \overset{*}{\vdash} (q, \epsilon, \alpha), q \in F\}$$

for some state q in F and any input stack string α.

Starting in the initial ID with w waiting on the input, P consumes w from the input and enters an accepting state. **The contents of the stack at that time is irrelevant**.

PDA- Acceptance by empty stack

Let

$$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

be a PDA. The **language accepted by P by empty stack is**:

$$N(P) = \{w \mid (q_0, w, Z_0) \overset{*}{\vdash} (q, \epsilon, \epsilon)\}$$

where q is any state

$N(P)$ is the set of inputs w that P can consume an at the same time empty the stack.

From final state to empty stack

- Let P_N be a PDA by empty stack.
- Let P_F be a PDA by final state.

Theorem:
If $L = N(P_N)$ for some PDA P_N, then there exist a PDA P_F, such that

$$L = L(P_F)$$

Equivalence of PDA and CFG

A language is

generated by a CFG

iff it is

accepted by a PDA by empty stack

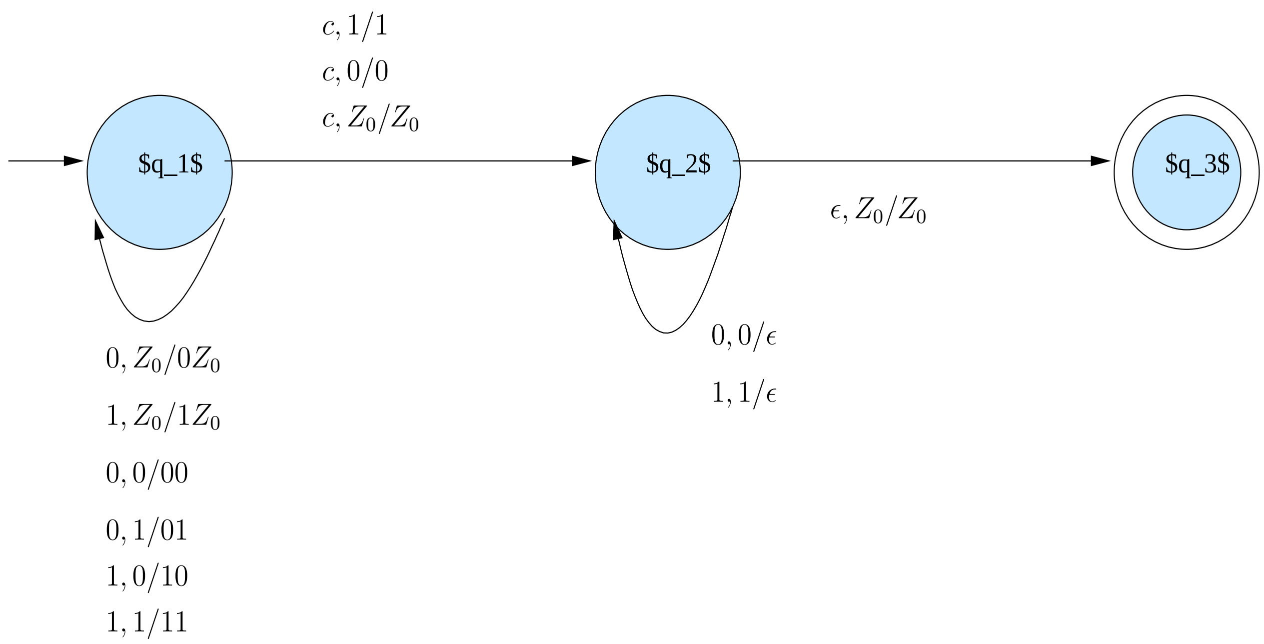
iff it is

accepted by a PDA by final state

Deterministic PDA

- A PDA $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ is **deterministic** iff
  1. $\delta(q, a, X)$ is always empty or a singleton
  2. If $\delta(q, a, X)$ is nonempty, then $\delta(q, \epsilon, X)$ must be empty

Example. $L_{wcw} = \{wcw \mid w \in \{0, 1\}^*\}$ is recognized by the following PDA



Importance of DPDA

- While PDA are by definition nondeterministic, the deterministic subcase is quite important.
- **Parses in general behave like DPDA**
- A DPDA can accept languages like L_{wcw} that are not regular, but there are CFL (like L_{wwr}) that cannot be accepted by a DPDA.
- Languages accepted by DPDA all have unambiguous grammar

  Theorem: If L is the language accepted by some DPDA P, then L has an unambiguous CFG.
- The DPDA languages are <u>not exactly equal</u> the subset of CFL that are **not** inherently ambiguous.
  Example: L_{wwr} has an unambiguous grammar and it is not a DPDA language.

Important properties

Let

- NFA: Nondeterministic finite automaton
- DFA : Deterministic finite automaton
- RE: Regular expression
- DDPA: Deterministic pushdown automaton
- DPA: Nondeterministic pushdown automaton
- CFL: Context free language

1. We know that if $L_{Regular}$ is a regular language, then there exist NFA, DFA and RE such that

$$\textbf{L(NFA) = L(DFA) = L(ER) = } L_{Regular}$$

2. **L(PDA) = CFL**
3. **L(DPDA) =** L_1 such that L_1 has an unambiguous CFG
4. **Regular languages** $\subset$ **L(DPDA)** $\subset$ **CFL**