Riga, Latvia

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Two-State Bilinear Intrinsically Universal Cellular Automata

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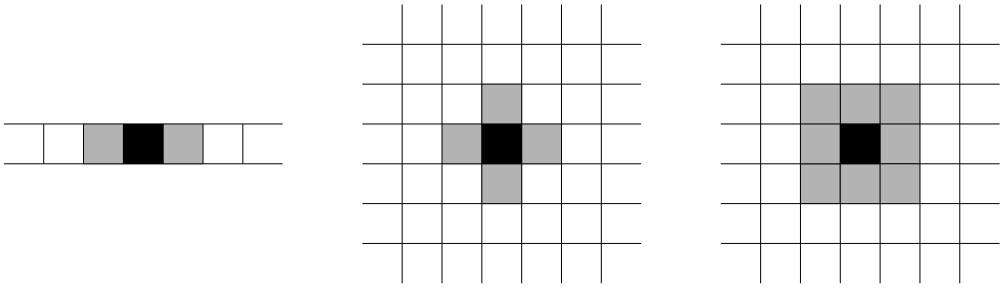
Cellular Automata

Definition. A *d-CA* $\mathcal{A}$ is a 4-uple $\left(\mathbb{Z}^d, S, N, \delta\right)$ with:

- S the finite set of states of $\mathcal{A}$,
- $N \subset \mathbb{Z}^d$, finite, the neighborhood of $\mathcal{A}$,
- $\delta : S^{|N|} \to S$ the local rule of $\mathcal{A}$.



▶ A *configuration* C is a mapping from $\mathbb{Z}^d$ to S.

▶ The *global rule* applies δ uniformly according to N:

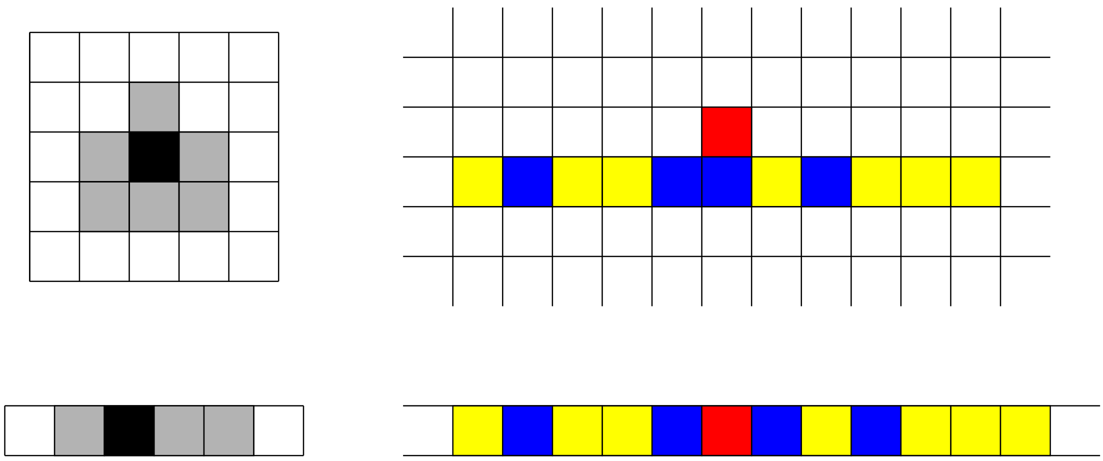
$$G_{\mathcal{A}}\left(C\right)_p = \delta\left(C_{p+N_1}, \ldots, C_{p+N_n}\right).$$

Computation Universality

Idea. A CA is *computation universal* if it can **compute** any partial recursive function.

▶ This notion is difficult to formalize.

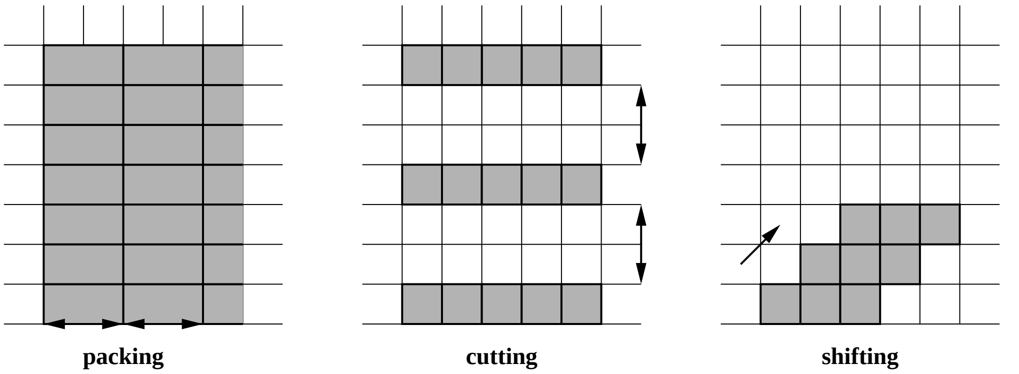
▶ In practice: step-by-step Turing simulation.

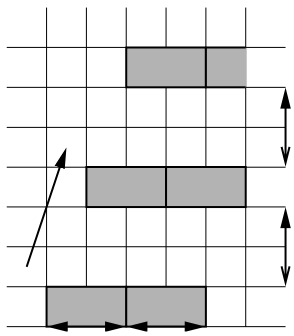


A. R. Smith III. Simple Computation-Universal Cellular Spaces. 1971

Intrinsic Simulation

► Geometrical transformations on space-time diagrams:





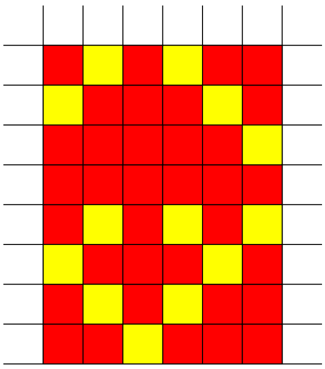
Definition. A d-CA $\mathcal{A}$ *simulates* a d-CA $\mathcal{B}$ if, up to geometrical transformations on both sides, space-time diagrams from $\mathcal{B}$ can be considered as space-time diagrams from $\mathcal{A}$.

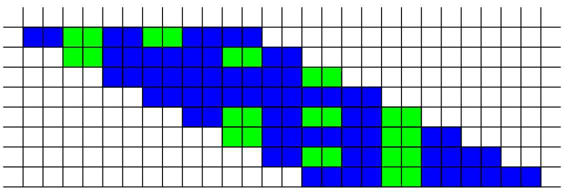
Intrinsic Universality

Definition. A d-CA $\mathcal{A}$ is *intrinsically universal* if $\mathcal{A}$ can simulate any d-CA $\mathcal{B}$.

► This definition is equivalent to the following one.

Definition. A d-CA $\mathcal{A}$ is *strongly intrinsically universal* if, for each d-CA $\mathcal{B}$, space-time diagrams from $\mathcal{B}$ can be considered as space-time diagrams from a geometrical transformation of $\mathcal{A}$.





Linear and Bilinear CA

Definition. A *linear* d-CA is a polynomial d-CA of degree 1. S is a finite commutative ring.

$$\delta(s_1, \ldots, s_n) = \sum_{i=1}^{n} a_i s_i$$

► Linear d-CA exhibit simple behavior (Martin *et al.* 1993)

Definition. A *bilinear* d-CA is a polynomial d-CA of degree 2. S is a finite commutative ring.

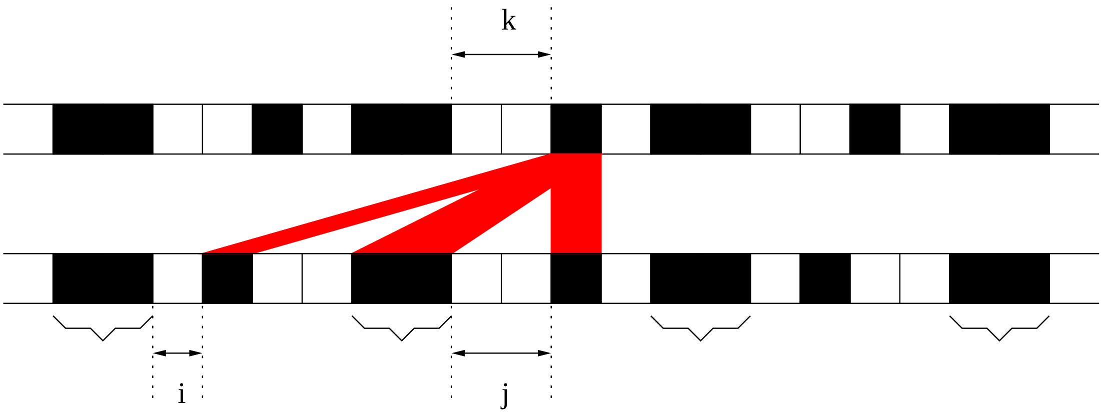
$$\delta(s_1, \ldots, s_n) = \sum_{i=1}^{n} \sum_{j=1}^{n} b_{i,j} s_i s_j$$

► There are intrinsically universal bilinear d-CA

At least over the ring $\mathbb{Z}_{211}^{211}$ (Bartlett and Garzon 1995)

Two-state Universal CA

► We construct a simple polynomial 1-CA over $\mathbb{Z}_2$ which simulate a given one-way 1-CA (*i.e.* $N = \{-1, 0\}$).

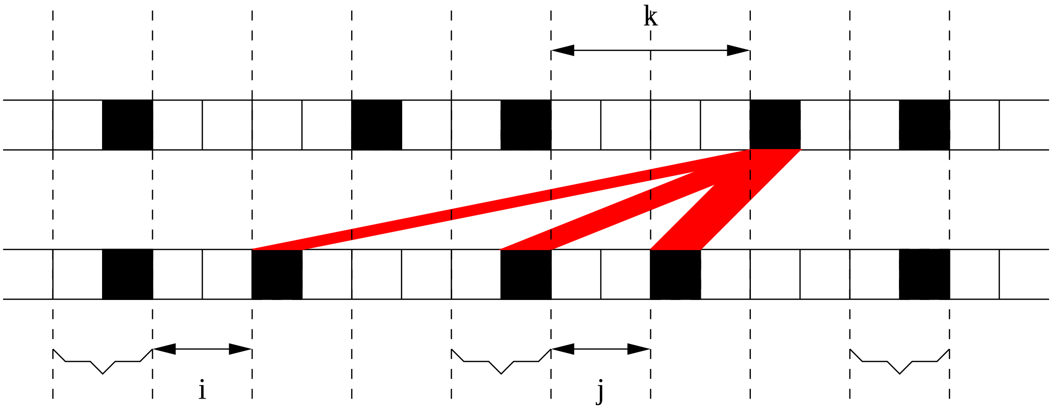


$$\delta\left(s_1, \ldots, s_n\right) = \sum \sum s_{-k} s_{-k-1} s_{-k-b+i} s_{-k+j}$$

► This polynomial has degree 4.

Degree 3

► Positional information can be encoded into the monomials. Here we use position of state 1 modulo 2.

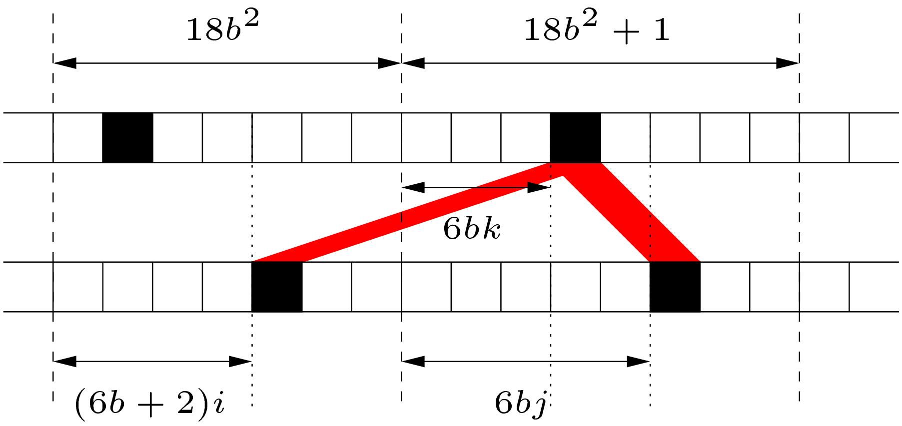


$$\delta\left(s_1,\ldots,s_n\right)=\sum\sum s_{-2k-1}s_{-2k-2b+2i}s_{-2k+2j}$$

► This polynomial has degree 3.

Two-State Universal Bilinear CA

- The idea is to forget the cell border information.



$$\delta(s_1, \ldots, s_n) = \sum \sum s_{18b^2-(6b+2)i+6bk}\, s_{6bj-6bk} + s_{18b^2+1-6bi+(6b+2)k}\, s_{(6b+2)j-(6b+2)k}$$

Going Further

► The notion of simulation and the way it links with universality is worth studying.

► Investigate other kind of “simple” (like here bilinear) universal cellular automata.

► Is it possible to define a kind of acceptable programming system on CA where universal programs would be intrinsically universal CA ?