On a directed variation of the 1-2-3 and 1-2 Conjectures

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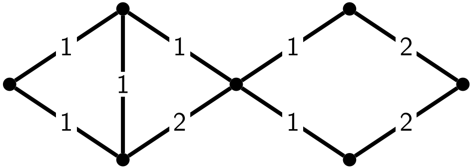
November 5th, 2015

1-2-3 Conjecture

Sum-colouring edge-weightings

G: simple graph
w: k-edge-weighting of G



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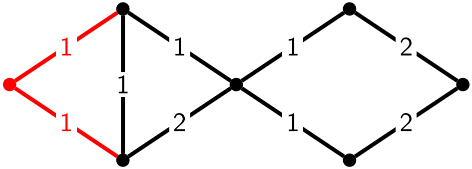

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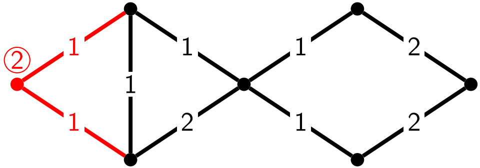

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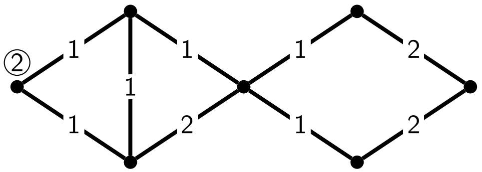

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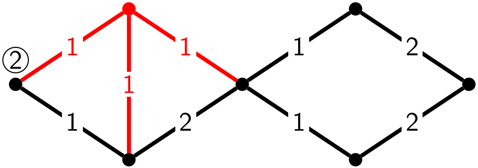

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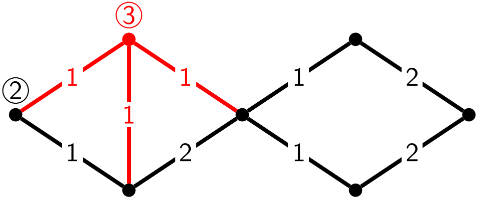

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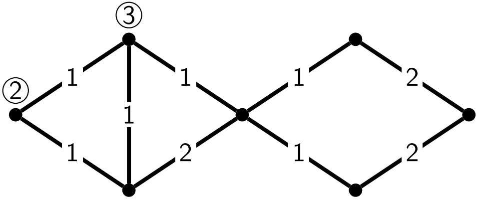

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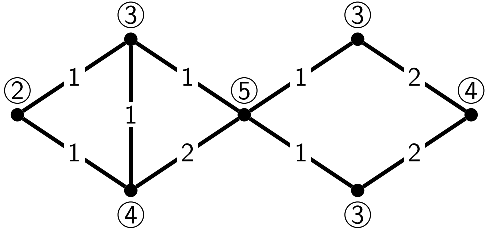

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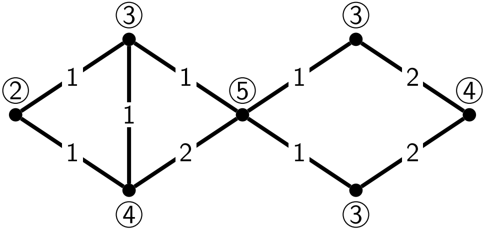

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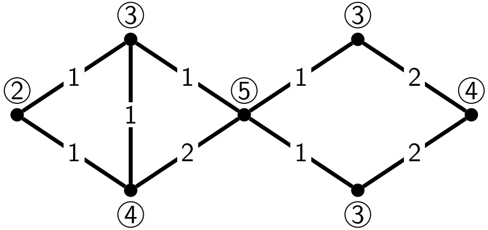

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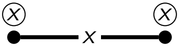
$\chi^e_\Sigma(G)$: $\min\{k : G$ has a sum-colouring k-edge-weighting$\}$

1-2-3 Conjecture

Note: $\chi^e_\Sigma(K_2)$ undefined



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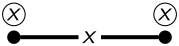

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1-2-3 Conjecture – Karoński, Łuczak, Thomason (2004)

For every *nice* graph G, we have $\chi^e_\Sigma(G) \leq 3$.

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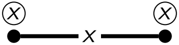

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$\chi_\Sigma^e(G) \leq 5$ [Kalkowski, Karoński, Pfender (2010)]

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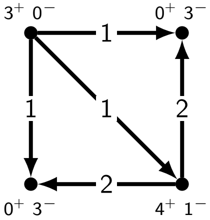

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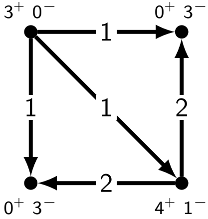

Distinguishing neighbours via σ^+ and σ^-?

Playing around with σ^+ and σ^-

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- *1-2-3 Conjecture*: induction possible, no exceptions, etc.
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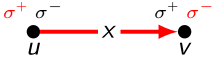
So what would be satisfying?

A new candidate directed variant

Łuczak's question and condition

What about requiring $\sigma^+(u) \neq \sigma^-(v)$ whenever $\overrightarrow{uv}$ is an arc?



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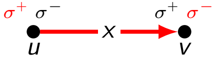

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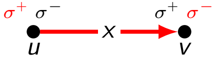

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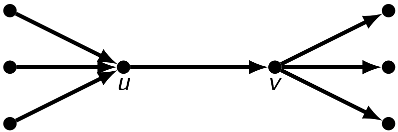

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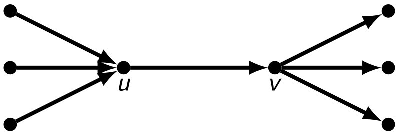

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D *nice*: no such configuration

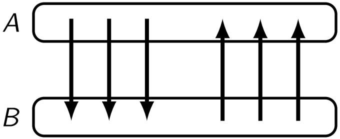
Directed 1-2-3 Conjecture – Łuczak

For every *nice* digraph D, we have $\chi^e_Ł(D) \leq 3$.

Tightness of $\{1, 2, 3\}$ – Bipartite digraphs

Theorem – Barme, B., Przybyło, Woźniak (2015+)

For every nice *bipartite* digraph D, we have $\chi^e_Ł(D) \leq 3$.



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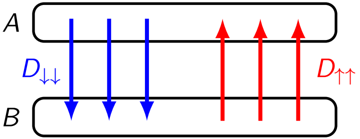

$$\forall v \in A,\ d^+_D(v) = d^+_{D_{\downarrow\downarrow}}(v) + 0 \text{ and } d^-_D(v) = 0 + d^-_{D_{\uparrow\uparrow}}(v)$$
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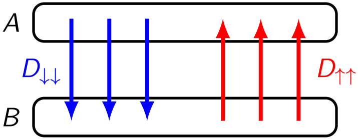

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$\Rightarrow$ $D_{\downarrow\downarrow}$ and $D_{\uparrow\uparrow}$ can be weighted independently

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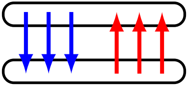

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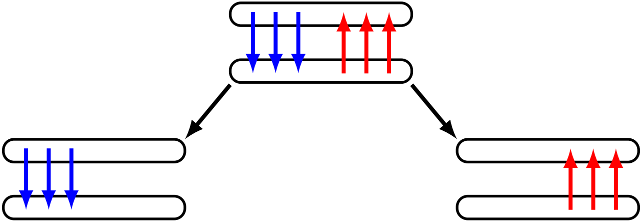

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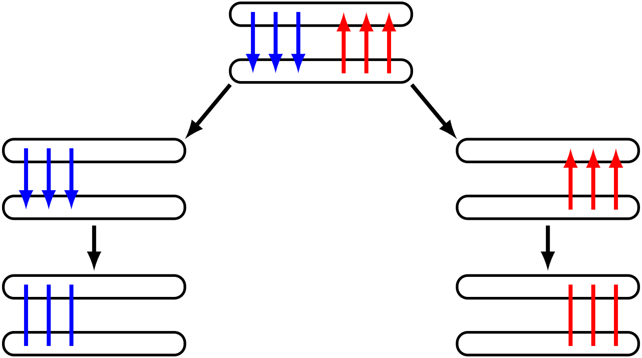

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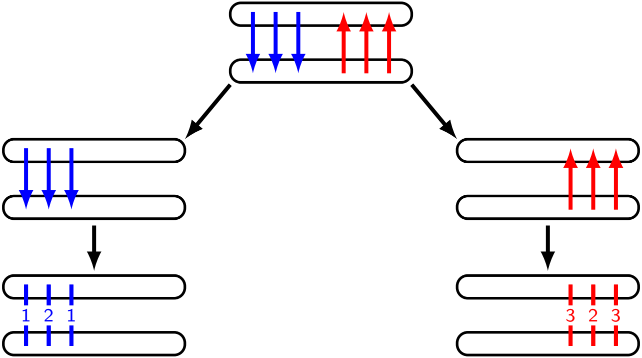

$\forall uv \in E(\mathrm{und}(D_{\downarrow\downarrow})), \sigma(u) \neq \sigma(v)$ $\forall uv \in E(\mathrm{und}(D_{\uparrow\uparrow})), \sigma(u) \neq \sigma(v)$

Karoński, Łuczak, Thomason $\Rightarrow$ 1-2-3 Conjecture holds for bipartite graphs

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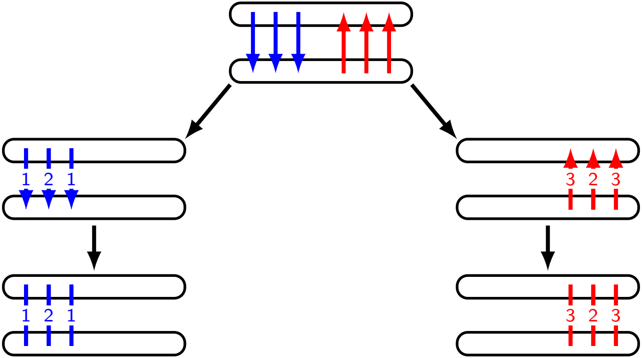

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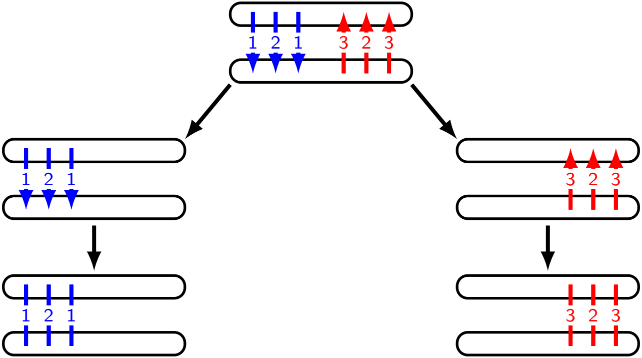

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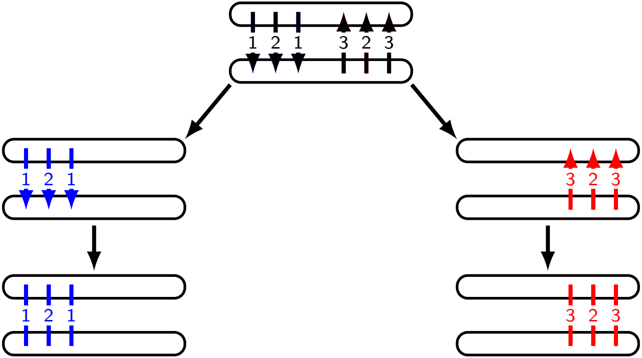

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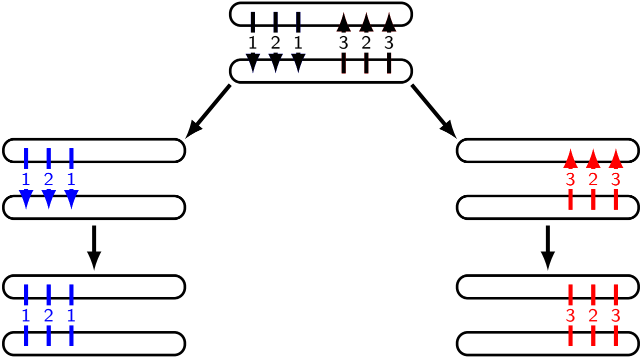

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$\blacksquare$

So $\chi^e_{\text{Ł}}(D) = \max\{\chi^e_{\text{Ł}}(D_{\downarrow\downarrow}), \chi^e_{\text{Ł}}(D_{\uparrow\uparrow})\}$ and *e.g.* $\chi^e_{\text{Ł}}(D_{\downarrow\downarrow}) = \chi^e_{\Sigma}(\mathrm{und}(D_{\downarrow\downarrow}))$
$\Rightarrow$ If $D = D_{\downarrow\downarrow}$ and $\chi^e_{\Sigma}(\mathrm{und}(D_{\downarrow\downarrow})) = 3$, then $\chi^e_{\text{Ł}}(D) = 3$

A solution to the Directed 1-2-3 Conjecture

Idea: treat σ^+ and σ^- separately

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Theorem – Barme, B., Przybyło, Woźniak (2015+)

The following two problems are equivalent:

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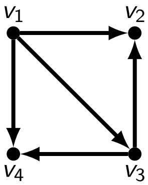

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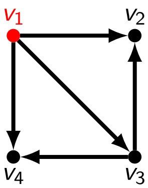

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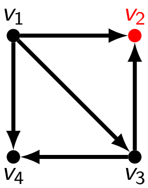

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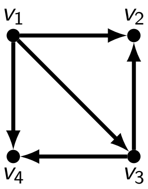

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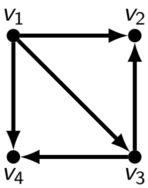

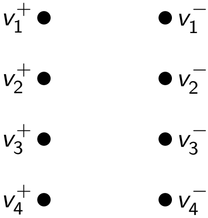

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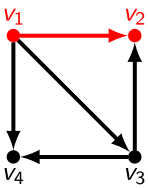

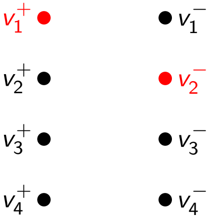

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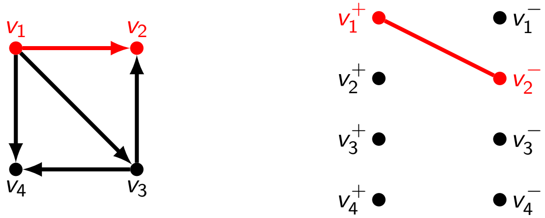

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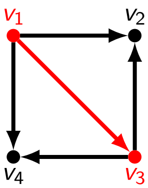

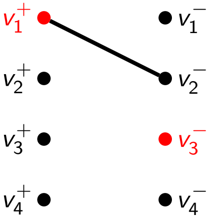

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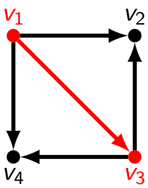

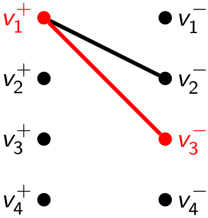

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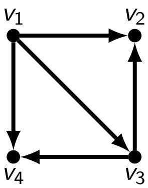
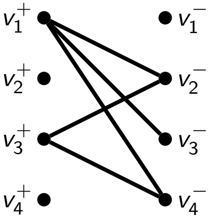
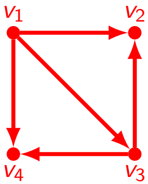
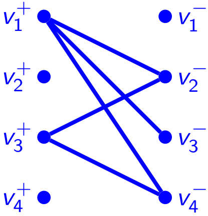




Relation between D and $\overleftrightarrow{G}(D)$





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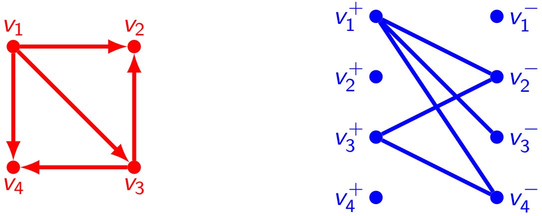

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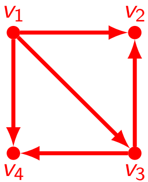

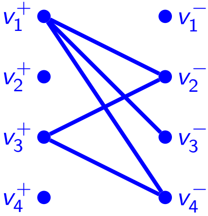

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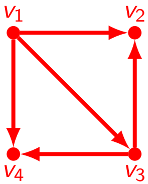

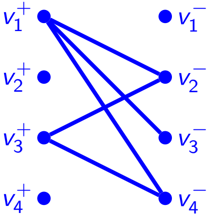

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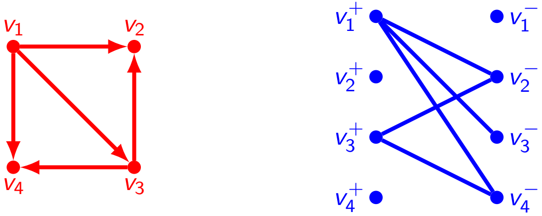

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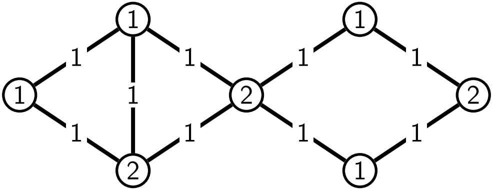
The Directed 1-2-3 Conjecture then follows from KŁT's result

1-2 Conjecture

Sum-colouring arc-weightings

w: (k, k)-total-weighting of G



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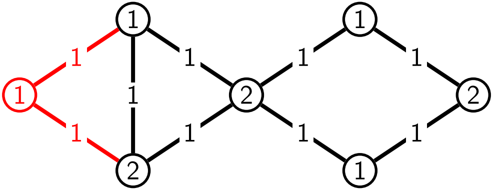

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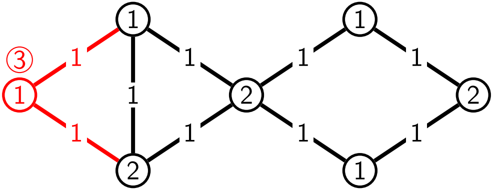

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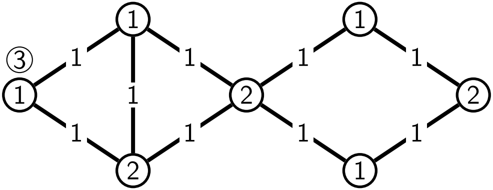

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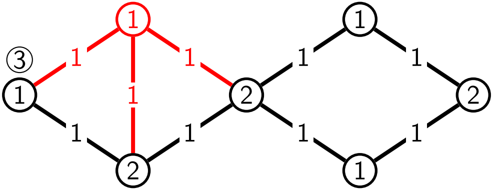

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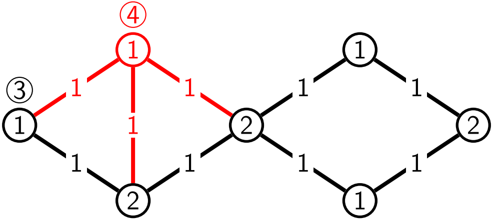

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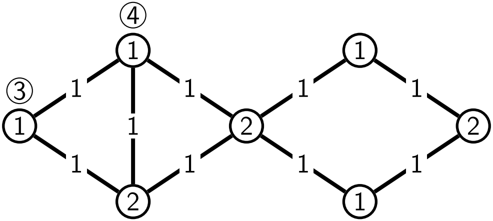

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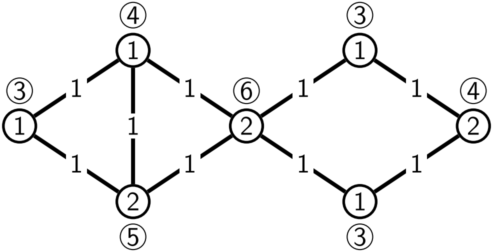

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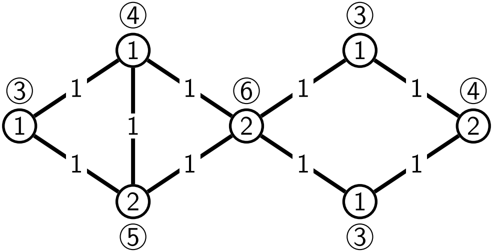

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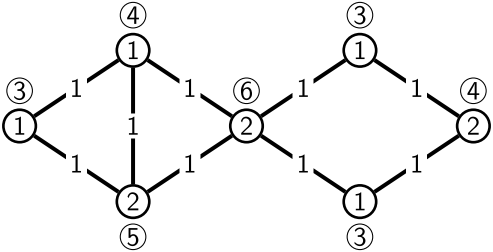

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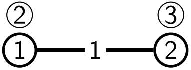
1-2 Conjecture

Note: $\chi^t_\Sigma(K_2) = 2$



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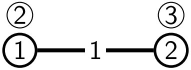

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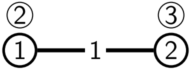

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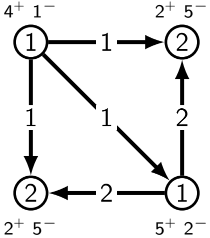

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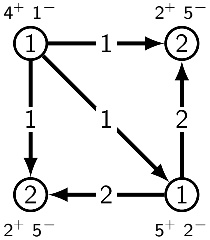

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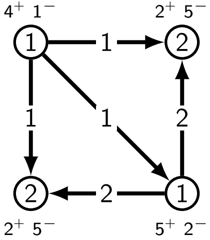

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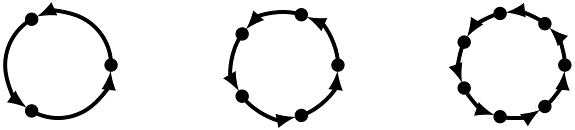
Answer: No! Odd directed cycles



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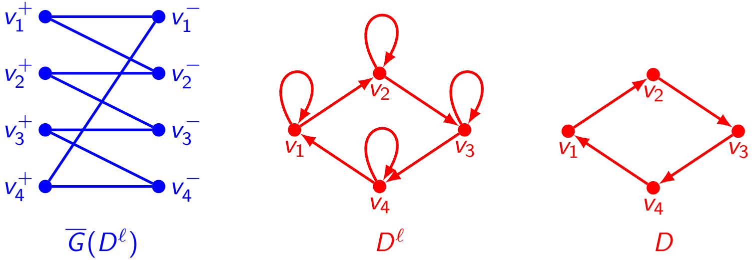

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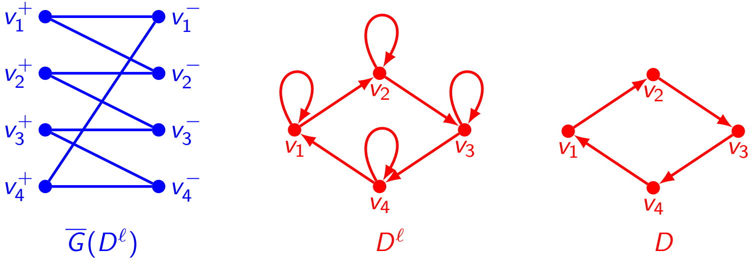

$\chi_\Sigma^e(\overline{G}(D^\ell)) \leq 3 \Rightarrow \chi_Ł^e(D^\ell) \leq 3 \Rightarrow \chi_Ł^t(D) \leq 3$

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Characterization? **Not clear...**

Theorem – Chang, Lu, Wu, Yu (2011)

For $G = (A, B)$ *nice bipartite* with $|A|$ *or* $|B|$ *even*, we have $\chi^{\mathsf{e}}_{\Sigma}(G) \leq 2$.

$\Rightarrow$ Directed 1-2 Conjecture true for nice digraphs with *even order*

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Thanks!