

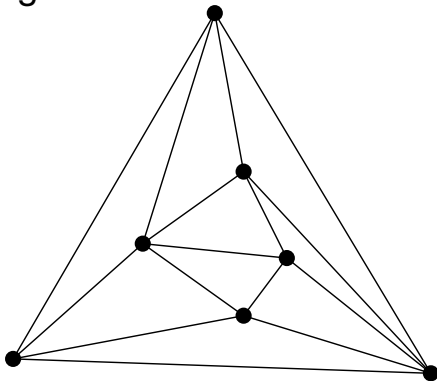
# Encoding Toroidal Triangulations

Vincent Despré, Daniel Goncalves and Benjamin  
Lévêque

**gipsa-lab**, G-SCOP, Grenoble  
**vincent.despre@gipsa-lab.fr**

5 November 2015

# Planar triangulations



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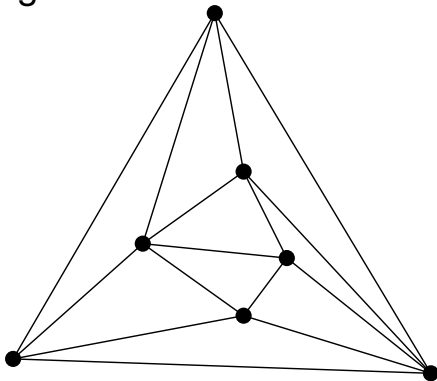
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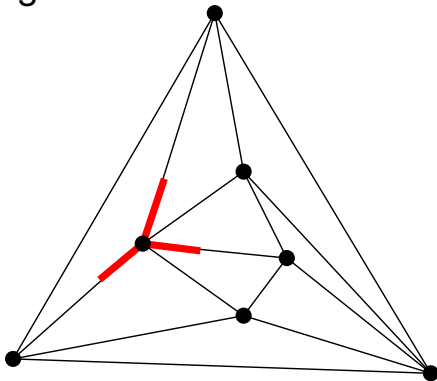
$$\text{Euler : } v - e + f = 2$$

$$\text{Triangulation : } 3f = 2e \implies (e - 3) = 3(v - 3)$$

$$e = 3v - 6$$

$$e_{int} = 3v_{int}$$

# Planar triangulations



$$\text{Euler : } v - e + f = 2$$

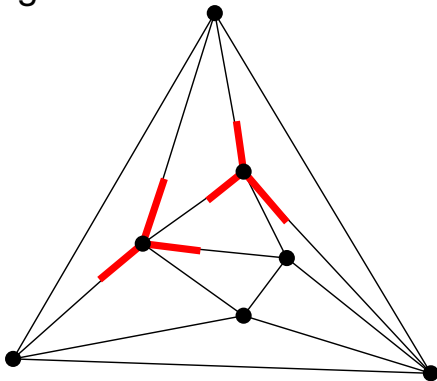
$$\text{Triangulation : } 3f = 2e \implies (e - 3) = 3(v - 3)$$

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↪ Associate to each internal vertex three incident edges

# Planar triangulations



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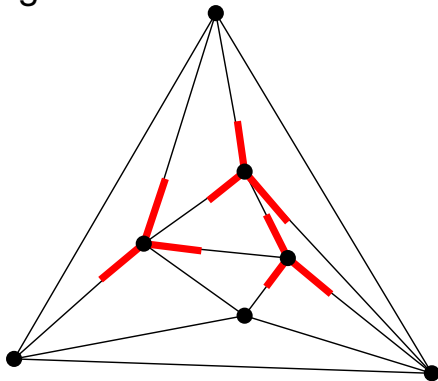
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# Planar triangulations



$$\text{Euler : } v - e + f = 2$$

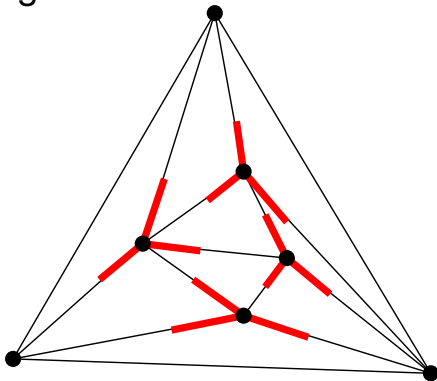
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# Planar triangulations



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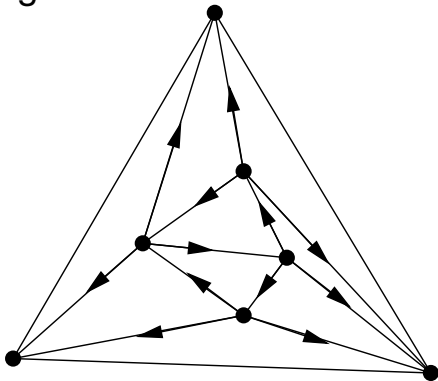
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# Planar triangulations



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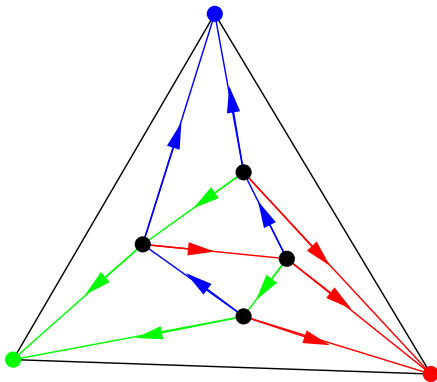
$$e = 3v - 6$$

$$e_{int} = 3v_{int}$$

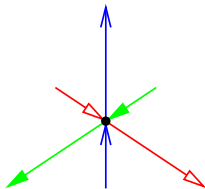
↪ Associate to each internal vertex three incident edges and deduce a 3-orientation



# Schnyder Woods



$\rightsquigarrow$  Schnyder wood



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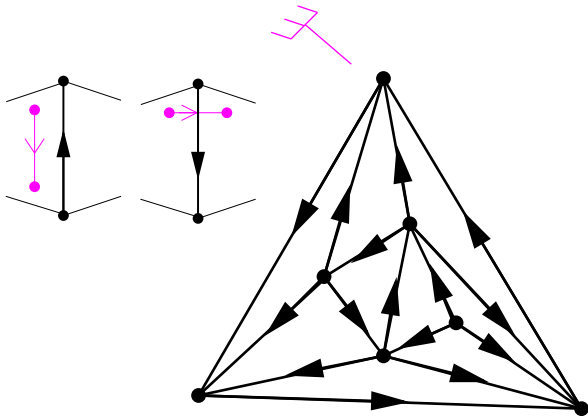
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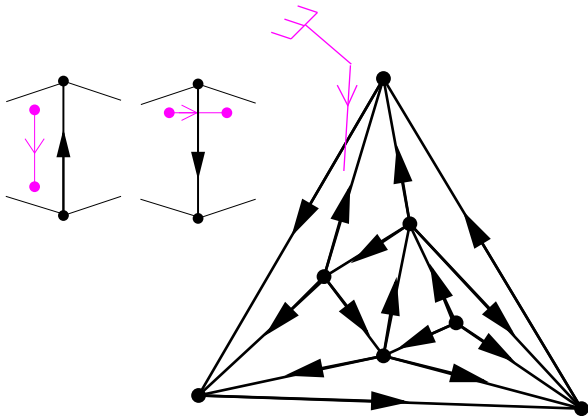
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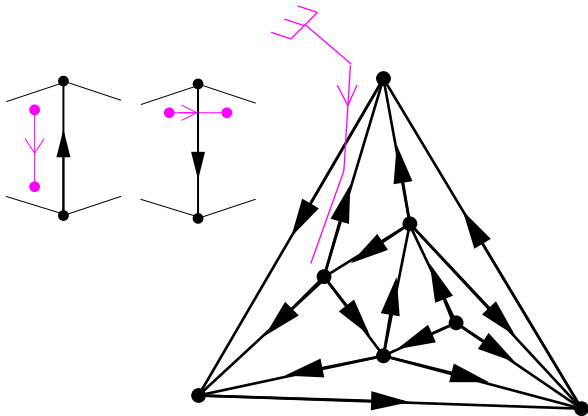
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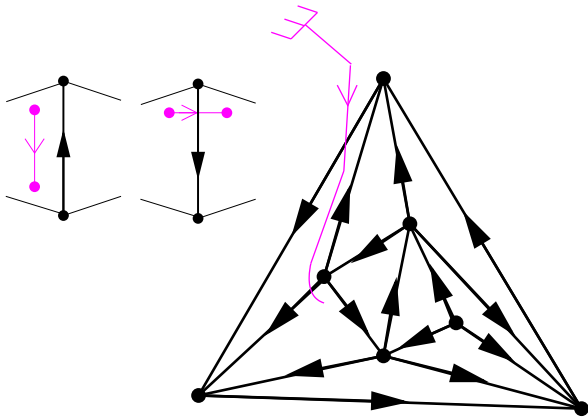
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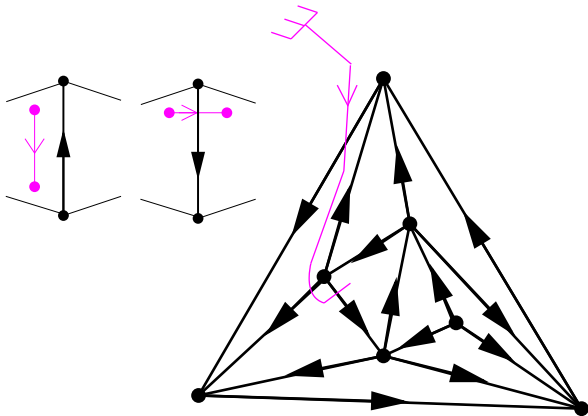
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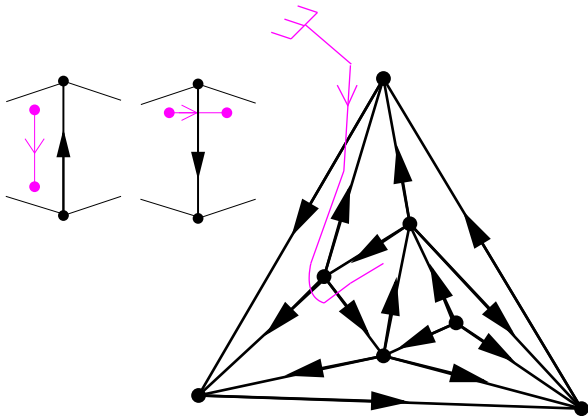
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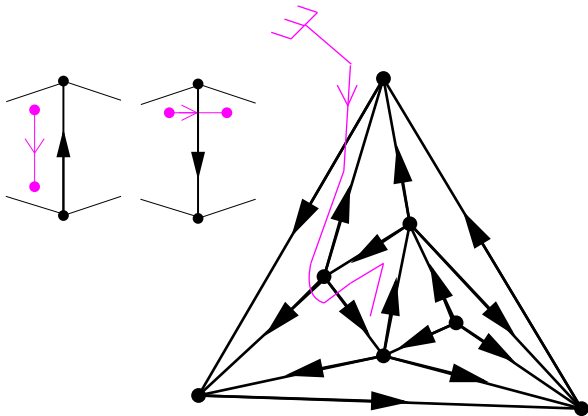
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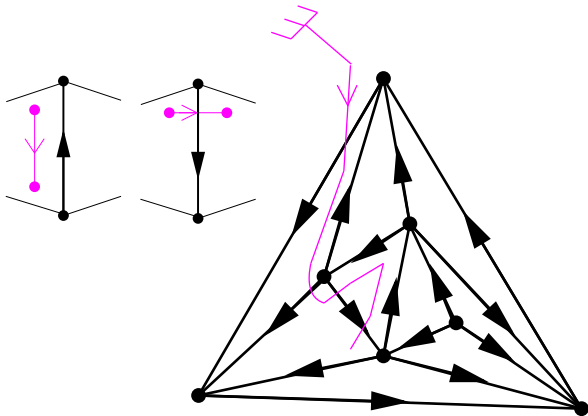
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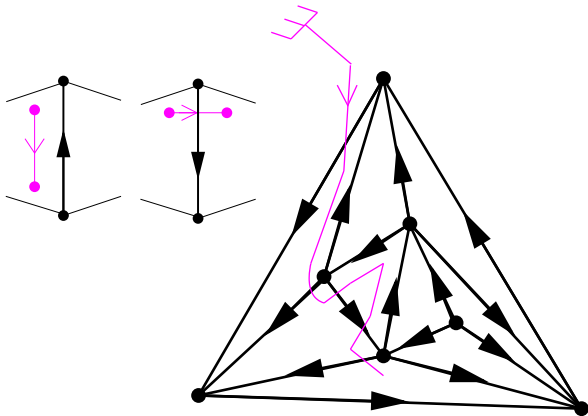
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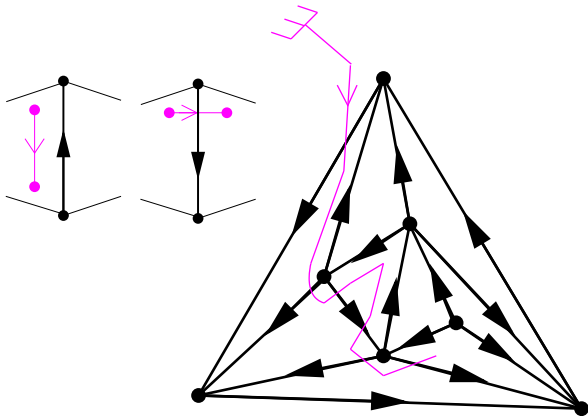
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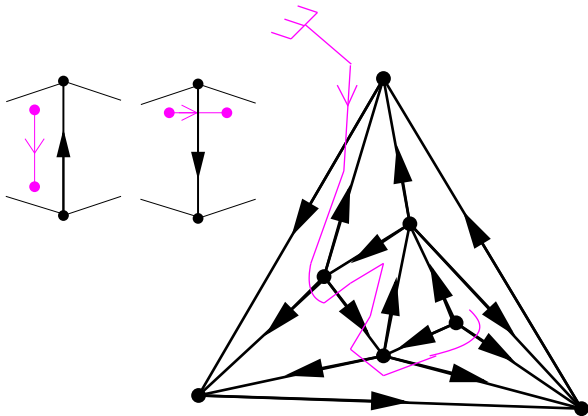
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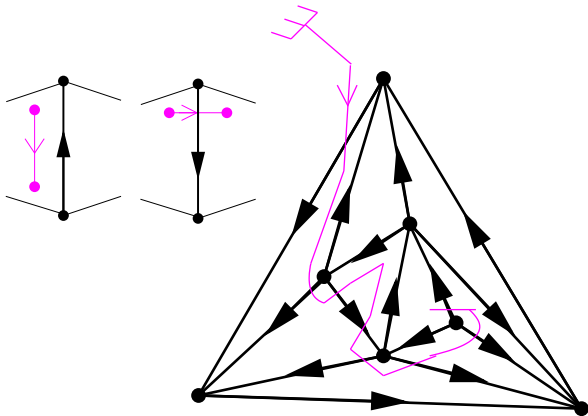
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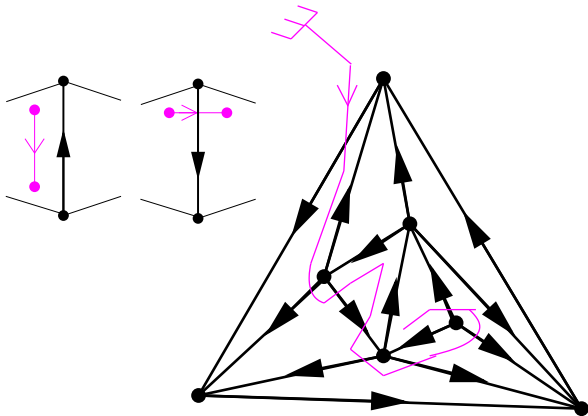
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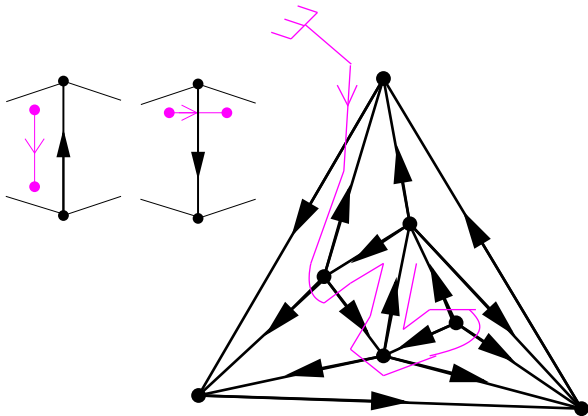
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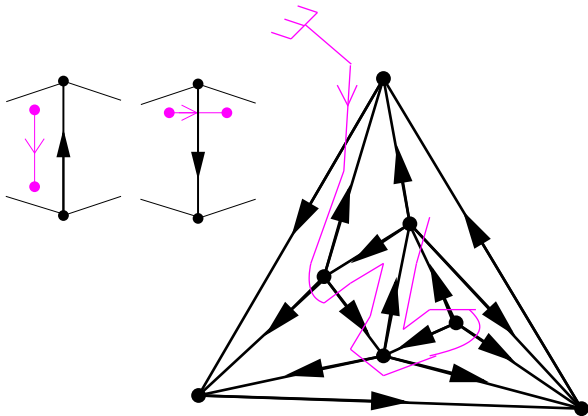
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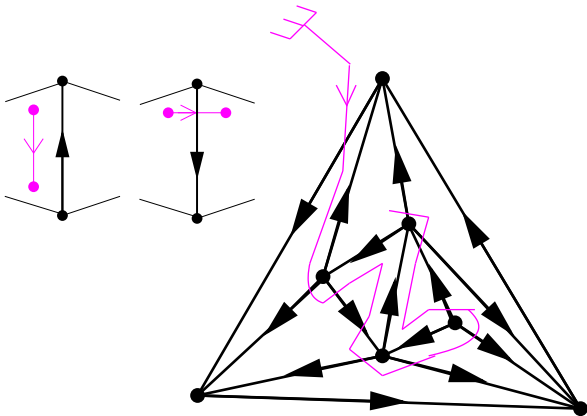
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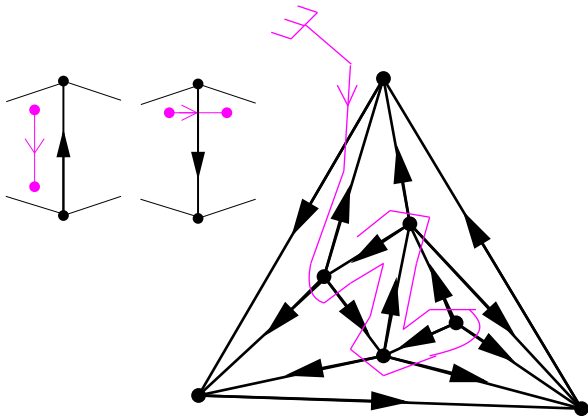
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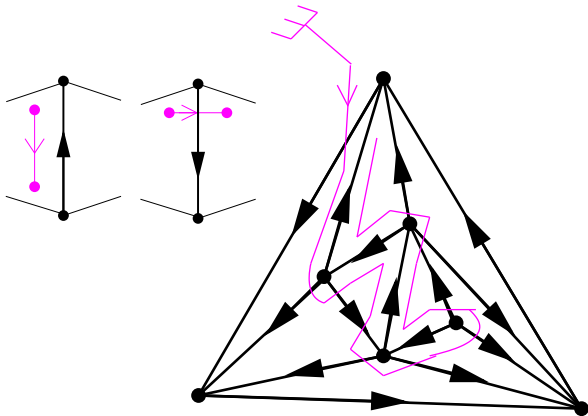
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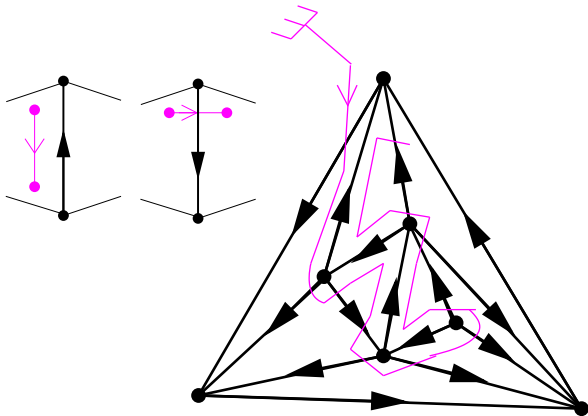
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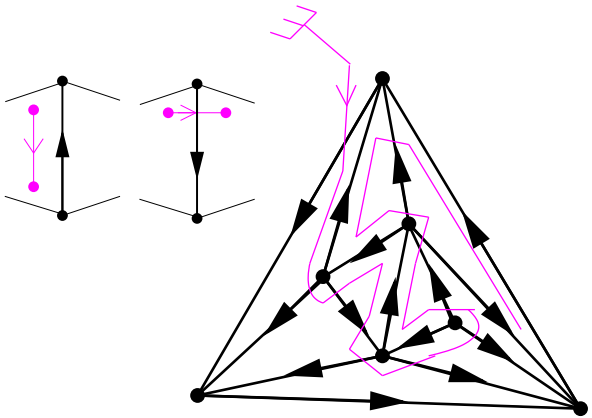
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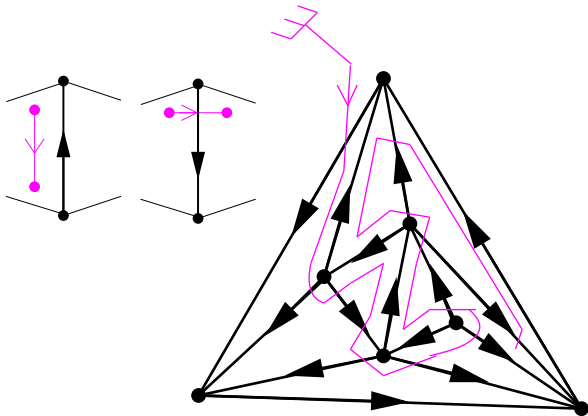
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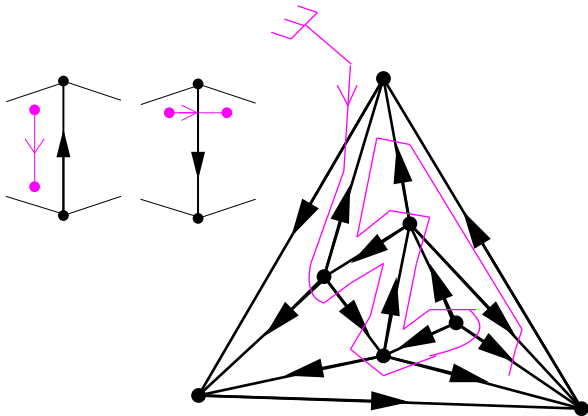
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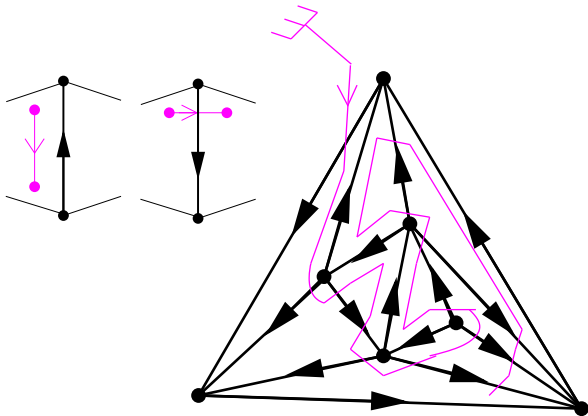
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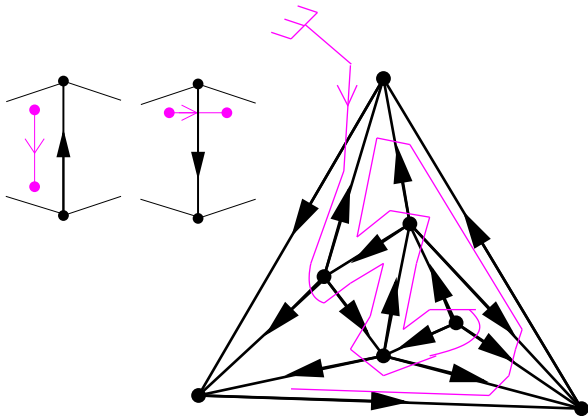
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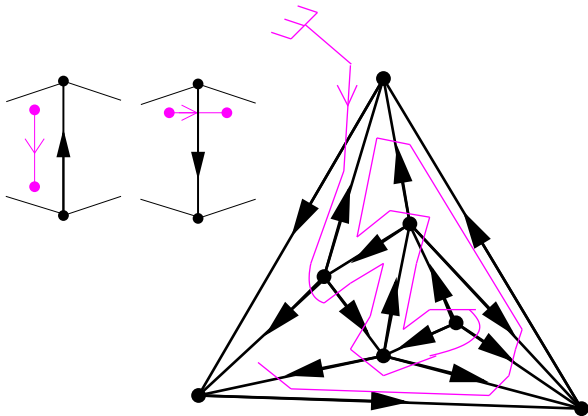
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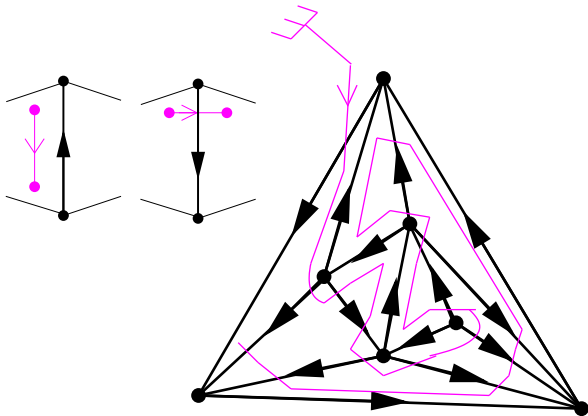
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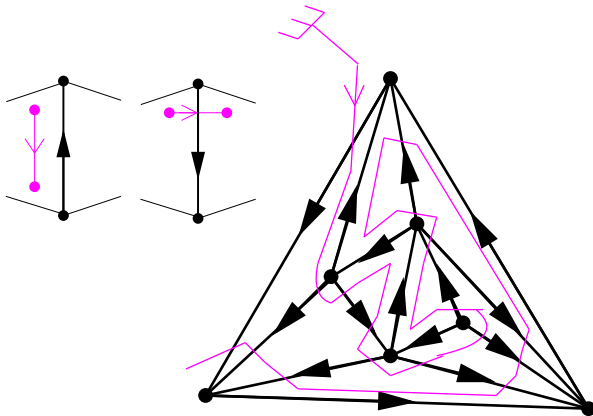
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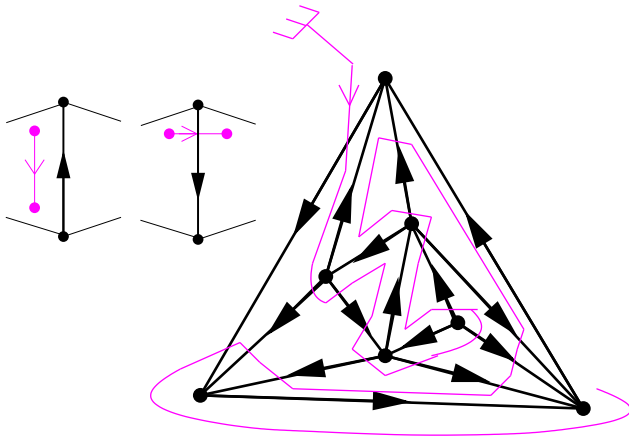
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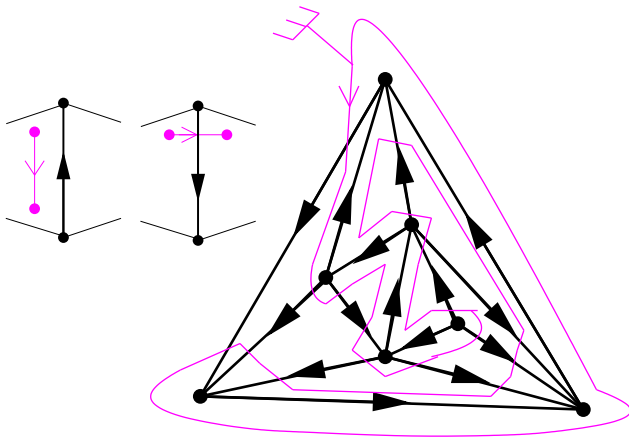
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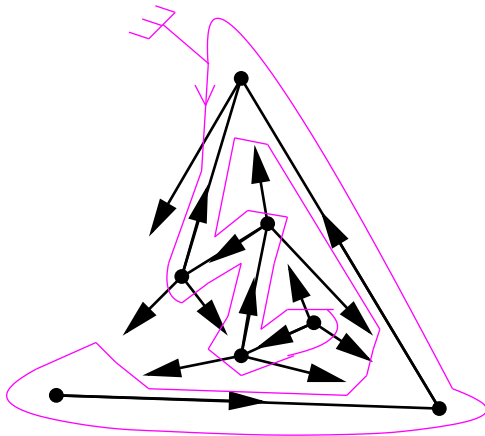
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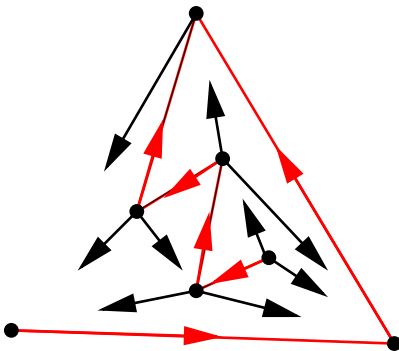
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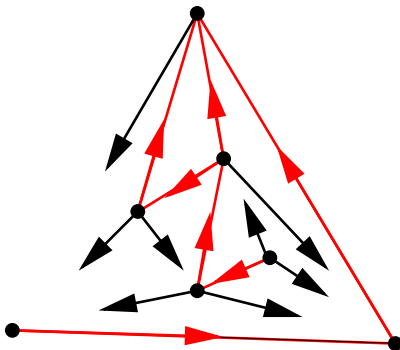
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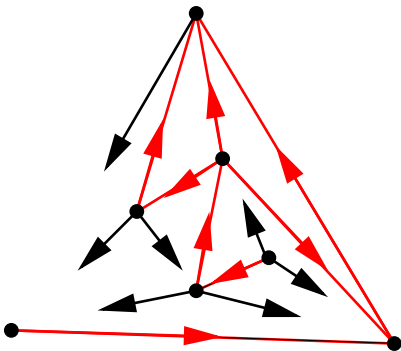
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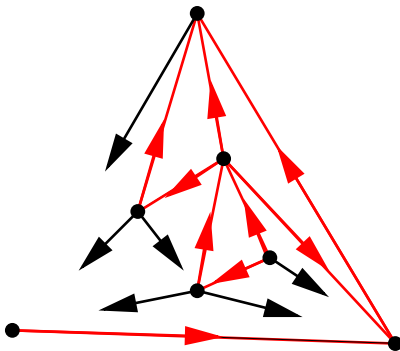
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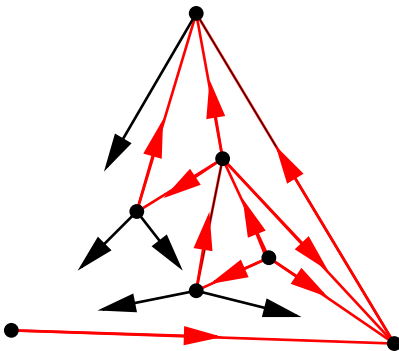
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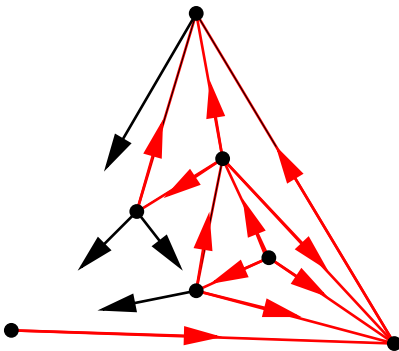
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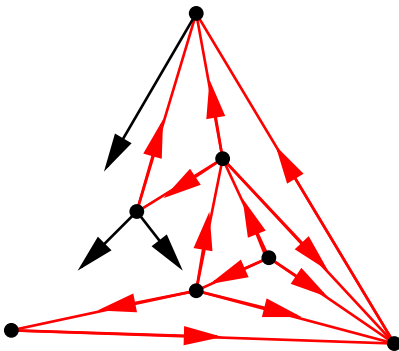
## Torus Case

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Triangulations

Vincent  
DESPRE

Planar Case

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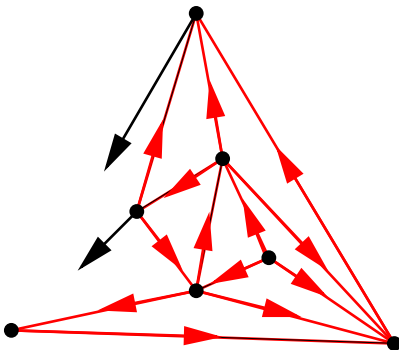
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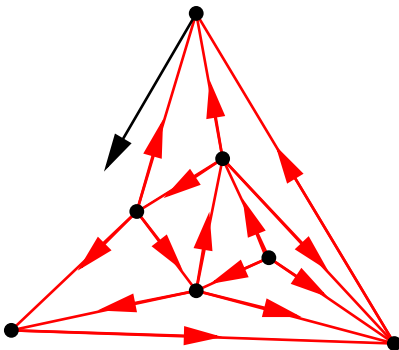
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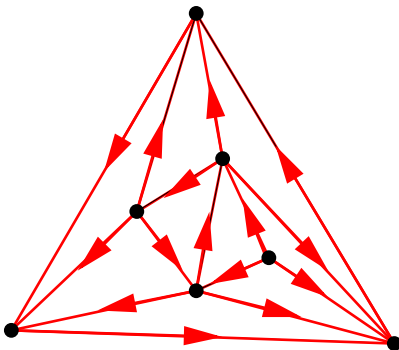
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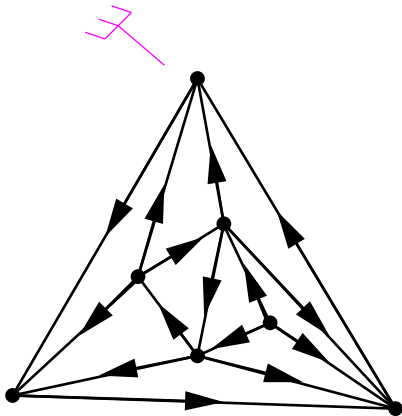
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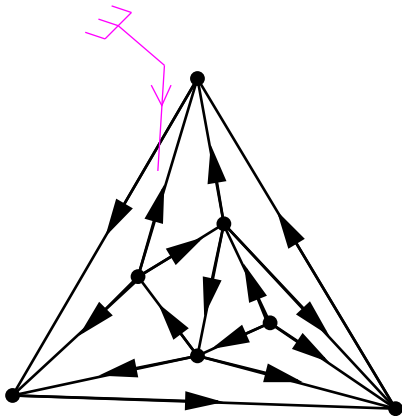
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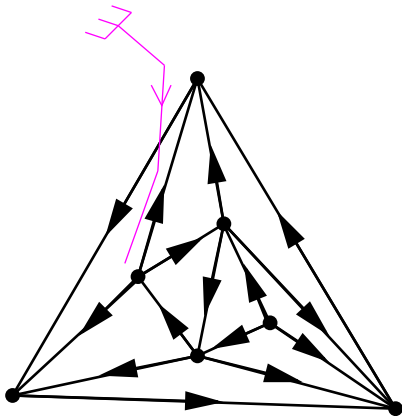
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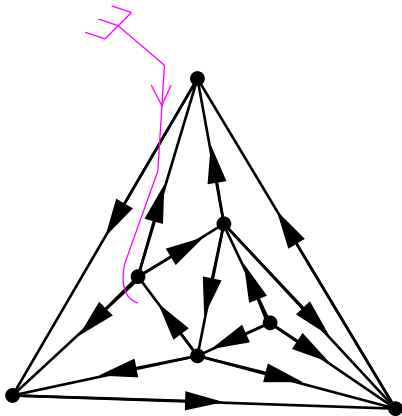
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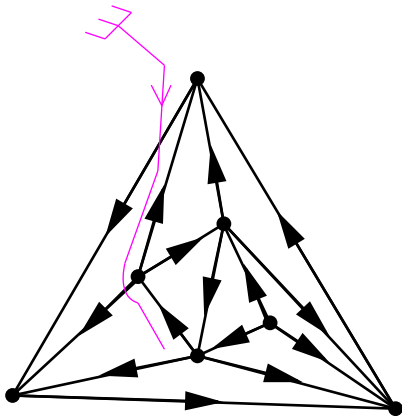
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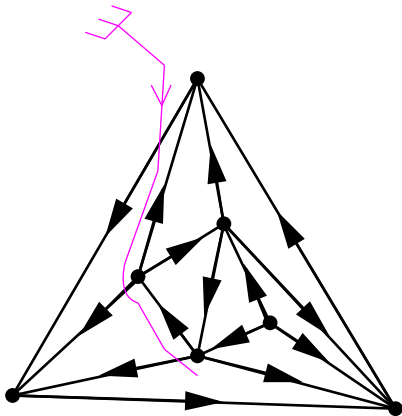
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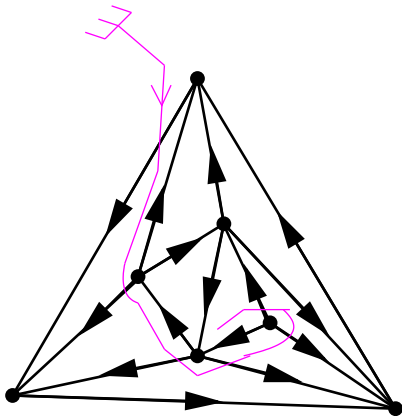
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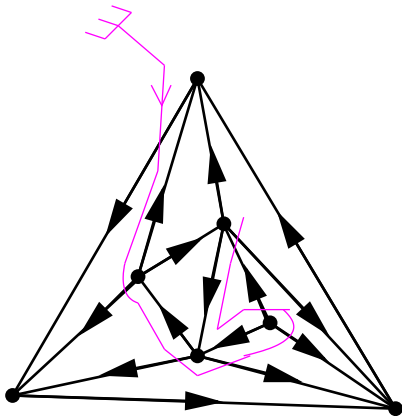
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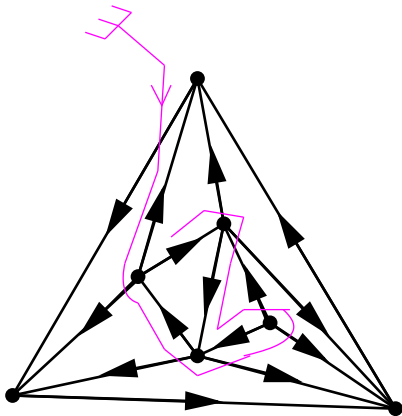
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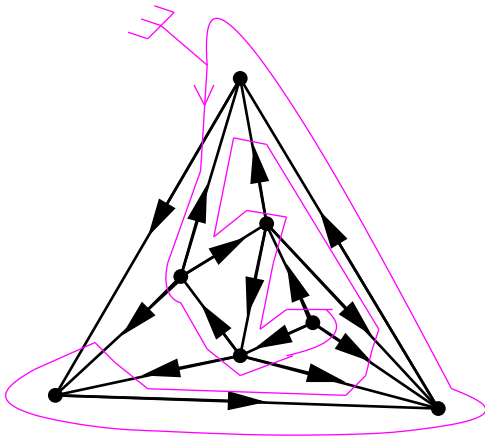
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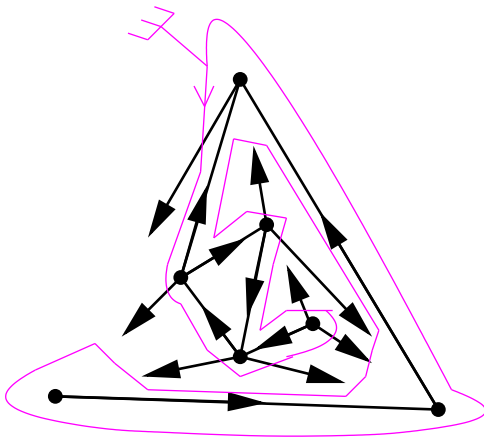
## Torus Case

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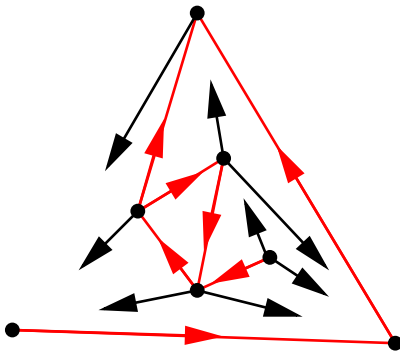
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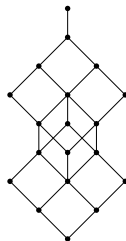
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# Conditions



Distributive lattice



# Conditions

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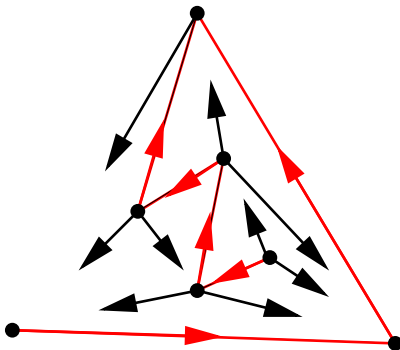
## Torus Case

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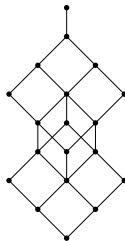
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Distributive lattice



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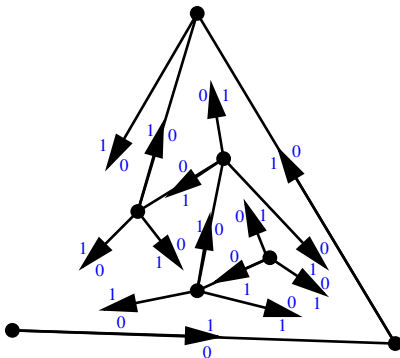
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101101011101011010001010001100

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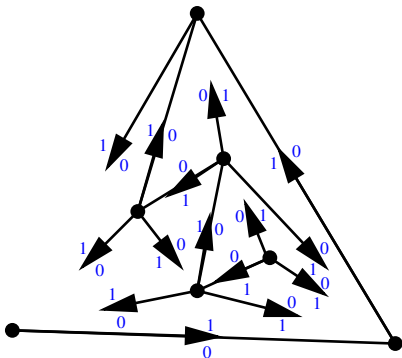
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101101011101011010001010001100  $\rightsquigarrow$  6n bits



# Encoding the Tree

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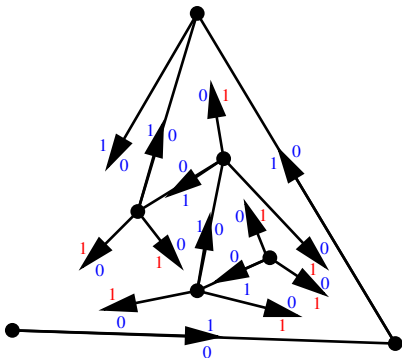
Torus Case

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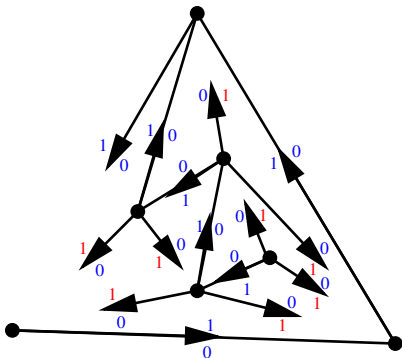
Encoding

Future Sight



101101011101011010001010001100  $\rightsquigarrow$  6n bits

# Encoding the Tree



101101011101011010001010001100

# Encoding the Tree

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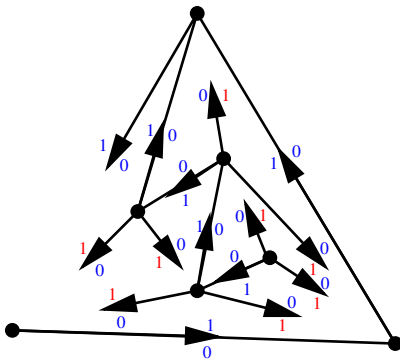
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1010011001000000001100

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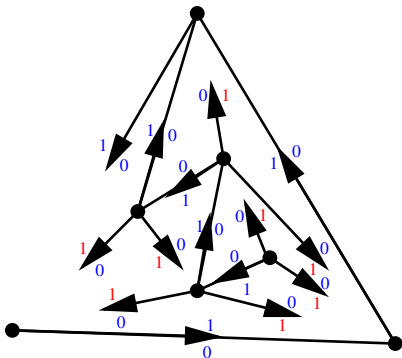
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1010011001000000001100  $\rightsquigarrow$   $4n$  bits

# Encoding the Tree

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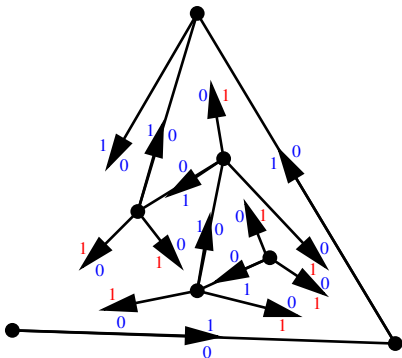
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1010011001000000001100  $\rightsquigarrow$   $4n$  bits ( $n \times 1$ )

# Encoding the Tree

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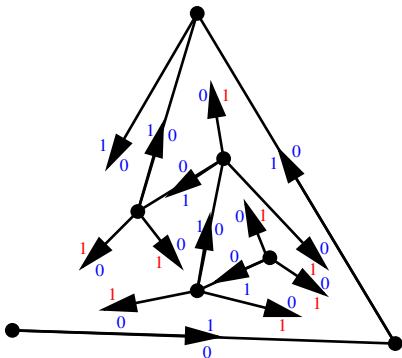
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1010011001000000001100  $\rightsquigarrow$   $4n$  bits ( $n \times 1$ )  $\rightsquigarrow$   $3,25n$  bits

**OPTIMAL !**

# Steps of the Algorithm

In addition it gives a new proof of Tutte's formula:

$$\mathcal{T}_{n+2} = \frac{2(4n-3)!}{n!(3n-1)!}$$

$$\log(\mathcal{T}_{n+2}) \sim \log\left(\frac{256}{27}\right)n \simeq 3.25n$$

**INPUT:** A triangulation of the plane.

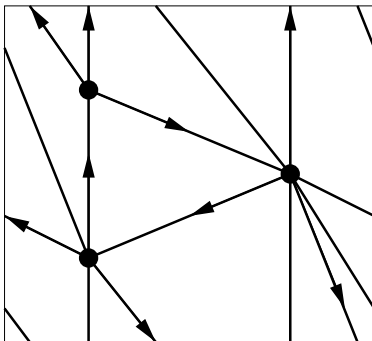
**OUTPUT:** The corresponding encoding.

- i/ Choose an orientation.
- ii/ Go down in the lattice to find the minimum element.
- iii/ Run Poulalhon and Schaeffer's algorithm.
- iv/ Encode.

# Triangulations on Oriented Surfaces

Euler's formula in genus  $g : v - e + f = 2 - 2g$

	Genus	Triangulation
Plane	0	$e = 3v - 6$
Torus	1	$e = 3v$



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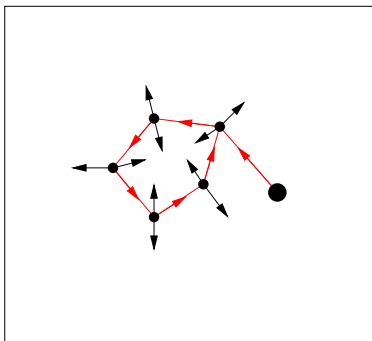
Future Sight



# Triangulations on Oriented Surfaces

Euler's formula in genus  $g : v - e + f = 2 - 2g$

	Genus	Triangulation
Plane	0	$e = 3v - 6$
Torus	1	$e = 3v$



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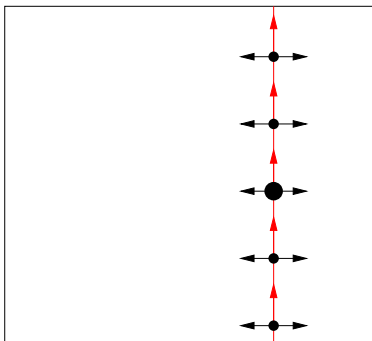
Encoding

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# Triangulations on Oriented Surfaces

Euler's formula in genus  $g : v - e + f = 2 - 2g$

	Genus	Triangulation
Plane	0	$e = 3v - 6$
Torus	1	$e = 3v$



# Triangulations on Oriented Surfaces

Euler's formula in genus  $g : v - e + f = 2 - 2g$

	Genus	Triangulation
Plane	0	$e = 3v - 6$
Torus	1	$e = 3v$

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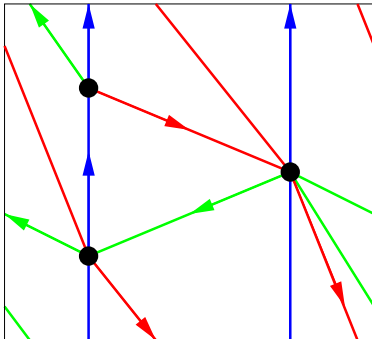
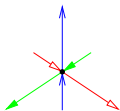
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# Triangulations on Oriented Surfaces

Euler's formula in genus  $g$  :  $v - e + f = 2 - 2g$

	Genus	Triangulation
Plane	0	$e = 3v - 6$
Torus	1	$e = 3v$

## Theorem (Barát and Thomassen 2006)

Triangulation on a surface  $\implies$   
orientation of the edges such that  $d^+(v) = 0 \pmod 3$ .

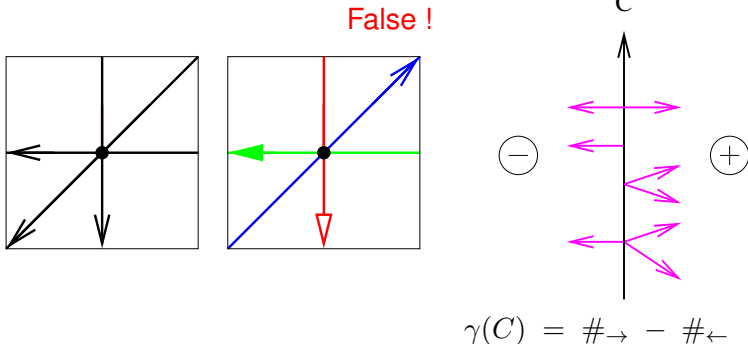
## Theorem (Albar, Gonçalves and Knauer 2014)

Triangulation on a surface  $g \geq 1 \implies$   
orientation of the edges such that  $d^+(v) = 0 \pmod 3$ ,  
 $d^+(v) > 0$

# Characterization

$g = 0$  : Schnyder wood  $\iff$  3-orientation

$g > 0$  : Schnyder wood  $\iff$   $(0 \bmod 3)$ -orientation ?



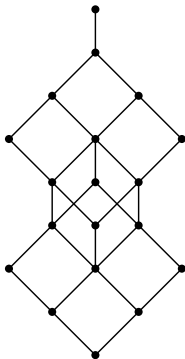
**Theorem (Gonçalves, Knauer and Lévêque 2014)**

Schnyder wood  $\iff$   $(0 \bmod 3)$ -orientation and  
for every cycle  $C$ ,  $\gamma(C) = 0 \bmod 3$

# Lattice structure

Theorem (Gonçalves, Knauer and Lévêque 2015)

Homologous orientations of a map on an orientable surface  
+ Fix a face  $\rightsquigarrow$  distributive lattice



Related to older results of Propp (1993), Ossona de Mendez (1994), Felsner (2004)

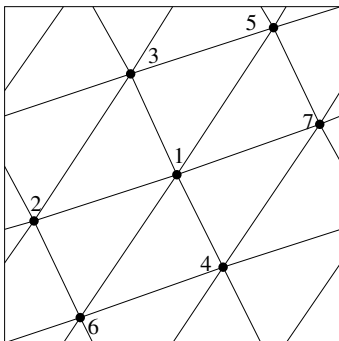
# Steps of the Algorithm

INPUT: A triangulation of the torus.

OUTPUT: The corresponding encoding.

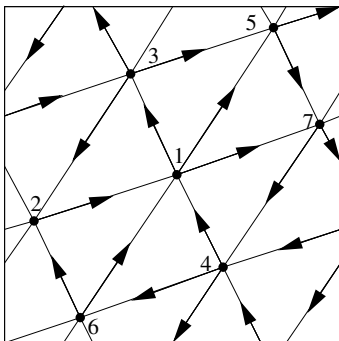
- i/ Choose an orientation.
- ii/ Go in the good lattice.
- iii/ Go down in the lattice to find the minimum element.
- iv/ Run Poulalhon and Schaeffer's algorithm.
- v/ Encode.

# Poulalhon-Schaeffer's Algorithm on the Torus

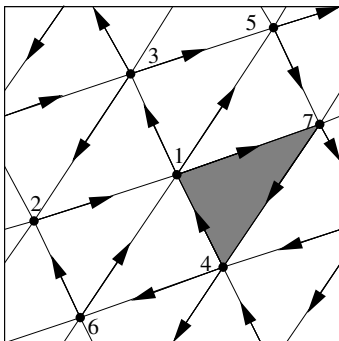




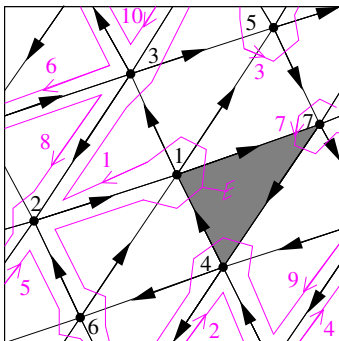
# Poulalhon-Schaeffer's Algorithm on the Torus



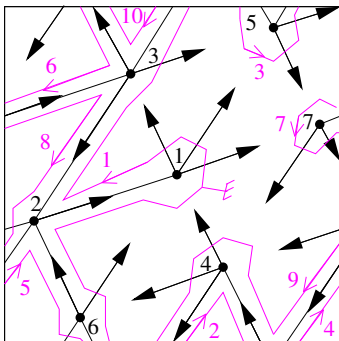
# Poulalhon-Schaeffer's Algorithm on the Torus



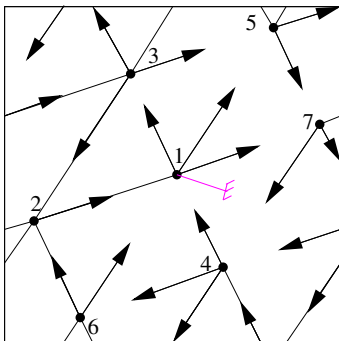
# Poulalhon-Schaeffer's Algorithm on the Torus



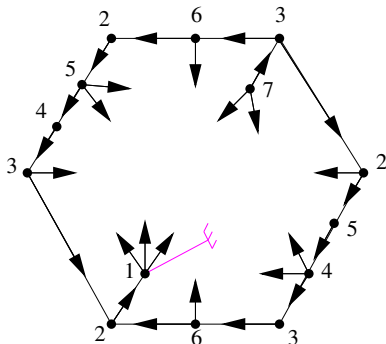
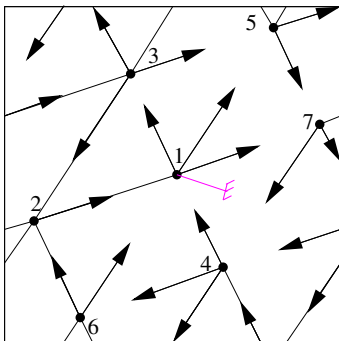
# Poulalhon-Schaeffer's Algorithm on the Torus



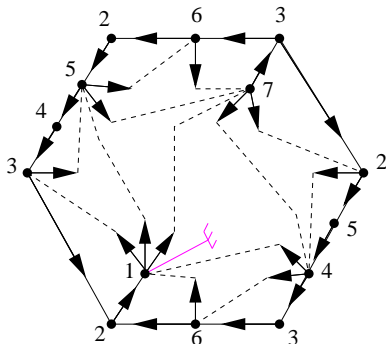
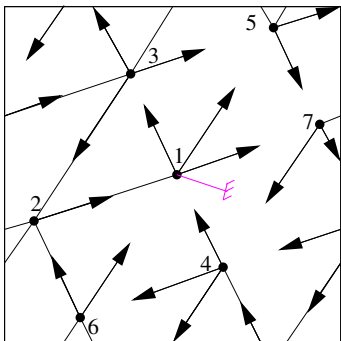
# Poulalhon-Schaeffer's Algorithm on the Torus



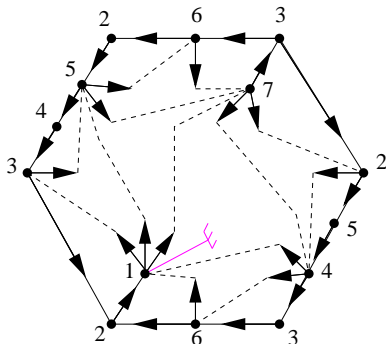
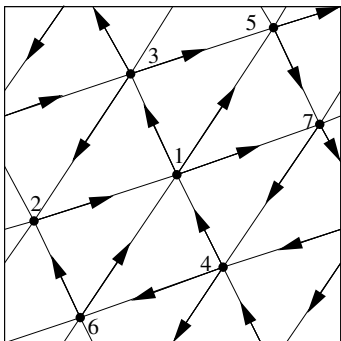
# Poulalhon-Schaeffer's Algorithm on the Torus



# Poulalhon-Schaeffer's Algorithm on the Torus

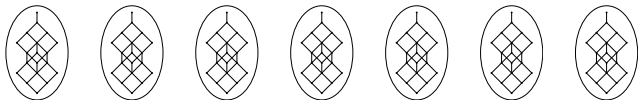
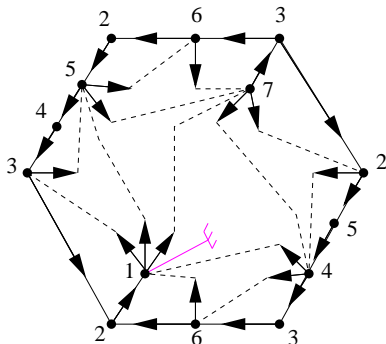
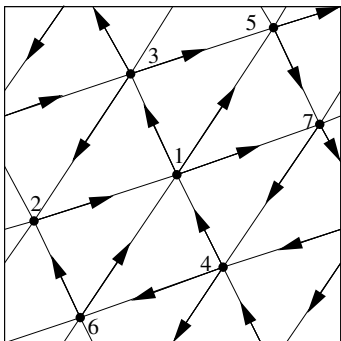


# Poulalhon-Schaeffer's Algorithm on the Torus

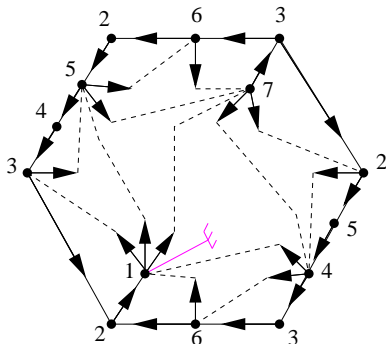
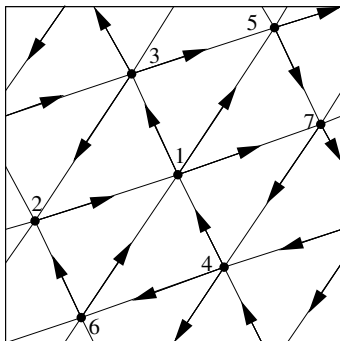




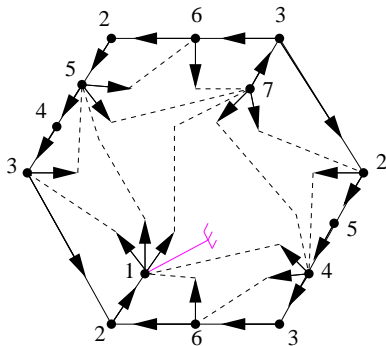
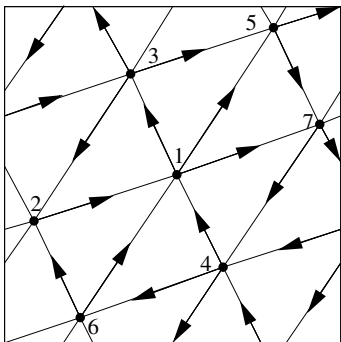
# Poulalhon-Schaeffer's Algorithm on the Torus



# Poulalhon-Schaeffer's Algorithm on the Torus



# Poulalhon-Schaeffer's Algorithm on the Torus



**Theorem (D., Gonçalves and Lévêque 2015)**

Applied on the minimal  $\gamma_0$ -Schnyder wood, Poulalhon and Schaeffer's algorithm outputs a spanning unicellular map.

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# Counter-examples

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Summary

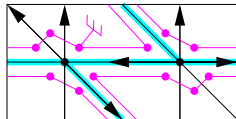
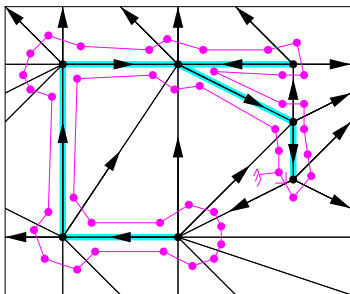
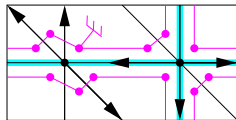
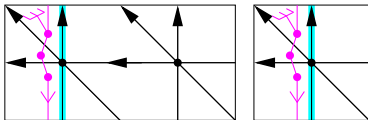
## Torus Case

Algorithm

**Structure**

Encoding

## Future Sight



# Encoding

Encoding  
Toroidal  
Triangulations

Vincent  
DESPRE

Planar Case

3-Orientations

Algorithm

Bijection

Encoding

Summary

Higher Genus

Orientations

Structure

Summary

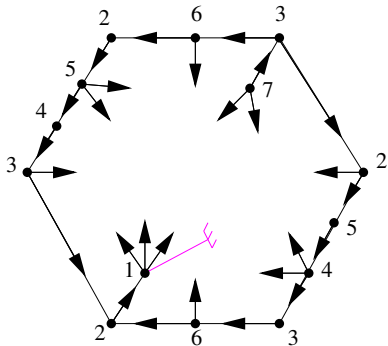
Torus Case

Algorithm

Structure

**Encoding**

Future Sight



# Encoding

Encoding  
Toroidal  
Triangulations

Vincent  
DESPRE

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Summary

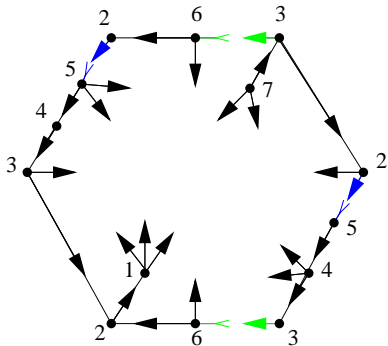
Torus Case

Algorithm

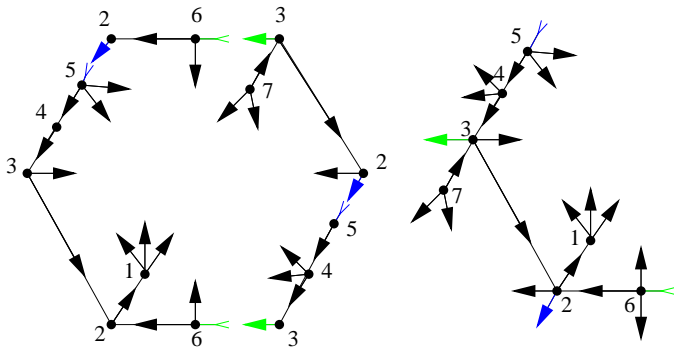
Structure

Encoding

Future Sight



# Encoding



Vincent  
DESPRE

Planar Case

3-Orientations

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Torus Case

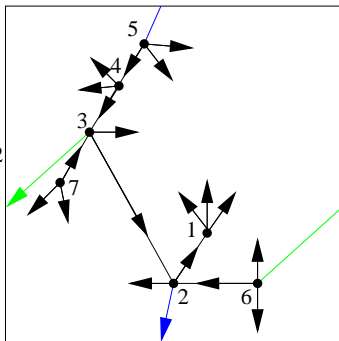
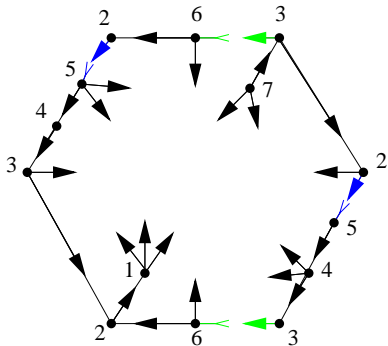
Algorithm

Structure

Encoding

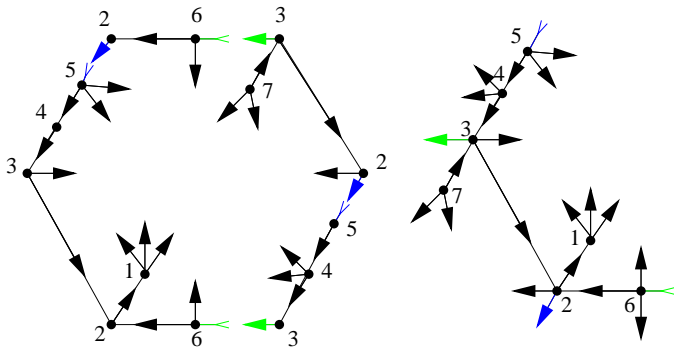
Future Sight

# Encoding

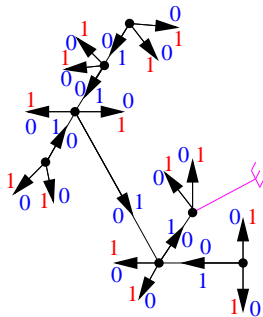
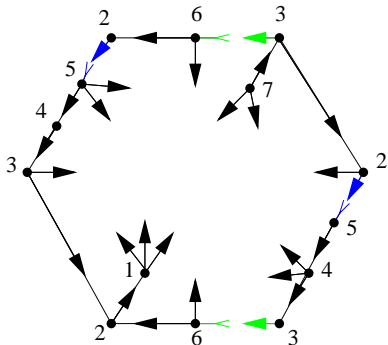




# Encoding



# Encoding



1010111011101001010010110100010101101000

↪ 6n bits

# Encoding

Planar Case

3-Orientations

Algorithm

Bijection

Encoding

Summary

Higher Genus

Orientations

Structure

Summary

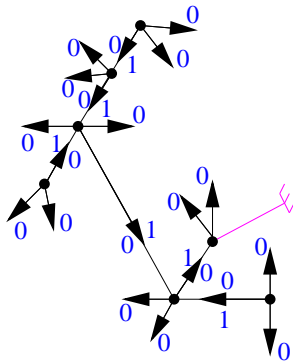
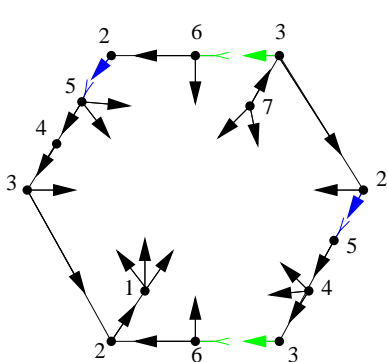
Torus Case

Algorithm

Structure

Encoding

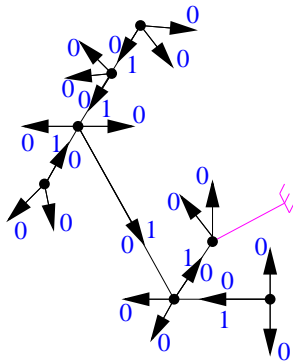
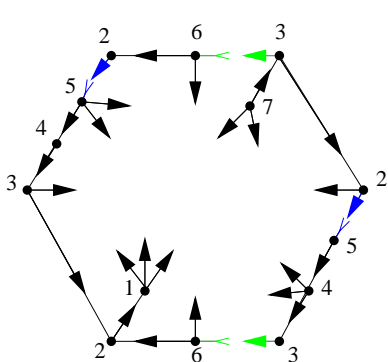
Future Sight



00110110000000100000010000

↪ 4n bits (n bits 1) ↪ 3,25n bits

# Encoding



00110110000000100000010000

$\rightsquigarrow$   $4n$  bits ( $n$  bits 1)  $\rightsquigarrow$   $3,25n$  bits

Optimal, linear and bijective !

# Future Sight

- ⇒ Counting and sampling.
- ⇒ Bijections for other toroidal maps :  $d$ -angulations, 3-connected maps, 4-connected triangulations, etc.
- ⇒ Higher genus : Is the generalization of the  $\gamma_0$  property possible?

## Conjecture

Triangulation on a surface  $g \geq 1 \implies$  orientation of the edges such that  $d^+(v) = 0 \pmod 3$ ,  $d^+(v) > 0$  and no oriented non-contractible cycle in the dual.