

# *The legacy of Fourier, Poincaré and Einstein about Relative Computation*

Françoise CHATELIN

*Ceremath, Université Toulouse 1  
CERFACS, 42 avenue G. Coriolis, 31057 Toulouse Cedex 1, France  
chatelin@cerfacs.fr*

Hypercomputation, that is nonlinear computation in real multiplicative Dickson algebras  $A_k \cong \mathbb{R}^{2^k}$ , is developed in (Chatelin 2011 a). For  $k \geq 2$  (resp.  $k \geq 3$ ) multiplication is not commutative (resp. associative). However addition remains both associative and commutative.

The situation changes in an essential way when computation is merely linear but there exists a *relator* which rules the way any two numbers are to be *added*. This kind of relating computation will be defined in precise terms. It includes the special case of an *explicit metric reference* consisting of a positive *finite* number  $\lambda$ ,  $0 < \lambda < \infty$ . The classical structure of an abelian additive group is weakened by considering an addition whose commutativity and associativity are controlled by the relator. A physically meaningful example was provided a century ago by 3D-Special Relativity (Einstein) where the role of  $\lambda$  as a metric reference is played by  $c$ , the speed of light, and the relator is a plane rotation.

In the early days of Special Relativity it was soon recognised that hyperbolic geometry underlies Einstein's law of addition for admissible velocities (Varičák 1910, Borel 1914) creating the relativistic effect known today as *Thomas precession* (Silberstein 1914, Thomas 1926).

But, despite Maxwell's valiant efforts (Maxwell 1871), Hamilton's noncommutative  $\times$  of 4-vectors was still unacceptable for most scientists at the dawn of the 20th century. Therefore Einstein's noncommutative  $+$  of 3-vectors (representing relativistically admissible velocities) was fully inconceivable: Einstein's vision was much ahead of its time! A more understandable version of Special Relativity was conceived by Minkowski in 1907, by dressing up as physical concepts the Lorentz transformations in the field of quaternions  $\mathbb{H}$  which had been introduced by Poincaré in 1905, see (Walter 1999, Auffray 2005). This is the version adopted until today in physics textbooks.

Einstein's intuition was left dormant for 80 some years until it was brought back to a new mathematical life in the seminal paper (Ungar 1988). During 20 years, Ungar has crafted an algebraic language for hyperbolic geometry which sheds a *natural light* on the physical theories of Special Relativity and Quantum Computation (Ungar 2008). We export some of Ungar's tools developed for mathematical physics into mathematical computation in a *relating* context. The reward of the shift of focus from physics to computation is to gain insight about the *geometric* ways by which information can be organically processed in the mind when *relation* prevails. This processing exemplifies the computational thesis posited in (Chatelin 2011 a,c).

Then we show how the seminal ideas of Fourier, Poincaré and Einstein blend together harmoniously to explain many features of computation in Nature and in the human mind.

1) The Fourier analysis of complex signals leads to the Fourier transform whose complex spectrum lies on the unit circle, with 4 eigenvalues  $+1, -1, i, -i$  which are the 4 units of the Gaussian ring of complex integers.

2) The Poincaré approach to relativity bears on the Lorentz transformations in the algebra  $\mathbb{H}$  of quaternions, using a noncommutative  $\times$ . Relative significant computation evolves from  $\mathbb{H}$  to  $\mathbb{G}^2$ , where  $\mathbb{G}$  is the algebra of octonions.

3) The Einstein perspective on relativity is based on a noncommutative  $\oplus$ , yielding a geometric 2-fold information potential with 2 types of geodesics. The potential lies in a  $\mathbb{R}^3$ -framed metric cloth, a non-euclidean space which is a computational construct exhibiting some features attributed to either hyperbolic or elliptic axiomatically defined geometries. By mixing these 3 views with the quadratic logistic iteration, we are able to propose an algorithmic mechanism which can serve as a working model for the law of causality that we experience in Nature and mind (Chatelin 2011 d).

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