

Points, Distances, and Cellular Automata: Geometric and Spatial Algorithmics

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Spatial computing aims at providing a scalable framework where computation is distributed on a uniform computing medium and communication happens locally between the nearest neighbors. Contrary to classical computation models based on Von-Neumann architecture, such a framework integrates from the start the fact that the communication time between two computation units is proportional to the distance separating them. We study the particular case of cellular automata, using a regular grid of computation units that update synchronously. As a first step towards generic computation, we propose to develop primitives allowing to structure the medium around a set of particles.

We consider three problems of geometrical nature. The first problem is analogous to load-balancing problems and considers moving the particles on the grid in order to uniformize their density [5]. This problem is solved specifically for the one dimensional grid and shows similarities with physical systems. Indeed implicit signals, corresponding to quantity of movements traveling through space, can be identified. They disappear either by cancellation with opposite signals or by leaving the space on the border.

The second problem is the construction of the convex hull of a set of particles [2, 6]. While this problem has already been tackled in the cellular automata framework, all solutions consider either that the space or the distance between the particle is bounded. They are also specific to particular cellular grids. The solution that we propose considers neither spatial nor distance bound, but uses nonetheless a finite number of states and a bounded neighborhood for any regular grid with an arbitrary number of dimensions.

The last problem is the construction of a connected proximity graph establishing connections between nearest particles [7]. The constructed graph is a generalization of the so-called Gabriel graphs, well-known in domains like clustering and wireless communication. Again, the solution that we propose considers no bound, uses a finite number of state, and works any regular multidimensional grids. In fact, the construction of this graph is a building block for the convex hull algorithms.

All these solutions are obtained by following the same global methodology. Our approach is to consider the metric space underlying the cellular automata topology and construct generic mathematical objects based solely on this metric [3]. This is the reason why the algorithms derived from the properties of those objects generalize over arbitrary regular grids. We implemented the usual ones, including hexagonal, 4 neighbors, and 8 neighbors square grids.

Also, all the solutions are based on the same basic component: the distance field, which associates to each site of the space its distance to the nearest particle [1, 4]. While the distance values are not bounded, it is shown that the difference between the values of neighboring sites is bounded, enabling encoding of the gradient into a finite state field. Our algorithms are expressed in terms of movements according to such gradient, and/or in terms of detection of local patterns in the gradient. This is the reason why our solutions use only a small finite number of states.

References

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