

Recent Discussions in Cosmological Computation

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Abstract

Andréka, Némethi and their colleagues have developed a series of first order relativity theories, in which they consider the extent to which results from relativistic physics can be derived formally from various sets of basic axioms; some of the associated models are thought to permit systems that can solve formally undecidable problems by exploiting black hole geometries. In related work, we have recently started joint work looking at the occurrence of closed timelike curves (CTCs) in general relativistic models, as might exist for example as a consequence of traversable wormholes. In this talk I will discuss some of the consequences of CTCs, and their relevance to physical computation.

Why does ‘time’ seem so different to ‘space’? This difference is fundamental to the way we experience the world, yet in view of relativity theory it seems almost paradoxical. Suppose we model spacetime as a $4d$ Minkowski space with coordinates (t, x, y, z) . The effect of boosting an object’s x -velocity by an amount v can then be represented in the usual way as a Lorentzian transformation

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cosh \phi & -\sinh \phi & 0 & 0 \\ -\sinh \phi & \cosh \phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix} \quad \text{where } e^\phi = \sqrt{\frac{(1+v/c)}{(1-v/c)}}$$

or in other words, as a kind of *rotation* in spacetime. But consider what this implies. If an object can appear to *rotate* in spacetime, simply as a result of an observer changing their own relative motion, then space and time must be fundamentally similar in nature. Apart from the appearance of the negative sign in the metric, time should be no different to space.

But how different these seem. We appear to move through space at will, happily revisiting favourite locations as often as we wish. Yet time seems to flow inexorably forwards; the past is forever fixed, the future unknowable. The question arises, then, which interpretation (if either) is correct? Is time a unidirectional flow of a fundamentally different nature to space, or are time and space intrinsically identical types of thing?

This question was thrown into sharp relief by Gödel [Göd49], who demonstrated a rotating solution to Einstein’s general relativistic equations which included *closed timelike curves* (CTCs). An observer traversing a CTC behaves like any other, moving always forwards in time, but the geometry of the spacetime in which they move leads them around a loop, so that they eventually revisit an event in their own past. For such an observer, the concepts of *past* and *present* become largely indistinguishable, since every future event on a CTC lies also in the past, and vice versa. While Gödel’s solution appears to be unphysical (the exceptional isotropy of the cosmic background suggests the absence of a preferred rotation axis), it nonetheless highlights the possibility that CTCs exist within physically realistic universes, and maybe even our own. Unsurprisingly, given that CTCs appear to offer the possibility of ‘time travel’ and causality violations, their consequences have been the focus of considerable research in cosmology [Haw92, Tho93, Vis03], and more recently, in computer science [AW09].

In this talk we discuss the consequences of CTCs for computer science, taking as our theme various recent conversations with members of the geometric logic in Budapest.³ The Budapest group have developed a series of first-order relativity theories, which provide a solid mathematical foundation for reasoning about cosmological behaviours [AMNS11]. We note that the nature of CTC ‘time travel’ is itself unclear, since it can be interpreted in two very different ways. In neither case, however, do CTCs, of themselves, necessarily entail causality violations. We also discuss the possibilities raised by CTCs both in terms of increased computational efficiency [Sta11], and regarding physical hypercomputation [ANS11].

References

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