Spatial Programming for Musical Representation and Analysis

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mgs.spatial-computing.org

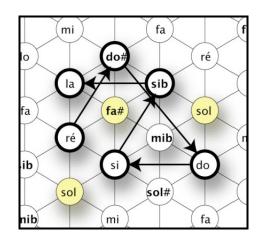
LACL, University Paris Est Créteil LIFO, University of Orléans – Mai 23-24, 2011

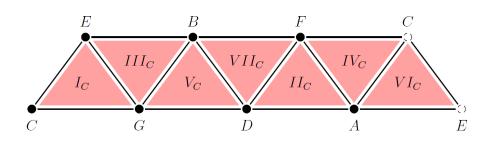
Context

- Spatial computing
 - Compute in space
 - Compute space
- MGS: a programming language for spatial computing
 - Introduction of topological concepts in a programming language
 - Two main principles
 - Data structure: topological collection
 - Control structure: transformation
- Musical analysis

Outline

- Background on MGS spatial-computing
- Music and spatial computing





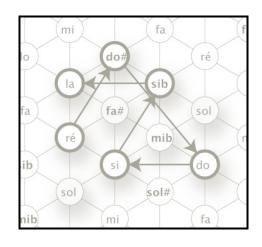
Space for musical representation

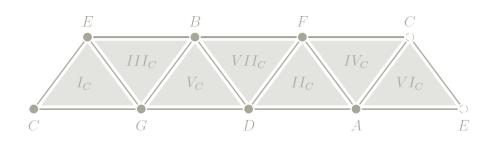
Space as musical representation

Conclusion

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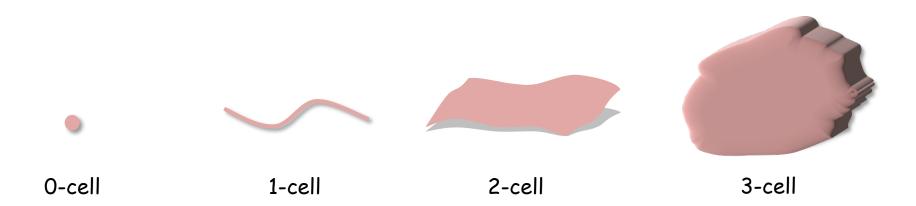


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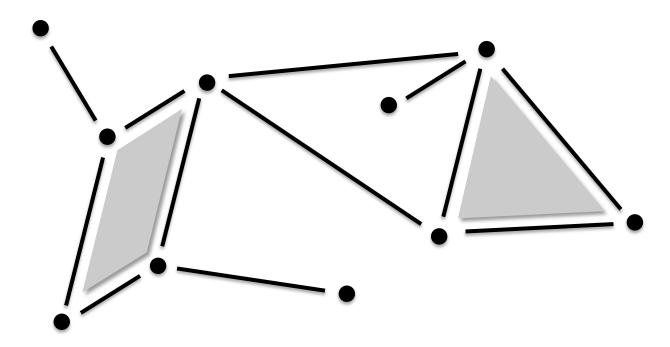
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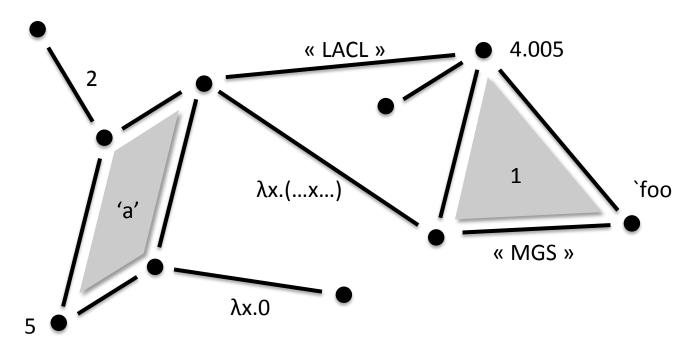
- Topological collections
 - Structure
 - A collection of *topological cells*



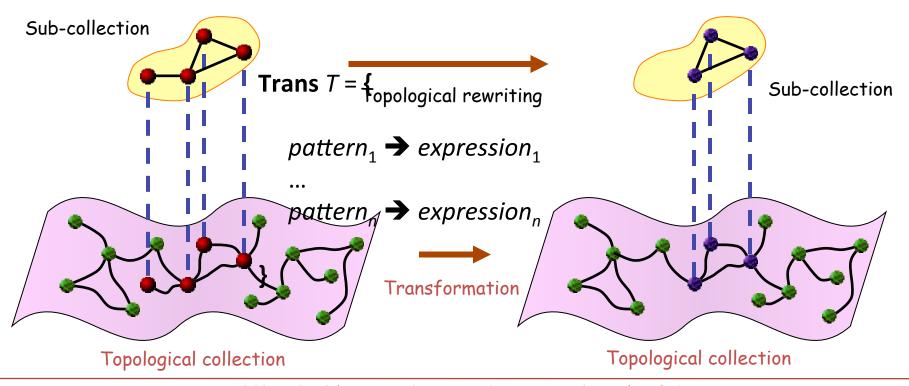
- Topological collections
 - Structure
 - A collection of topological cells
 - An *incidence relationship*



- Topological collections
 - Structure
 - A collection of topological cells
 - An incidence relationship
 - Data: association of a value with each cell

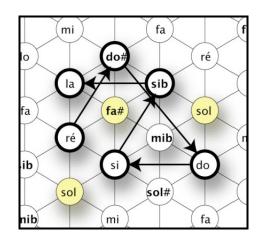


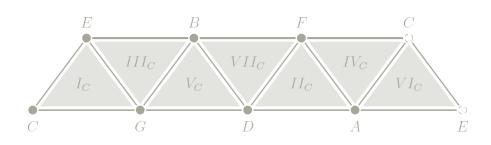
- Transformations
 - Functions defined by case on collections
 Each case (pattern) matches a sub-collection
 - Defining a rewriting relationship: topological rewriting



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Neo-Riemannian Problematic

- Traditional western music representation
 - Based on the use of a staff
 - Main drawbacks for visualization of harmony
 - Contrapuntal proximity
 The spatial distance between notes is not relevant for harmonic purpose
 - Spatially close patterns can sound very different

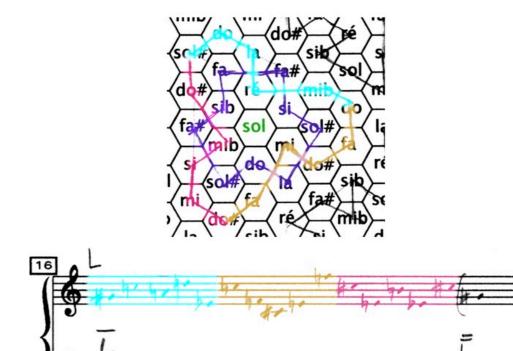


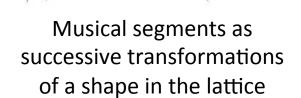


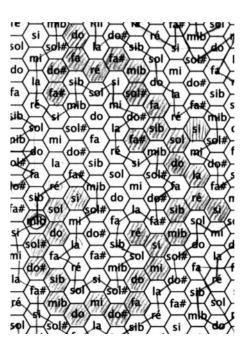
- Neo-Riemannian representation of music
 - Graphical representation of harmony rules
 - Consonance used as a graphical criterion
 Two notes that sound well must be spatially close

Neo-Riemannian Problematic

Musical composition (J.-M. Chouvel)



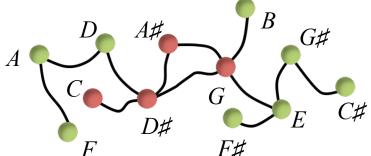




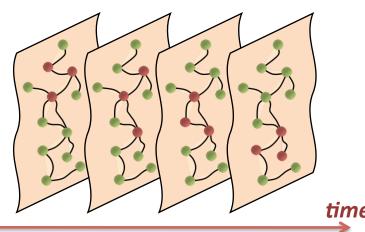
Musical melody as a path in the lattice

Spatial Representation of Musical Sequences

- Temporal succession of musical events
- Musical event as a topological collection
 - Positions are notes
 - Labels represent played notes



Succession of events as a stream of collections

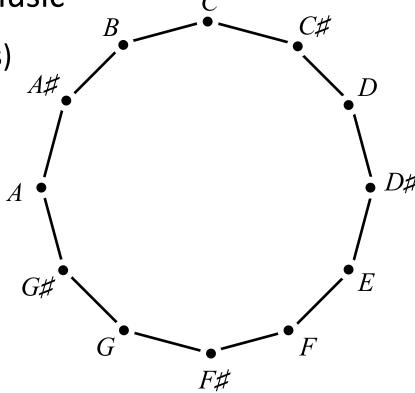


Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)

Which neighborhoods for significant visualization?

Strong algebraic structure of music

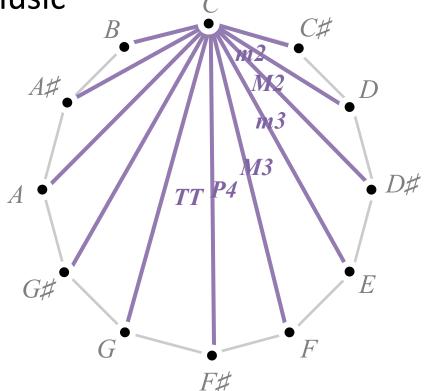
 \square Set $\mathcal N$ of notes (i.e. pitch classes) We do not consider octaves



 $\mathcal{N} = \{ C, C \#, D, D \#, E, F, F \#, G, G \#, A, A \#, B \}$

- Which neighborhoods for significant visualization?
- Strong algebraic structure of music

 - Group (I,+) of intervals
 - Relative difference between notes

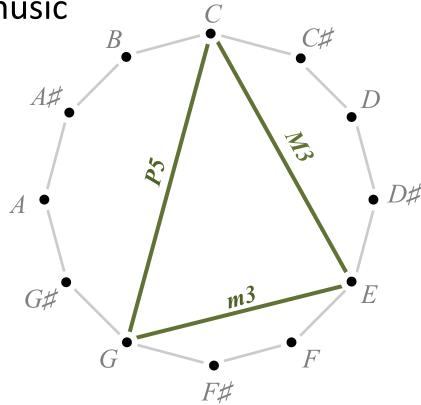


 $I = \{ P1, m2, M2, m3, M3, P4, TT, P5, m6, M6, m7, M7 \}$

- Which neighborhoods for significant visualization?
- Strong algebraic structure of music

- Group (I,+) of intervals
 - Relative difference between notes
 - $(I,+) \cong (Z_{12},+)$ (isomorphism)
 - Example

$$M3 + m3 = P5$$



 $I = \{ P1, m2, M2, m3, M3, P4, TT, P5, m6, M6, m7, M7 \}$

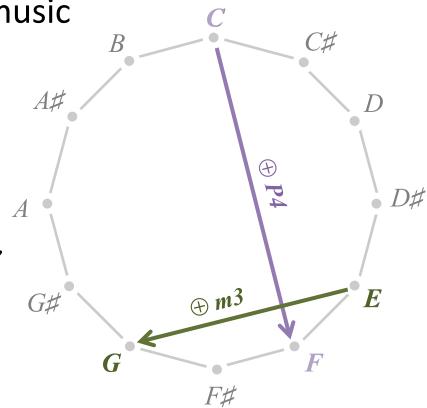
- Which neighborhoods for significant visualization?
- Strong algebraic structure of music

$$\square \quad \mathsf{Set} \ \mathcal{N} = \{ \ \textit{C, C\#, ..., A\#, B} \ \}$$

- Group $(I,+) = \{ P1, ..., M7 \}$
- Transposition ⊕ of notes
 - $n \oplus i = n$ '
 if i is the interval between n and n'
 - Example

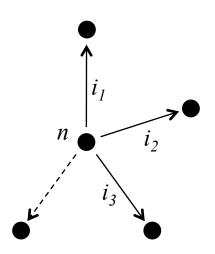
$$C \oplus P4 = F$$

$$\Box E \oplus m3 = G$$



 \oplus action of I over ${\mathcal N}$

- Consonance as a neighborhood relationship
 - $S \subset \mathcal{N} \times \mathcal{N}$
 - $(n_1, n_2) \in S$ if n_1 "sounds well" with n_2
- \blacksquare Assumptions on S
 - \square S is symmetric
 - $(n_1, n_2) \in S \Rightarrow (n_2, n_1) \in S$
 - \square S is defined up to a transposition
 - $\forall i \in I$, $(n_1, n_2) \in S \Rightarrow (n_1 \oplus i, n_2 \oplus i) \in S$
 - \blacksquare (C, G) sounds well \Rightarrow (E= C \oplus M3, B = G \oplus M3) sounds well
- lacksquare S characterized by a subset I of I
 - $I = \{ i_1, i_2, ..., i_n \}$
 - $\forall n \in \mathbb{N}, \forall i \in I, (n, n \oplus i) \in S$



- Spatial representation of S
 - I as a set of group generators
 - \blacksquare < I > subgroup of I generated by the elements of I
 - **Example** $I = \{ M2 \}$

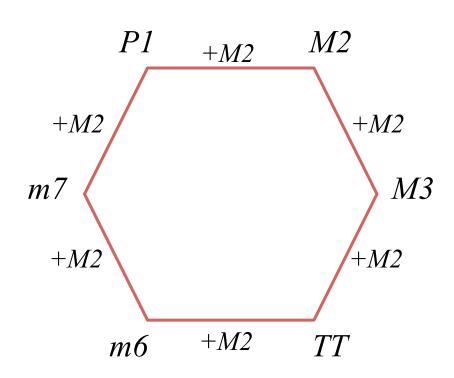
$$S = \{ (C,D), (C\#,D\#), ... \}$$

$$I = \{ M2 \}$$

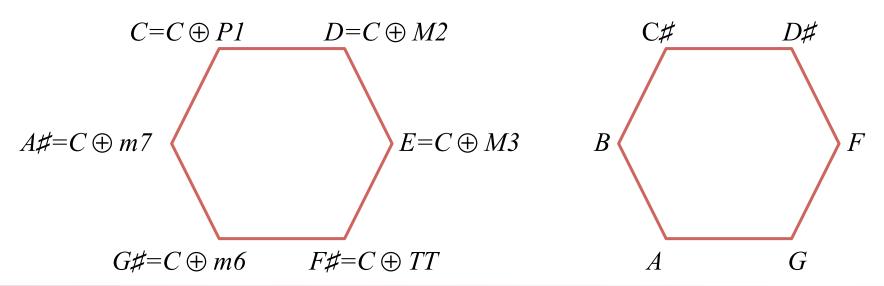
$$< I > = \{ P1, P1 + M2, P1 + 2.M2, ... \}$$

$$= \{ P1, M2, M3, TT, m6, m7 \}$$

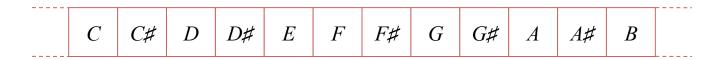
- Spatial representation of S
 - I as a set of group generators
 - \square Graph representation of < I >
 - Cayley's graph
 - \square Vertices: intervals of < I >
 - Edges: generators of I
 - **Example** with $I = \{ M2 \}$



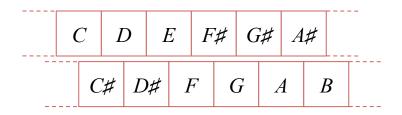
- Spatial representation of S
 - \square I as a set of group generators
 - \square Graph representation of < I >
 - Representation of S based on Cayley graph
 - \blacksquare Action of < I > on N
 - **Example** with $I = \{ M2 \}$

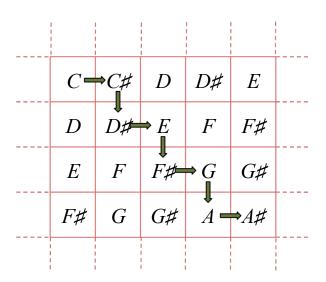


- Scale representations



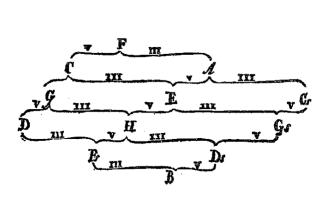
□ Whole-tone scale $I = \{ M2 \}$





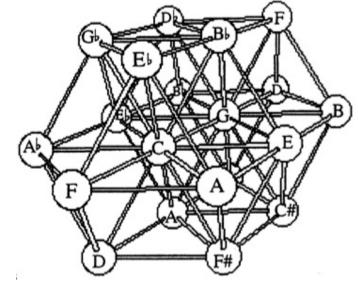
- Traditional harmony representation
 - $I = \{ M3, P5 \}$ (Euler's Tonnetz)
 - $I = \{ m3, M3, P5 \}$ (Harmonic table)
 - $I = \{ m3, M3, P5, m7 \}$ (3D-Tonnetz)

mi



fa fa# sol sol sol# fa

fa

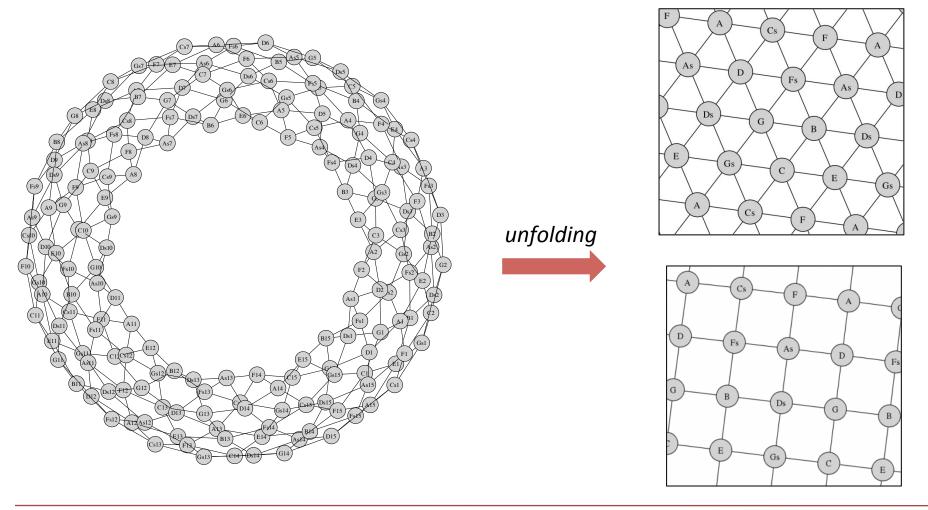


(Euler)

(J.-M. Chouvel)

(E. Gollin)

Automatic graph generation



Instruments conception

$$I = \{ m2, P4 \}$$
 (Guitar)





 $I = \{ m2, P5 \}$ (Violin)

$$I = \{ m2, M2, m3 \}$$
 (Accordion)

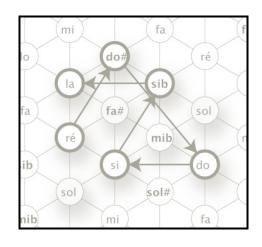


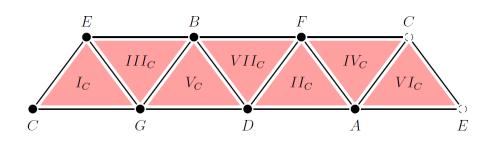
- Analysis example
 - Signature of a piece
 - Example : F. Chopin Prelude

Extract of the Prelude N.4 Op28 of F. Chopin

Outline

- Background on MGS spatial-computing
- Music and spatial computing





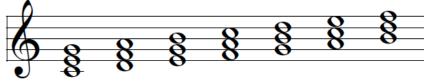
Space for musical representation

Space as musical representation

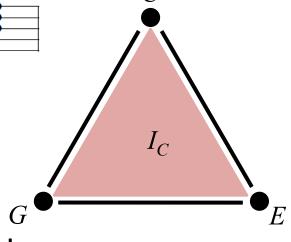
Conclusion

Tonality and Möbius Strip

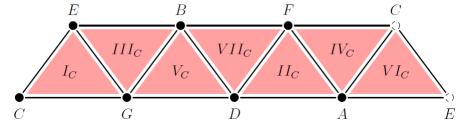
- Motivation: spatial visualization of tonality
- Association of a chord set with the tonality: the degrees
 - Example: C-major tonality



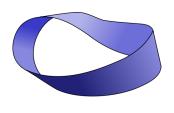
- Spatial representations
 - Note = vertex
 - Chord = surface



Fusion of the common notes for the 7 degrees







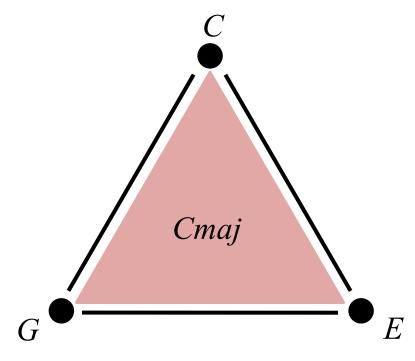
Self-Assembly of Chords

- Automation of the process for the analysis of other chords sequences
- Reaction of the chords between themselves
- Simplicial representation of musical objects

Note: 0-simplex

2-note chord: 1-simplex

3-note chord: 2-simplex



Self-Assembly of Chords

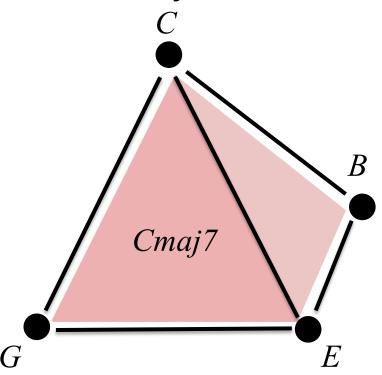
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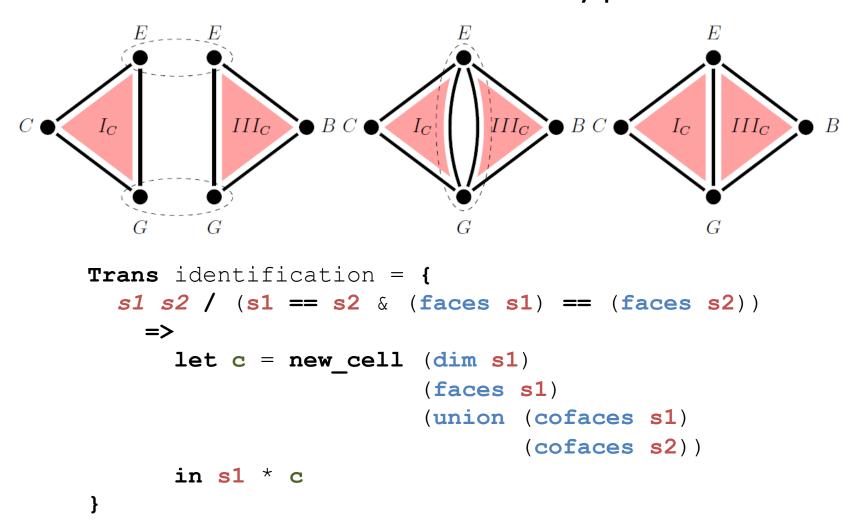
3-note chord: 2-simplex

4-note chord: 3-simplex

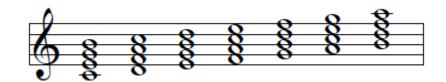


Self-Assembly of Chords

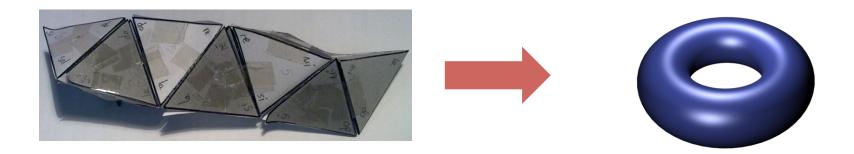
MGS transformation for self-assembly process



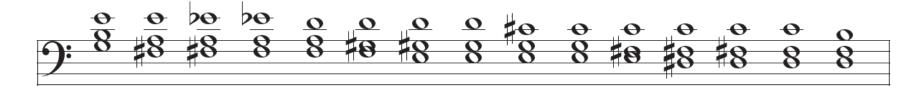
Four-note degrees of C-major tonality

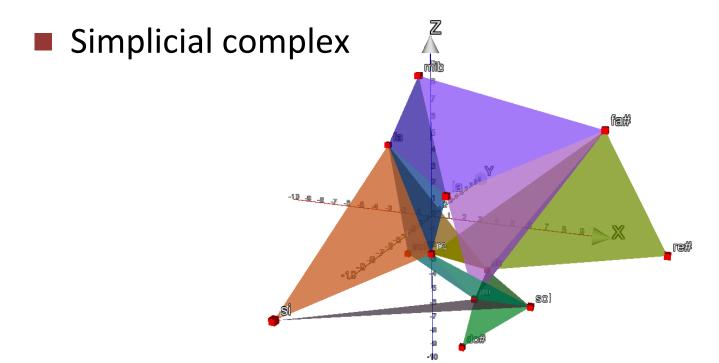


- Chord = 3-simplex (tetrahedrons)
- Self-assembly

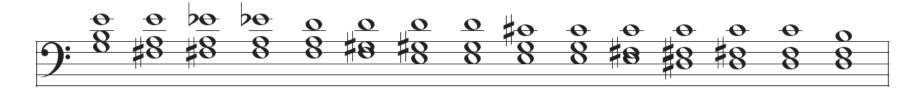


Extract of the Prelude No. 4 Op. 28 of F. Chopin

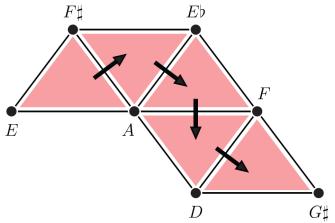




Extract of the Prelude No. 4 Op. 28 of F. Chopin



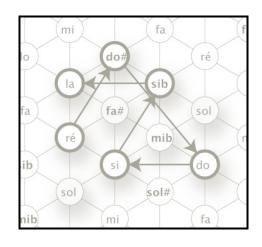
Analysis of the path under the chords

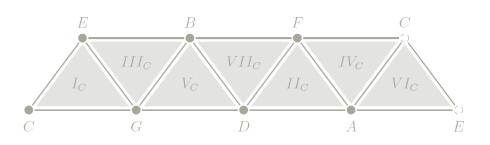


■ The path chosen by F. Chopin is associated with the smallest movements on the chords

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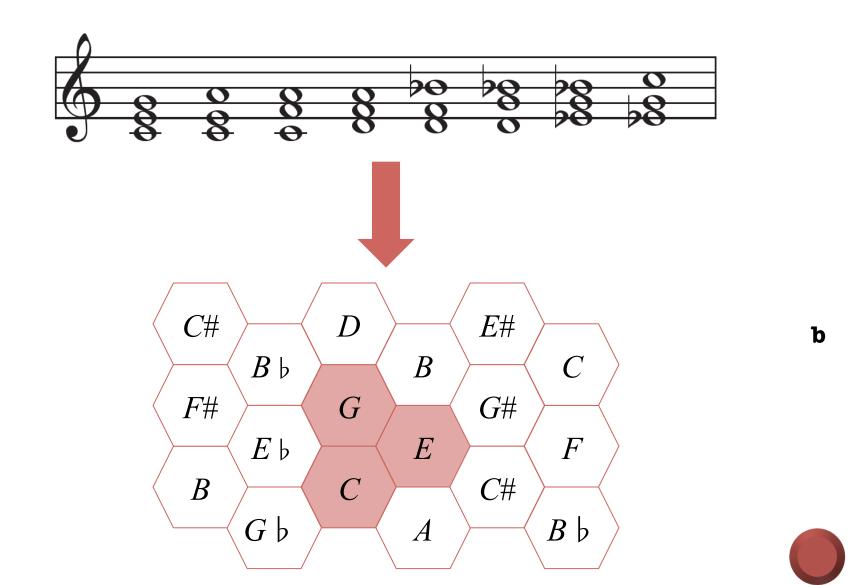
Conclusion & Perspectives

- Preliminary work
- Strong collaborations with composers / musicologists
- Extend the validation on more musical problems
 - Spatial representations of other musical properties (timbre, rythm, fingering etc.)
- Extension to study musical styles
- Spatial properties ⇔ musical properties
- Acknowledgements

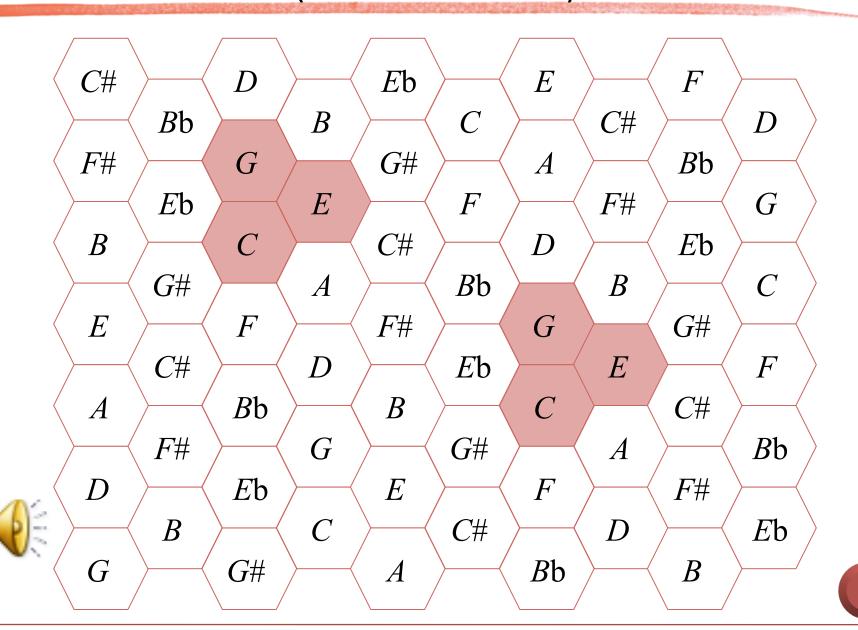
Jean-Louis Giavitto, Moreno Andreatta, Carlos Agon, Jean-Marc Chouvel



Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)



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Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)

⟨ C# ⟩	\sim Eb	E	\	\overbrace{F}
\rightarrow Bb	$\langle B \rangle$	$\langle C \rangle$	C#	\longrightarrow D
$\langle F\# \rangle \longrightarrow \langle G \rangle$	G#	$\stackrel{\cdot}{=}$ A		Bb
Eb	$\langle E \rangle$	$\langle F \rangle$	$\langle F\# \rangle$	$ angle \hspace{-0.5cm} - \hspace{-0.5cm} \left\langle \hspace{-0.2cm} \hspace{-0.2cm} \hspace{-0.2cm} \hspace{-0.2cm} \hspace{-0.2cm} G \hspace{-0.2cm} \hspace{-0.2cm} ight angle$
$\langle B \rangle \longrightarrow C$	\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	$\rightarrow \qquad D$		Eb
\rightarrow G #	$\langle A \rangle$	$\langle Bb \rangle$	$\langle B \rangle$	$ angle \qquad \left\langle C ight angle$
$\langle E \rangle \longrightarrow \langle F \rangle$	F#	$\langle G \rangle$		G#
\ \ \ C\# \ \	$\langle D \rangle$	\leftarrow Eb	$\langle E \rangle$	ightharpoons F
$\langle A \rangle \longrightarrow \langle Bb \rangle$	$\rightarrow B$	\rightarrow C		C# >
$ ightarrow \left\langle F^{\#} \right angle$	$\langle G \rangle$	$\langle G^{\#} \rangle$	$\langle A \rangle$	\rightarrow Bb \rightarrow
$\langle D \rangle \longrightarrow \langle Eb \rangle$	\rightarrow E	$\rightarrow \qquad \qquad F$		F#
\rightarrow B	$\langle C \rangle$	$\langle C\# \rangle$	$\langle D \rangle$	angle lacksquare Eb $ angle$
$\langle G \rangle - \langle G \#$	\nearrow A	Bb		B

Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)

$\langle C\# \rangle \longrightarrow \langle D \rangle \longrightarrow \langle E \rangle$	$E \longrightarrow F \longrightarrow$
\nearrow Bb \nearrow B	$C \rightarrow C \rightarrow C \rightarrow D$
$\langle F\# \rangle \longrightarrow \langle G \rangle \longrightarrow \langle G \rangle$	$H \rightarrow A \rightarrow Bb \rightarrow A$
\nearrow Eb \nearrow E	$\overline{}$ $F \longrightarrow F $
$\langle B \rangle \sim C \rangle \sim C$	
\longrightarrow G # \longrightarrow A	$\overline{}$ Bb $\overline{}$ C
$\langle E \rangle \longrightarrow \langle F \rangle \longrightarrow \langle F \rangle$	
\sim C# \sim D	\leftarrow Eb \rightarrow \leftarrow F \rightarrow
$\langle A \rangle \longrightarrow \langle Bb \rangle \longrightarrow \langle Bb \rangle$	
\nearrow $F\#$ G	G# A Bb
$\langle D \rangle \longrightarrow \langle Eb \rangle \longrightarrow \langle Eb \rangle$	
\nearrow B \nearrow C \nearrow	C# D Eb
$\langle G \rangle \langle G \# \rangle \langle A \rangle$	Bb

Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)

⟨ C# ⟩	Eb E	\nearrow \nearrow \nearrow \nearrow
\nearrow Bb \nearrow B		$\langle C\# \rangle - \langle D \rangle$
$\langle F\# \rangle \longrightarrow \langle G \rangle$	\longrightarrow $G\#$ \longrightarrow A	\rightarrow Bb
\rightarrow Eb \rightarrow E	$E \rightarrow F \rightarrow$	$\langle F\# \rangle \hspace{-1em} \langle G \rangle$
$\langle B \rangle \longrightarrow \langle C \rangle$	C# D	\rightarrow Eb \rightarrow
G#	$Bb \rightarrow$	$\langle B \rangle \longrightarrow \langle C \rangle$
$\langle E \rangle \longrightarrow \langle F \rangle$	F# G	G#
\rangle $C\#$ \rangle L	Eb	$\langle E \rangle \longrightarrow \langle F \rangle$
$\langle A \rangle \longrightarrow \langle Bb \rangle$	$B \rightarrow C$	C# >
\rightarrow $F\#$	$G \longrightarrow G\#$	$\langle A \rangle \longrightarrow \langle Bb \rangle$
$\langle D \rangle \longrightarrow \langle Eb \rangle$	$-\langle E \rangle -\langle F \rangle$	F#
\rightarrow $B \rightarrow$ C	$C \rightarrow C\#$	$\langle D \rangle \sim \langle Eb \rangle$
$\langle G \rangle - \langle G \# \rangle$	A Bb	B

Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)

C# D Eb F
\nearrow Bb \nearrow C $C\#$ D
$\langle F\# \rangle \longrightarrow \langle G \rangle \longrightarrow \langle G\# \rangle \longrightarrow \langle Bb \rangle \longrightarrow \langle $
\nearrow E b \nearrow E \nearrow F # \nearrow G \nearrow
$\langle B \rangle \langle C \rangle \langle C\# \rangle \langle D \rangle \langle Eb \rangle \langle C\# \rangle \langle D \rangle \langle Eb \rangle \langle D \rangle \langle Eb \rangle \langle D \rangle$
G# A Bb B C
$\langle E \rangle \hspace{1cm} F \hspace{1cm} \langle F \rangle \hspace{1cm} \langle F \rangle \hspace{1cm} \langle G \rangle \hspace$
C# D E E F
$A \rightarrow Bb \rightarrow C \rightarrow C\#$
F# G $G#$ A Bb
$\langle D \rangle \langle Eb \rangle \langle E \rangle \langle F \rangle \langle F$
\nearrow B \nearrow C \nearrow $C\#$ D \nearrow E b
$\langle G \rangle \langle G \# \rangle \langle A \rangle \langle Bb \rangle \langle B \rangle$

Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)

⟨ <i>C</i> #⟩	D		<i>E</i> b	\	E	\	F	<u></u>
\rightarrow Bb	\	B	\longrightarrow	C	\longrightarrow	C#	\longrightarrow	$\left\langle D \right\rangle$
$\langle F\# \rangle$	G		$G^{\#}$	\longrightarrow	A	\rightarrow	<i>B</i> b	
\rightarrow Eb		E	\rangle — \langle	F	\rangle —— \langle	F#	\longrightarrow	G
$\langle B \rangle$	C	$\overline{}$	<i>C</i> #	\longrightarrow	D	\longrightarrow	<i>E</i> b	\rightarrow
ightarrow G#		A	\longrightarrow	<i>B</i> b	\rangle —— \langle	B	\longrightarrow	$\langle C \rangle$
$\langle E \rangle$	F		$F^{\#}$	\longrightarrow	G	\rightarrow	$G^{\#}$	
\ \ \ C#		D	\rightarrow	<i>E</i> b	\	E	\rightarrow	$\langle F \rangle$
$\langle A \rangle$	Bb	$\overline{}$	B	\rightarrow	C	\rightarrow	<i>C</i> #	
\nearrow $F\#$		G	\rightarrow	$G^{\#}$	\rightarrow	A	\rightarrow	Bb
$\langle D \rangle$	Eb		E		F		$F^{\#}$	—
\rightarrow B	—	C	\rightarrow	<i>C</i> #		D		$\langle Eb \rangle$
$\langle G \rangle$	G#		A		Bb		B	

Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)

⟨ <i>C</i> #⟩	$\langle D \rangle$		<i>E</i> b	\	E	\	F	
\rightarrow Bb		B	\rightarrow	C	\longrightarrow	C#	\longrightarrow	$\left\langle D\right\rangle$
$\langle F\# \rangle$	$\left G \right $		G#	\longrightarrow	A	\rightarrow	<i>B</i> b	
\rightarrow Eb		E	\rangle — \langle	F	\rangle —— \langle	F#	\longrightarrow	G
$\langle B \rangle$	$\langle C \rangle$	$\overline{}$	<i>C</i> #	\longrightarrow	D	\longrightarrow	<i>E</i> b	>
\rightarrow G #		A	\rangle — \langle	<i>B</i> b	\rangle —— \langle	B	\longrightarrow	$\langle C \rangle$
$\langle E \rangle$	$\langle F \rangle$	·	F#	\longrightarrow	G	\rightarrow	$G^{\#}$	>
C#		D	\rightarrow	<i>E</i> b	\	E	\rightarrow	$\left\langle F \right\rangle$
$\langle A \rangle$	Bb		B	\rightarrow	C	\rightarrow	<i>C</i> #	
\nearrow F #		G	\rightarrow	$G^{\#}$	\rightarrow	A	\rightarrow	Bb
$\langle D \rangle$	$\langle Eb \rangle$		E		F	<u> </u>	$F^{\#}$	—
\nearrow B		C	\\	<i>C</i> #		D		$\langle Eb \rangle$
$\langle G \rangle$	$race{G\#}$		A		Bb		B	

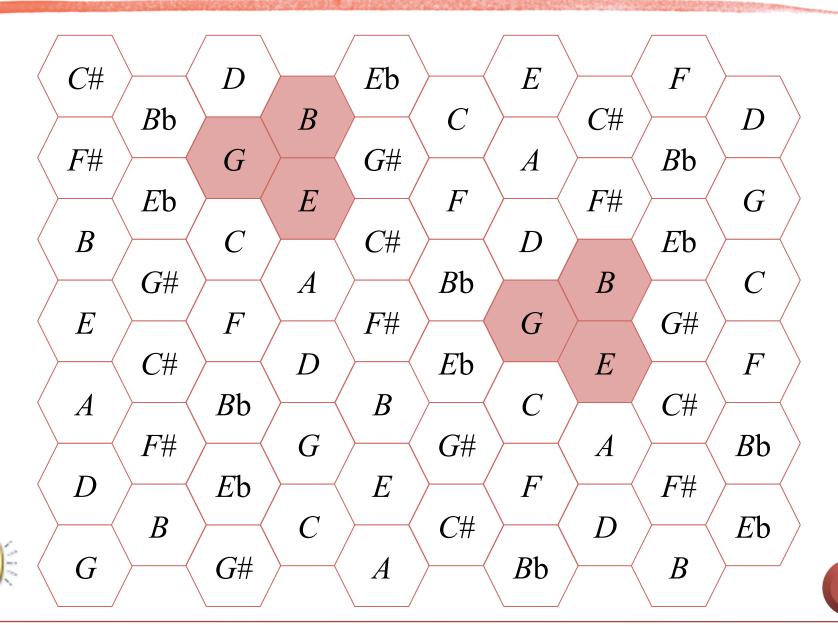
Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)

⟨C#⟩ D	\sim Eb	\nearrow E	\	\overbrace{F}	
\rightarrow Bb	$B \rightarrow $	$\begin{pmatrix} & C & \end{pmatrix}$	C#	/	D
$\langle F\# \rangle \longrightarrow \langle G \rangle$	\longrightarrow G #	$\rightarrow A$	\rightarrow	Bb	$\overline{}$
\rightarrow Eb	$E \rightarrow -$	$\langle F \rangle$	$\left\langle F^{\#} ight angle$	\rightarrow	G
$\langle B \rangle \sim \langle C \rangle$	<i>C</i> #	\rightarrow D		Eb	
\rightarrow G # \rightarrow	$A \rightarrow -$	Bb	$\langle B \rangle$	\	$C \mid \rangle$
$\langle E \rangle \longrightarrow \langle F \rangle$	\longrightarrow F #	\rangle G		G#	
\ \ C\# \	$D \rightarrow $	Eb	$\langle E \rangle$		F
$\langle A \rangle \longrightarrow \langle Bb \rangle$	\rightarrow B	\rangle \subset C		C#	
\nearrow $F#$	$G \rightarrow \overline{}$	G#	$\langle A \rangle$	\longrightarrow \vdash \vdash	3b
$\langle D \rangle \longrightarrow \langle Eb \rangle$	\longrightarrow E	\rangle F		F#	
\rightarrow B	$C \rightarrow \frown$	C#	$\langle D \rangle$	$\longrightarrow $ \vdash	Eb
$\langle G \rangle - \langle G \# \rangle$	\nearrow A	\nearrow Bb		B	/

Extract of the 2nd movement of the Symphony No. 9 (L. van Beethoven)

/		\	D	\	Eb	>	E	\	F	<u></u>
\	>	Bb	\	B	\	C	>	C#	>	$\langle D \rangle$
	F#	\rightarrow	G	\rightarrow	G#	\rightarrow	A	\rightarrow	Bb	
/	\	<i>E</i> b	\	E	\longrightarrow	$\langle F \rangle$	>	$\stackrel{/}{iggr}F\#$	\	$\left\langle \begin{array}{c} G \end{array} \right\rangle$
	B	\rightarrow	C	\longrightarrow	<i>C</i> #	\longrightarrow	D	\rangle	Eb	
	\	$G^{\#}$	> <	A	\rightarrow	Bb /	>	$\setminus B$	>	$\langle C \rangle$
	E		F	>	$F^{\#}$	\rightarrow	G	—	$G^{\#}$	
/	/	C# /	D1	D	\nearrow	Eb	\rangle	ackslash		$\left\langle \begin{array}{c} F \end{array} \right\rangle$
	A	T :#	Bb	G	B	<u>C</u> #	C	1	<i>C</i> #	Dla
	$\stackrel{\wedge}{D}$	F#	<i>E</i> b		$\stackrel{/}{E}$	<i>G</i> #	$\stackrel{/}{F}$	A	$\stackrel{/}{F}\#$	Bb
	\rightarrow	B	\rightarrow \leftarrow	C	\\	/ C#	\\	$\stackrel{/}{D}$	1'π·	$\left\langle Eb \right\rangle$
	G		G#		$\stackrel{\wedge}{A}$		<i>B</i> b		B	
\	/	/		/	/	/		/		/





	<i>C</i> #	\	D	\	Eb	\	E	\	F	<u></u>
/	\	Bb	\longrightarrow	В	\rangle — \langle	C	\rangle —— \langle	C#	\longrightarrow	$\left\langle D \right\rangle$
	F#	\rangle \langle	G	_ _	$G^{\#}$	<u> </u>	A		Bb /	
	D	Eb		E		F	\rightarrow	${}^{igwedge}F^{\#}$		$\langle G \rangle$
	B	G#	C	A	<i>C</i> #	Bb	D	B	Eb	C
	$\stackrel{\wedge}{E}$	\)\(\frac{\pi}{\pi}\)	$\stackrel{/}{F}$	\rightarrow \rightarrow	$\stackrel{/}{F}\#$	\longrightarrow	G		$G^{\prime\prime}$	\
)	C#	\rightarrow	D		<i>E</i> b		$\stackrel{/}{E}$		$\left\langle F \right\rangle$
	\boldsymbol{A}	\longrightarrow	<i>B</i> b	\longrightarrow	$\begin{pmatrix} B \end{pmatrix}$	\longrightarrow	C	\rightarrow	<i>C</i> #	\
/	\	F#	\longrightarrow	G	\longrightarrow	<i>G</i> #	\rangle ——($\langle A \rangle$		Bb
	D	\rangle — \langle	Eb /	>	E	\	F	\rightarrow	F#	
	\	B		C	\rightarrow	<i>C</i> #	>	$\setminus D$	_ _	$\langle Eb \rangle$
	G		G#	>	A		Bb_		B	 /

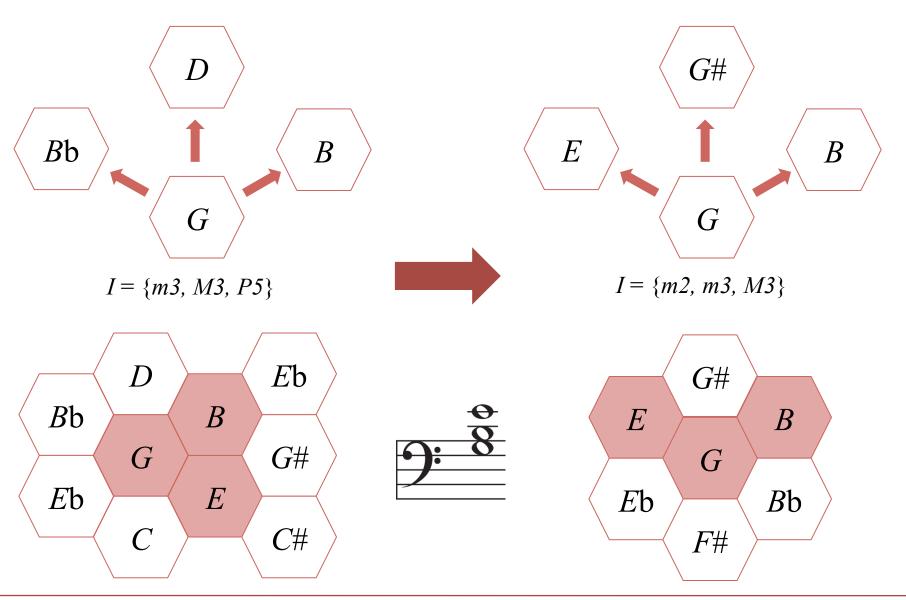
	<i>C</i> #	\	D	\	<i>E</i> b	\	E	\	F	<u></u>
		<i>B</i> b	\longrightarrow	B	\rangle — \langle	C	\longrightarrow	C#	\longrightarrow	$\left\langle D\right\rangle$
	F#	\rangle — \langle	G	\	G#	\longrightarrow	A	\rightarrow	<i>B</i> b	>
		Eb	\rangle —— \langle	E	\rightarrow	F	\rangle —— \langle	F#	\rangle —— \langle	G
	B	\rangle — \langle	C	\	<i>C</i> #	\longrightarrow	D	\rightarrow	<i>E</i> b	
		G#	\rightarrow	A	\rightarrow	<i>B</i> b	\rangle — \langle	B	\rangle — \langle	C
	E	\rangle — \langle	F	\longrightarrow	F#	\rangle — \langle	G	\rightarrow	$G^{\#}$	>
\rightarrow	$\overline{}$	<i>C</i> #	\rangle — \langle	D	\rightarrow	Eb	\rightarrow	E	\rangle — \langle	F
	\boldsymbol{A}	\rightarrow	<i>B</i> b	\	B	\rangle	C		<i>C</i> #	>
	\langle	F#	\rangle — \langle	G	\rightarrow	$G^{\#}$	\rangle —— \langle	A	\rightarrow	Bb
	D	\rangle — \langle	Eb	\longrightarrow	(E)	\rangle —— \langle	F	\rightarrow	F#	
	$\overline{}$	B	\rangle — \langle	C	\rightarrow	<i>C</i> #	\rangle —— \langle	D	\rightarrow	Eb
	G	>	G#	\	A	>	Bb_	>	B	

C#	£ \	D	\	<i>E</i> b	\rightarrow \land \rightarrow \land \rightarrow \right	E	<u></u>	F	<u></u>
	$-\langle Bb \rangle$	\longrightarrow	B	\longrightarrow	$\begin{pmatrix} C \end{pmatrix}$	\longrightarrow	C#	\longrightarrow	$\left\langle \begin{array}{c} D \end{array} \right\rangle$
$\langle F $	<u> </u>	G	\longrightarrow	$G^{\#}$	\rangle — \langle	A	\rightarrow	<i>B</i> b	\
<u> </u>	\prec Eb	\rangle —— \langle	E	\rightarrow	F	\rangle —— \langle	F#	\rangle — \langle	G
$\langle B \rangle$	\rightarrow	C	\rangle —— \langle	<i>C</i> #	\rangle — \langle	D	\rightarrow	Eb	>
<u> </u>	\prec G #	\rangle — \langle	A	\rightarrow	<i>B</i> b	\	$\setminus B$	\rangle — \langle	$C \rangle$
$\langle E$		F	\longrightarrow	F#	\rangle — \langle	G	\rightarrow	$G^{\#}$	>
<u> </u>	← C#	\setminus	D	\rightarrow	Eb	\rightarrow	$\langle E \rangle$	\rangle — \langle	F
$\langle A$	\rightarrow	<i>B</i> b	\longrightarrow	B	\rangle	C		<i>C</i> #	>
	$ \leftarrow$ F#	\rangle — \langle	G	\rightarrow	G#	\rangle —— \langle	A	\rightarrow	Bb
$\langle D$		<i>E</i> b	\longrightarrow	$\langle E \rangle$	\rangle — \langle	F	\rightarrow	F#	
	\prec B	\rangle —— \langle	\boldsymbol{C}	\rangle — \langle	<i>C</i> #	\rangle —— \langle	D	\rangle —	Eb
$\langle G \rangle$		G#	\	A	\	Bb_		B	

(C#)			<i>E</i> b	\	E	\	$\stackrel{/}{F}$	<u> </u>
	Bb >	$- \langle B \rangle$	\rightarrow	C	\longrightarrow	C#	\longrightarrow	$\left\langle \begin{array}{c} D \end{array} \right\rangle$
$\langle F\# \rangle$	$- \langle C$,	G#	\rangle —— \langle	A	\rightarrow	<i>B</i> b	\rightarrow
	E b \nearrow	$-\langle E \rangle$	\rightarrow	F	\longrightarrow	F#	\longrightarrow	G
$\langle B \rangle$	$- \langle$ ϵ		<i>C</i> #	\setminus	D	\rightarrow	Eb	\
	$\mathcal{G}^{\#}$	\neg A		Bb	\longrightarrow	(B)	\longrightarrow	C
$\langle E \rangle$	$- \langle F$	· -	F#	\rightarrow	G	\rightarrow	$G^{\#}$	\
	C#	$\neg \langle D \rangle$	\rightarrow	<i>E</i> b	\longrightarrow	(E)	\rightarrow	$ F\rangle$
$\langle A \rangle$	$ \langle B \rangle$	b ><	B	\rightarrow	C		<i>C</i> #	\
	F# >	$\neg \langle G \rangle$	\rightarrow	$G^{\#}$	\rangle — \langle	A	\rightarrow	Bb
$\langle D \rangle$	$\langle E $	b ><	$\langle E \rangle$		F	\rightarrow	F#	
	$B \nearrow$	$- \langle C \rangle$	\rightarrow	<i>C</i> #	\rangle —— \langle	D	\longrightarrow	Eb
$\langle G \rangle$	$- \langle G \rangle$	#	A	\	Bb_		B	

C#	#	D	\	<i>E</i> b	\	E	<u></u>	F	<u></u>
	$-\langle Bb \rangle$	\rightarrow	B	\longrightarrow	C	\longrightarrow	C#	\longrightarrow	$\left\langle \begin{array}{c} D \end{array} \right\rangle$
$\langle F^{\sharp}$	<i>‡</i>	G	\longrightarrow	$G^{\#}$	\rangle —— \langle	A	\rightarrow	<i>B</i> b	>
<u> </u>	\prec E b	\rangle —— \langle	E	\rightarrow	F	\rangle —— \langle	F#	\rangle —— \langle	G
$\langle B \rangle$		C	\longrightarrow	<i>C</i> #	\rangle — \langle	D		Eb	\
<u> </u>	- G#	\rightarrow	A	\rightarrow	<i>B</i> b	\rangle —— \langle	$\langle B \rangle$	\rangle —— \langle	C
$\langle E$		F	\rightarrow	F#	\rangle — \langle	G	\rightarrow	$G^{\#}$	\
	\prec C #	\rightarrow	D	\rightarrow	Eb	\rangle —— \langle	$\langle E \rangle$	\rangle —— \langle	$\left F \right\rangle$
$\langle A$	\rightarrow	<i>B</i> b	\longrightarrow	B	\rangle — \langle	C	\rightarrow	<i>C</i> #	>
	egreen F#	\rangle — \langle	G	\rightarrow	$G^{\#}$	\rangle — \langle	A	\rightarrow	Bb
$\langle D$	\rightarrow	<i>E</i> b	\longrightarrow	E	\longrightarrow	F		F#	
	$\overline{}$ B	\rangle —— \langle	\boldsymbol{C}	\rangle — \langle	<i>C</i> #	\rangle —— \langle	D	\rightarrow	Eb
$\langle G \rangle$		G#	\	A	>	Bb_		B	

	<i>C</i> #	\	D	<u> </u>	<i>E</i> b	> (E	<u></u>	F	<u></u>
/	\	<i>B</i> b	\longrightarrow	B	\rangle — \langle	C	\rangle — \langle	C#	\longrightarrow	$\left\langle D\right\rangle$
	F#	\rightarrow	G	> <	G#	\longrightarrow	A	\rightarrow	<i>B</i> b	
/	\longrightarrow	<i>E</i> b	\rangle —— \langle	E	\rightarrow	F	\rangle —— \langle	F#	\rangle —— \langle	G
	B	\rightarrow	C	\longrightarrow	<i>C</i> #	\rangle —— \langle	D	\rightarrow	Eb	\rightarrow
\	\longrightarrow	G#	\rightarrow	A	\rightarrow	Bb	\rangle — \langle	B	\rangle — \langle	$C \rangle$
	E	\rightarrow	F	\	F#	\rangle — \langle	G	\rightarrow	$G^{\#}$	\
	\longrightarrow	<i>C</i> #	\rightarrow	D	\rightarrow	Eb	\longrightarrow	$\stackrel{/}{_{\sim}} E$	\rightarrow	$ F\rangle$
	A	\rightarrow	<i>B</i> b	\longrightarrow	B	\rightarrow	C	\rightarrow	<i>C</i> #	>
\	\longrightarrow	F#	\longrightarrow	G		G#	\rightarrow	A	\rightarrow	Bb
	D	\rightarrow	<i>E</i> b	\rangle ——($\left(egin{array}{ccc} E \end{array} ight]$	\longrightarrow	F		F#	
\	\longrightarrow	B	\rightarrow	\boldsymbol{C}		<i>C</i> #	\rightarrow	D	\rightarrow	Eb
	G	>	G#	\	A	\	Bb		B	



	G	\rightarrow \land \rightarrow \land \rightarrow \right	D	\	A	\	$\int E$		B		
\	\longrightarrow	Bb	\rightarrow	F	\rightarrow	\boldsymbol{C}	\longrightarrow	G	\rightarrow	$\left\langle D \right\rangle$,
	F#	\rangle —— \langle	<i>C</i> #	\longrightarrow	G#	\longrightarrow	<i>E</i> b		<i>B</i> b	\	
\	\longrightarrow	A	\rightarrow	E	\rightarrow	B	\longrightarrow	$\stackrel{/}{F}$ #	\rightarrow	$\left\langle C^{\#}\right\rangle$	<i>)</i>
	F	\rangle —— \langle	C	\longrightarrow	G	\longrightarrow	D	\rightarrow	A		
\	\longrightarrow	$G^{\#}$	\rightarrow	Eb	\rightarrow	Bb	\rightarrow	$\langle F \rangle$	\rightarrow	$\left\langle \begin{array}{c} C \end{array} \right\rangle$	<i></i>
	E	\rightarrow	B	\longrightarrow	F#	\longrightarrow	<i>C</i> #	\rightarrow	$G^{\#}$	\	
\	\longrightarrow	G	\rightarrow	D	\rightarrow	A	\rightarrow	$\stackrel{/}{E}$	\rightarrow	$\left\langle \hspace{0.1cm} B \hspace{0.1cm} \right angle$,
	<i>E</i> b	\longrightarrow	<i>B</i> b	\longrightarrow	F	\longrightarrow	C	\rightarrow	G	\	
\	\longrightarrow	F#	\rightarrow	<i>C</i> #	\rightarrow	G#	\rightarrow	Eb	\longrightarrow	$\langle Bb \rangle$	>
	D	\longrightarrow	A	\longrightarrow	$\stackrel{'}{E}$	\longrightarrow	B	\rightarrow	F#	\	
\	\longrightarrow	F	\rightarrow	\boldsymbol{C}	\rightarrow	G	\rightarrow	D	\rightarrow	$\left\langle A \right\rangle$)
	<i>C</i> #	\	G#	\	<i>E</i> b	\	Bb		$\langle F \rangle$		

$\langle G \rangle - \langle D \rangle - \langle A \rangle - \langle E \rangle$	B
F# $C#$ $C#$ $C#$ $C#$ $C#$ $C#$ $C#$ C	Bb
$ ightharpoonup A ightharpoonup \left\langle egin{array}{c} E \end{array} ightharpoonup \left\langle egin{array}{c} B \end{array} ightharpoonup \left\langle egin{array}{c} F\# \right\rangle \left\langle egin{array}{c} F\# \end{array} ightharpoonup \left\langle egin{array}{c} F\# \right\rangle \left\langle egin{array$	C# >
F C E B F	$A \subset C$
$E \rightarrow G \rightarrow D \rightarrow F\# \rightarrow C\# \rightarrow E$	G# B
$\left\langle Eb \right\rangle \longrightarrow \left\langle Bb \right\rangle \longrightarrow \left\langle F \right\rangle \longrightarrow \left\langle C \right\rangle \longrightarrow \left\langle$	$\left\langle \begin{array}{c} G \end{array} \right\rangle$
C# $C#$ $C#$ E B	F# Bb
C# C	F A

	\overline{G}	>	D	\	A	\	E	\	B	
/		Bb /		F		C	>	G	\	$\left \begin{array}{c}D\end{array}\right>$
	F# /	${\sim}$ A	<i>C</i> #	${F}$	<i>G</i> #	B	Eb	F#	Bb	C# \
	$\left\langle F ight angle$	>	C	\longrightarrow	$\stackrel{/}{G}$	\longrightarrow	D		A	
/	\rightarrow	<i>G</i> #	\longrightarrow	Eb	\rangle — \langle	Bb	\longrightarrow	$\left\langle \begin{array}{c} F \end{array} \right\rangle$		$\left\langle C \right\rangle$
	E	$\overset{ ightharpoonup}{G}$	B	${D}$	F#	/	<i>C</i> #	${\triangleright}$ E	$G^{\#}$	B
	$\stackrel{/}{E}$ b	<i>G</i>	Bb	\longrightarrow	$\stackrel{/}{F}$	A	$\stackrel{/}{C}$	angle	$\stackrel{/}{G}$	D
\	\rightarrow	F#	\longrightarrow	<i>C</i> #	\longrightarrow	G#	\longrightarrow	$\left(\begin{array}{c}Eb\end{array}\right)$	\longrightarrow	$\left\langle Bb \right\rangle$
	$\left D \right $	\leftarrow	A	\rangle	E	\rangle — \langle	B	>	$F^{\#}$	
	C#	F	G#	C	Eb	$\supset G$	Bb	D	F	$\langle A \rangle$

$\langle G \rangle$		D	\	A	\	E	\	B	<u></u>
	$\langle Bb \rangle$	\rangle — \langle	F	\rangle	C	\rightarrow	G	\longrightarrow	$\left\langle D \right\rangle$
$\langle F \#$		C# /	>	$G^{\#}$	>	Eb /		<i>B</i> b	
F	A	C	E	G	B	D	F#	1	<i>C</i> #
	G#	\rightarrow	Eb	\rightarrow \langle	Bb	\rightarrow	$\stackrel{/}{F}$	$\stackrel{A}{ ightarrow}$	C
$\langle E \rangle$	<u></u>	$\left\langle B\right\rangle$	\	$\stackrel{/}{F}\#$	\longrightarrow	C#	\rangle — \langle	<i>G</i> #	
	$\langle G \rangle$	\rangle — \langle	D	\rangle — \langle	A	\longrightarrow	E	\longrightarrow	$\langle B \rangle$
$\langle Eb \rangle$		Bb /	\	F		C		G	D1
D	F#	A	<i>C</i> #	E	<i>G</i> #	B	Eb	F#	<i>B</i> b
	$\checkmark F$	\rightarrow \langle \rightarrow \langle \rightarrow \langle \rightarrow \right	$\stackrel{'}{C}$	\rightarrow \leftarrow	$\stackrel{/}{G}$	\rightarrow \langle	$\stackrel{/}{D}$	1 11	$A \rightarrow$
C#		G#	\	<i>E</i> b	\	Bb	\	F	

$\langle G \rangle$	D		A	E	\	B	<u></u>
	Bb	$\langle F \rangle$	$\overline{}$ C		$\left\langle G \right\rangle$	\longrightarrow	D
$\langle F\# \rangle$	C#	'	G#	\leftarrow Eb		Bb /	
$\langle F \rangle$	$A \longrightarrow C$	E	$G \longrightarrow B$	D	F#	A	<i>C</i> #
	$G^{\#}$	$\stackrel{/}{E}$ b	Bb	\	$\stackrel{/}{F}$	\nearrow \land	$\left\langle \begin{array}{c} C \end{array} \right\rangle$
$\langle E \rangle$	$ \left(B \right)$		F#	\leftarrow C#	\longrightarrow	G#	\
	$G \rightarrow \langle$	$\langle D \rangle$	$\longrightarrow \langle A \rangle$		E	\	$\langle B \rangle$
$\langle Eb \rangle$	Bb		$F \nearrow C^{\mu}$	C		G	D1.
$\langle D \rangle$	F# A	<i>C#</i> \rightarrow	\overbrace{E} G #	B	\rightarrow Eb	F#	$\langle Bb \rangle$
	$F \rightarrow \langle$	$\left\langle \begin{array}{c} C \end{array} \right\rangle$	$- \langle G \rangle$	\rightarrow	$\left\langle \begin{array}{c} D \end{array} \right\rangle$	\longrightarrow	$\left\langle A \right\rangle$
C#	G#		<i>E</i> b	Bb_		F	> /

	\overline{G}	D	\	A	\	E	\	B	<u></u>
\rightarrow	\longrightarrow Bb		F	\rangle — \langle	C	\rangle — \langle	G	\longrightarrow	$\left\langle D \right\rangle$
	F# >	C# /	>	$G^{\#}$	\rightarrow	<i>E</i> b	\rightarrow	Bb /	
	A		E		B		F#		C#
	F	$\begin{pmatrix} C \end{pmatrix}$		G	Dla	D		A	
	$E \longrightarrow G^{\#}$	$\stackrel{/}{B}$	<i>E</i> b	F#	<i>B</i> b	/ C#	F	$G\!\#$	C
\	$\stackrel{L}{\longrightarrow} G$		D		$\stackrel{/}{A}$	C π	$\stackrel{/}{E}$	\rightarrow	$\left \begin{array}{c} B \end{array} \right\rangle$
	Eb \longrightarrow	Bb	> ($\stackrel{/}{F}$	\longrightarrow	C	\rightarrow	G	
\rightarrow	$-\!\!\!\!\!-\!$	\rightarrow	<i>C</i> #	\rangle — \langle	$G^{\#}$	\rangle — \langle	<i>E</i> b	\longrightarrow	Bb
	$D \nearrow \longrightarrow$	$\langle A \rangle$	\	E	\longrightarrow	B	\longrightarrow	F#	
\rightarrow	$\overline{}$ F		C	>	G		D	<u> </u>	$\langle A \rangle$
	C# >	G#	> (Eb_		Bb_		F	

	G	\	D	\	A	\	E	<u></u>	B	<u></u>
\	\longrightarrow	<i>B</i> b	\setminus	F	\rangle — \langle	C	\rangle — \langle	G	\longrightarrow	$\left\langle D\right\rangle$
	F#	\longrightarrow	<i>C</i> #	\rangle	<i>G</i> #	\longrightarrow	<i>E</i> b	\longrightarrow	<i>B</i> b	>
	\	A	\rangle — \langle	E	\rangle	B	\rightarrow	F#	\longrightarrow	C#
	F	\rangle — \langle	C		G	\rightarrow	D	\rightarrow	A	
/	> ($G^{\#}$	\longrightarrow	<i>E</i> b	\rightarrow	Bb /		F		C
	E	\	B	<u> </u>	$F^{\#}$	\	<i>C</i> #	<u> </u>	$G^{\#}$	
/	—	G	D1	D	\rightarrow	A		$\setminus E$		$\langle B \rangle$
	Eb		Bb /		F		C		G	701
/	\nearrow	$F^{\#}$		<i>C</i> #		$G^{\#}$	\nearrow	Eb	Bb
	D	$\stackrel{/}{F}$	A	C	E		B	\bigcap	F#	1
/	C#	Γ/	/ G#	\/	Eb	G	Bb	D	$\stackrel{/}{F}$	$\langle A \rangle$
	\/	/	\/	/		/	DU /	/	<i>I'</i>	/

	G	\	D	\	A	\	E	\	B	<u></u>
/		Bb /		F		C		$\langle G \rangle$	D1 .	$\setminus D$
	F#	A	<i>C</i> #	\overline{E}	<i>G</i> #	B	Eb	F#	<i>B</i> b	<i>C</i> #
	$\langle F \rangle$		$\begin{pmatrix} C \end{pmatrix}$		\bigcap G	>	\bigcap	\	A	
	E	$G^{\#}$	B	\succeq Eb	F#	\rightarrow Bb	<i>C</i> #	F	G#	C
/	\rightarrow	G	\rightarrow	D	\longrightarrow	$\stackrel{/}{\wedge} A$	\longrightarrow	$\stackrel{/}{E}$	\backslash	$\left\langle B \right\rangle$
	Eb	F#	Bb	<i>∕</i> —∕	F	\supset G #	C	Eb	G	Bb
	$\stackrel{/}{D}$	<i>Γ</i> π	$\stackrel{/}{A}$	\rightarrow	$\stackrel{/}{E}$	\rightarrow	B		$\stackrel{/}{F}\#$	
/		F		C		G)——($\bigcup_{i=1}^{n} D_{i}$		$\langle A \rangle$
	<i>C</i> #		G#		Eb_		Bb_		F	

	G	\	D	\	A	\	E	\	B	<u></u>
/		<i>B</i> b		F		$\begin{pmatrix} C \end{pmatrix}$		$\langle G \rangle$	D1	$\left\langle D \right\rangle$
	$F^{\#}$	A	<i>C</i> #	$\stackrel{/}{E}$	<i>G</i> #	B	$\nearrow E b$	F#	Bb	C# \
	F	\longrightarrow	$\langle C \rangle$		$\left\langle \begin{array}{c} G \end{array} \right\rangle$	\longrightarrow	$\begin{pmatrix} D \end{pmatrix}$	\rightarrow	$\stackrel{'}{\wedge} A$	\
/	${E}$	<i>G</i> #	${ ightharpoondown}$	$\nearrow Eb$	$F^{\#}$	Bb	 C#	F	G#	C
	\longrightarrow	$\stackrel{/}{G}$	\longrightarrow	D	\rightarrow \langle 111	A		$\stackrel{/}{E}$		$\left\langle B\right\rangle$
	Eb /		Bb		$\stackrel{'}{\setminus} F$		C		G	D1
	D	$F^{\#}$	A	<i>C</i> #	E	$G^{\#}$	B	$\operatorname{\mathcal{E}b}$	F#	$\langle Bb \rangle$
/	\	$\stackrel{/}{\setminus} F$	\longrightarrow	$\begin{pmatrix} C \end{pmatrix}$	\longrightarrow	$\langle G \rangle$	\	$\langle D \rangle$	\	$\left\langle \begin{array}{c} A \end{array} \right\rangle$
	<i>C</i> #		<i>G</i> #	>	Eb_		Bb		F	

	$\left G \right\rangle$	·	D	\	A	\	E	\	B	<u> </u>
/	\	<i>B</i> b	\rangle —— \langle	F	\rangle — \langle	C	\rangle — \langle	$\left(\begin{array}{c} G \end{array} \right)$	\longrightarrow	D
	F#	$\overline{}$	<i>C</i> #	\	$G^{\#}$	\longrightarrow	<i>E</i> b	\rightarrow	<i>B</i> b	
/		A		E	\rightarrow	B	\rightarrow	F#	\rightarrow	C#
	$\left F ight $		C	>	G	-	D	<u> </u>	A	
/		$G^{\#}$		<i>E</i> b		Bb		$\setminus F$		C
	$\left E ight angle$		B	\nearrow	$F^{\#}$		<i>C</i> #		$G^{\#}$	
	Eb	G	Bb	D	$\stackrel{/}{F}$	A	C	E	G	B
	$\succeq U$	F#		/ C#	\	G#		<i>E</i> b	\(Bb
	$\left\langle \begin{array}{c} D \end{array} \right\rangle$	Ι'π	A	C π	$\stackrel{/}{E}$	\rightarrow	B		$\stackrel{/}{F}\#$	
/	\rightarrow	F	\longrightarrow	C	\longrightarrow	G	\longrightarrow	D	\longrightarrow	$\langle A \rangle$
	<i>C</i> #		<i>G</i> #	>	Eb_	\	Bb_	>	F	

	G	\rightarrow \land \rightarrow \land \rightarrow \right	D	\	A	\rightarrow \land \rightarrow \land \rightarrow \right	E	\	B	<u></u>
/	\longrightarrow	<i>B</i> b	\rangle —— \langle	F	\rangle — \langle	C	\rangle —— \langle	G	\longrightarrow	$\left\langle D \right\rangle$
	$F^{\#}$	\rightarrow	C#	\	$G^{\#}$	\rightarrow	<i>E</i> b	\rightarrow	<i>B</i> b	
/	\rightarrow	A		E	\rightarrow	B	\rightarrow	$_{ackslash}F$ #	\rightarrow	C#
	F		C	\	G		D	<u> </u>	A	
/		$G^{\#}$		<i>E</i> b		<i>B</i> b		$\setminus F$		$\setminus C \setminus C$
	E	\bigcap	B	\bigcap	F#	1	<i>C</i> #	\overline{E}	$G^{\#}$	\mathcal{D}
	Eb	G	Bb	D	$\stackrel{/}{F}$	A	C	E	G	$\langle B \rangle$
	$\succeq U$	$\overset{\prime}{F}$ #		/ C#		$G^{\#}$	\rightarrow	<i>E</i> b		Bb
	D	\rightarrow \langle \rightarrow \langle \rightarrow \langle \rightarrow \right	A	\rightarrow \langle	$\stackrel{/}{E}$		$\stackrel{/}{B}$		F#	
/	\longrightarrow	F	\setminus	\boldsymbol{C}	\rangle — \langle	G	\rangle — \langle	D	\longrightarrow	$\langle A \rangle$
	<i>C</i> #	>	<i>G</i> #	>	Eb_	>	Bb_		F	

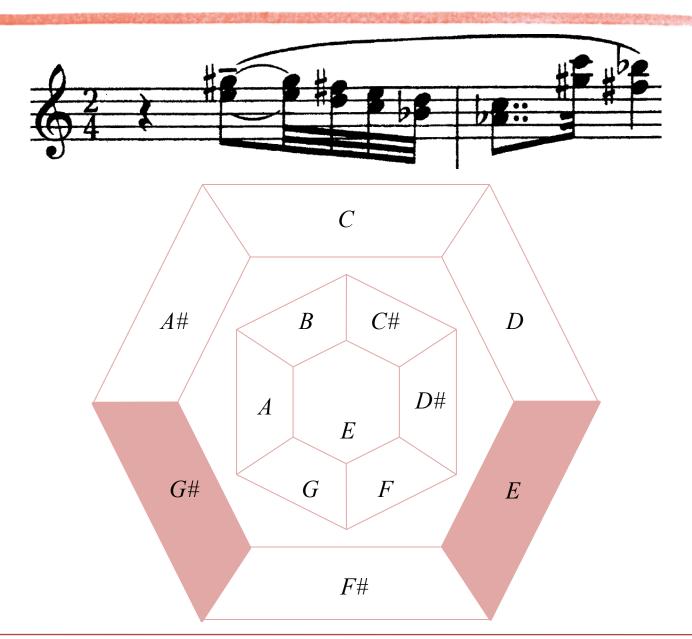
	G	>	D	\	A	\	E	\	B	<u></u>
\ /	\rightarrow	<i>B</i> b	\rangle —— \langle	F	\rangle — \langle	C	\rangle —— \langle	G	\longrightarrow	D
	F#	\longrightarrow	C#	\	G#	\longrightarrow	Eb	\longrightarrow	<i>B</i> b	
/	\rightarrow	A	\rightarrow	E	\rangle — \langle	B	\rightarrow	F#	\rightarrow	C#
	$\langle F \rangle$	\rightarrow	C	\	G	\longrightarrow	D	\rightarrow	A	
/	<u> </u>	$G^{\#}$		<i>E</i> b		<i>B</i> b	~	F	>	C
	$\left\langle E \right\rangle$		B	>	ackslash F#		<i>C</i> #	—	$G^{\#}$	
/		G	D1	D		A		E		$\langle B \rangle$
	Eb	T:44	Bb	<u> </u>	F	C#	C		G	Dla
/	$\stackrel{/}{D}$	F#	$\stackrel{/}{A}$	<i>C</i> #	$\stackrel{/}{E}$	<i>G</i> #	B	Eb	F#	<i>B</i> b
		$\stackrel{\wedge}{F}$	\rightarrow A	$\stackrel{/}{C}$	$\stackrel{E}{ ightarrow}$	$\stackrel{/}{G}$	\rightarrow \rightarrow	D	1 π	$\left\langle A\right\rangle$
	C#		G#	\	<i>E</i> b		Bb		F	71

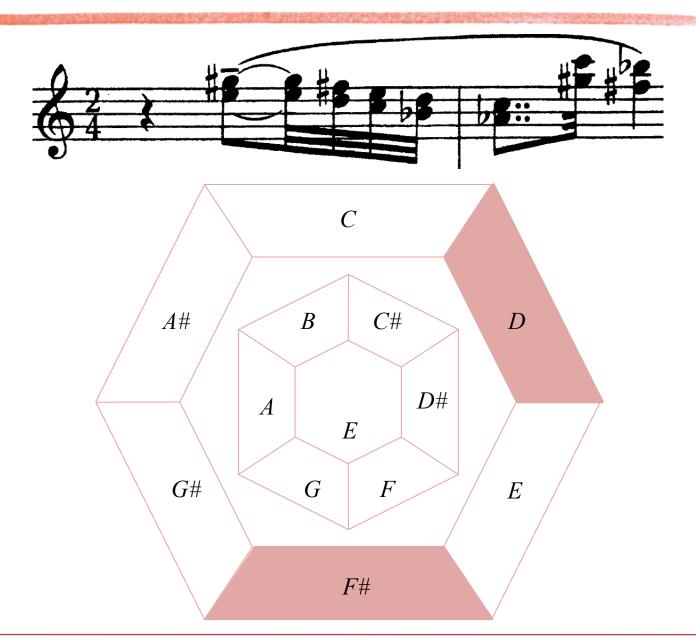
	G	\	D	\	A	\	E		B	
/	\longrightarrow	<i>B</i> b	\longrightarrow	F	\rangle — \langle	C	\rangle — \langle	G	\longrightarrow	$\left\langle D\right\rangle$
	F#	\longrightarrow	<i>C</i> #	> <	G#	\longrightarrow	Eb	\longrightarrow	<i>B</i> b	
/	\rightarrow	A	\rightarrow	E	\rightarrow	B	\rightarrow	F#	\rightarrow	C#
	F	\	C	>	G	\longrightarrow	D	\rightarrow	A	
/	<u> </u>	$G^{\#}$		<i>E</i> b		<i>B</i> b	~	F		$\left\langle C \right\rangle$
	$\langle E \rangle$		B		ackslash F#		<i>C</i> #		$G^{\#}$	
/		G	D1	D		A		E		$\langle B \rangle$
	<i>E</i> b	F#	Bb	<i>C</i> #	F	C#	C		G	Bb
	D	Γ# \	$\stackrel{/}{A}$	<i>C</i> #	$\stackrel{/}{E}$	<i>G</i> #	$\stackrel{/}{B}$	Eb	F#	DU
	\rightarrow	$\stackrel{\prime}{F}$	\backslash	C	\rightarrow \angle	$\stackrel{/}{G}$)(D	\rightarrow	$A \rightarrow$
	C#		G#	\	Eb	\	Bb		F	71

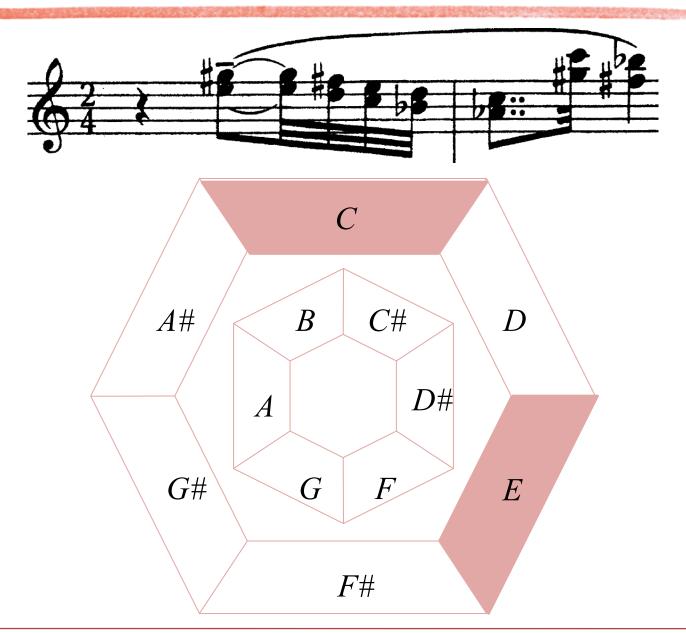
	G	\	D	\	A	\	E		B	<u></u>
/	\longrightarrow	<i>B</i> b	\longrightarrow	F	\rightarrow	C	\rangle — \langle	G	\longrightarrow	$\left\langle D \right\rangle$
	F#	\longrightarrow	<i>C</i> #	\longrightarrow	G#	\longrightarrow	Eb	\longrightarrow	<i>B</i> b	>
· /	\longrightarrow	A	\longrightarrow	E	\rangle — \langle	B	\longrightarrow	F#	\rightarrow	C#
	F	\rightarrow	C	\	G	\rightarrow	D	\longrightarrow	A	
/	\longrightarrow	$G^{\#}$	\rightarrow	Eb	\rangle — \langle	<i>B</i> b	\rightarrow	F	\rightarrow	$\langle C \rangle$
	E	\rightarrow	B		$F^{\#}$	\rightarrow	<i>C</i> #	\rightarrow	$G^{\#}$	
/	\	G	—	D	\rightarrow	A		E	\rightarrow	$\langle B \rangle$
	Eb	 (Bb	—	$\setminus F$	—	C	\rightarrow	G	
/	<u> </u>	$F^{\#}$	/	<i>C</i> #	<u> </u>	$G^{\#}$	—	Eb	— —	Bb
	D	—	A	\rangle	E		B		F#	
/		F		C		G	D 1	D		$\langle A \rangle$
	<i>C</i> #		G#	>	Eb_		Bb_		F	

	G	\	D	\	A	\	E		B	
\ /	\longrightarrow	<i>B</i> b	\rangle —— \langle	F	\rightarrow	C	\rangle —— \langle	G	\rangle — \langle	D
	F#	\rangle — \langle	<i>C</i> #	\longrightarrow	G#	\rangle — \langle	<i>E</i> b	\rightarrow	<i>B</i> b	\
\ /	\longrightarrow	A	\rangle —— \langle	E	\rightarrow	B	\rangle —— \langle	F#	\rangle — \langle	C#
	F	\rangle — \langle	C	\longrightarrow	G	\rangle — \langle	D	\rightarrow	A	
\	\longrightarrow	G#	\rightarrow	Eb	\rightarrow	<i>B</i> b	\rangle — \langle	F	\rangle — \langle	C
	E	\rangle —	В	\rightarrow	F#	\longrightarrow	<i>C</i> #	\rightarrow	<i>G</i> #	\rightarrow
\	\longrightarrow	G	\longrightarrow	D	\rightarrow	A	\longrightarrow	$\stackrel{/}{E}$	\rightarrow	B
	<i>E</i> b	\rightarrow	Bb	\longrightarrow	$\stackrel{/}{F}$	\rightarrow	C	\rightarrow	G	\rightarrow
\	\longrightarrow	F#	\rightarrow	<i>C</i> #	\rightarrow	<i>G</i> #	\rightarrow	<i>E</i> b	\rightarrow	Bb
	D	\rightarrow	A	\longrightarrow	$\stackrel{/}{E}$	\rightarrow	B	\rightarrow	F#	\rightarrow
\	\longrightarrow	F	\longrightarrow	\boldsymbol{C}	\rightarrow	G	\rightarrow	D	\rightarrow	$A \rightarrow$
	C#	\	igg(G#igg)	\	<i>E</i> b	\	Bb		$\left\langle \begin{array}{c} F \end{array} \right\rangle$	









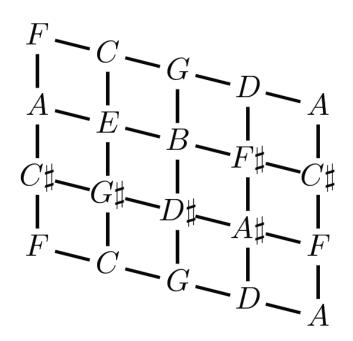
Spatial Computing & MGS

- Spatial computing
 - Compute in space
 - Compute space
- MGS: a programming language for spatial computing
 - Introduction of topological concepts in a programming language
 - Two main principles
 - Data structure: topological collection
 - Control structure: transformation

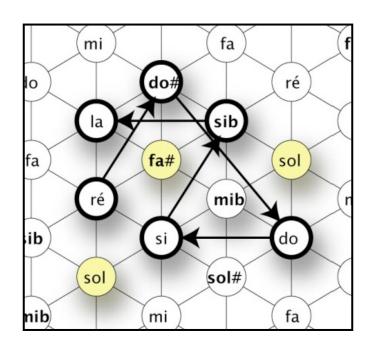
Neo-Riemannian Problematic

Examples

Neo-Riemannian approach for tonal music



Euler's tonnetz



Hexagonal network of notes (J.-M. Chouvel)