An Optical Approach to Computation

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Content

- Optical computing
- Optical 3-satisfiability
- Optical graph 3-colorability
- Future works

OPTICAL COMPUTING

Light Properties

- Special physical properties of light
  - High parallel nature
  - High speed
  - Splitting abilities
  - Many different wavelengths in a single ray

Optical Computing In Different Areas

- Data transmission
- Data storage
- Data processing
  - Optical logic gates
  - Processors based on light properties

Optical Data Processing

- Continuous space machine
    - Images as memory cells
    - Optical operations
        - Copy, Fourier transformation,…
    - Contributions on solving NPC problems
- Optical non-deterministic Turing machine
    - Light rays pass different computational path
    - Split light rays in decision points
    - Contributions on solving NPC problems

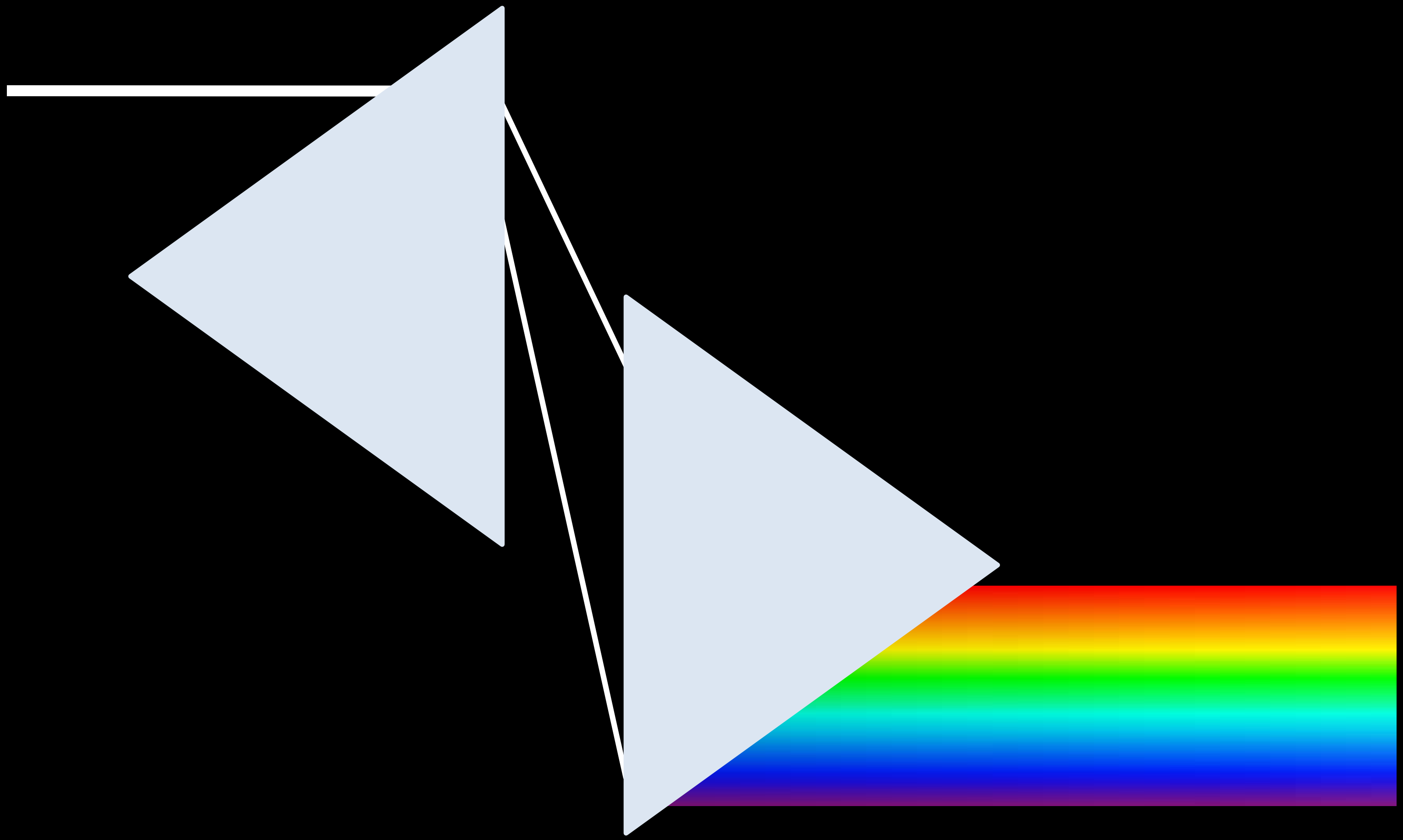
OPTICAL GRAPH 3-COLORABILITY

The 3-Sat Problem

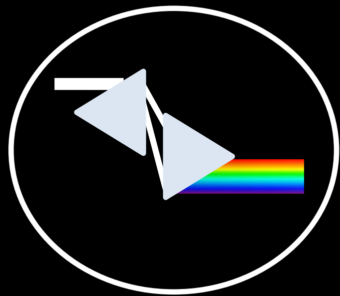
- Is the given Boolean formula satisfiable?
  - Conjunctive of some clauses
    - Clause: disjunction of some literals
    - Literal: variable or negation of a variable

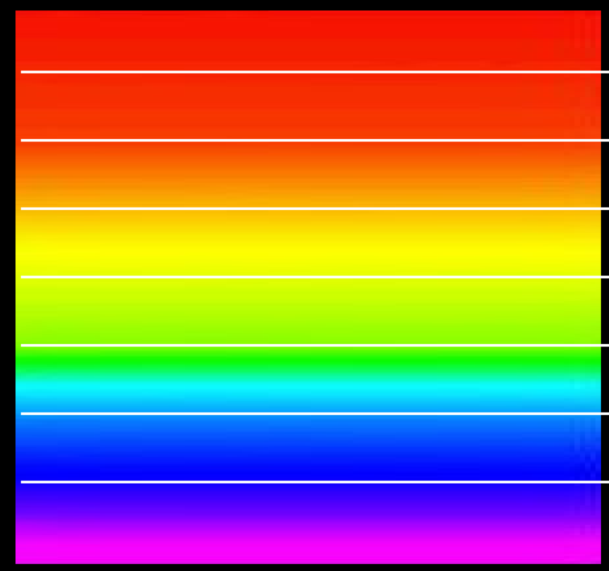
$$\underbrace{(\overbrace{x_1}^{\text{Literal}} \vee \overline{x_3} \vee x_4)}_{\text{Clause}} \wedge (\overline{x_2} \vee x_3 \vee x_5) \wedge (\overline{x_4} \vee \overline{x_5} \vee \overline{x_1}) \wedge (\overline{x_2} \vee x_1 \vee x_5)$$

Wavelngths as Value-Assignments



Wavelngths as Value-Assignments





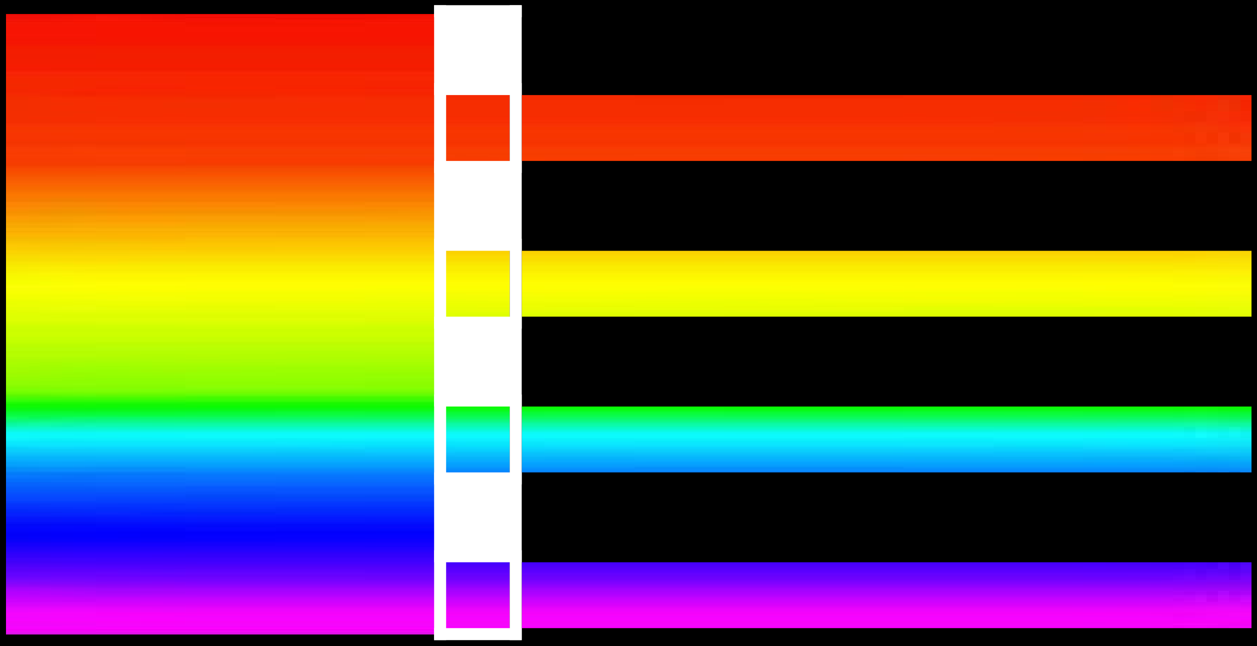
x_1	x_2	x_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Divide a wavelength spectrum into 2^n sections.
Consider each section as a possible value-assignment.

Filters

$\sigma_F(R)$ drops wavelengths not satisfying F from a given light ray R

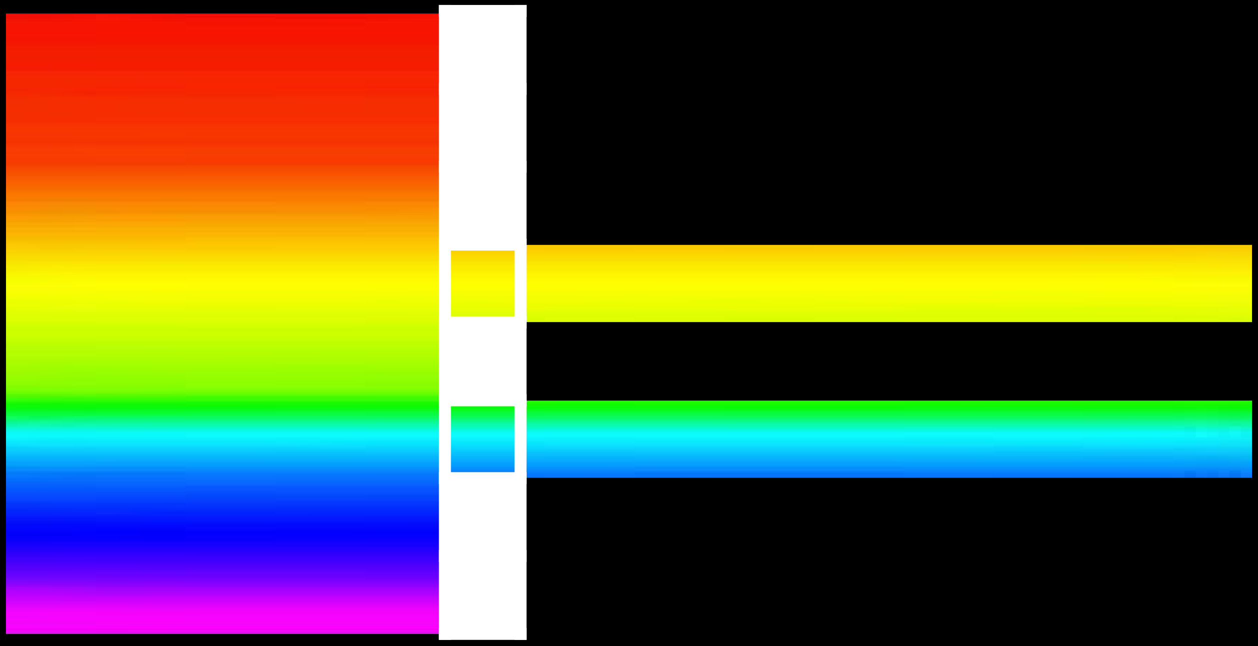
Literal Selector



σ_l

Punch an opaque ribbon where the literal is satisfied

Simple Clause Selectors

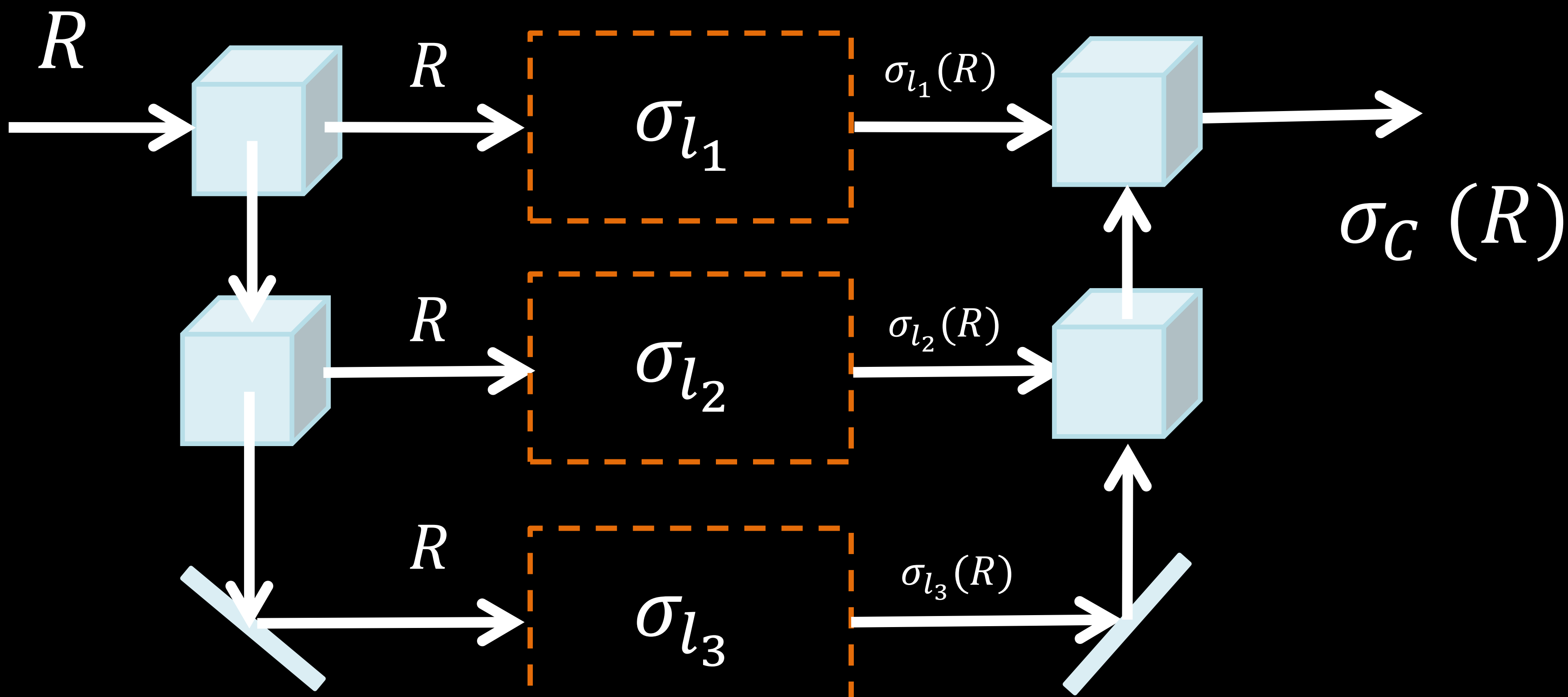


σ_C

Punch an opaque ribbon where the clause is satisfied.

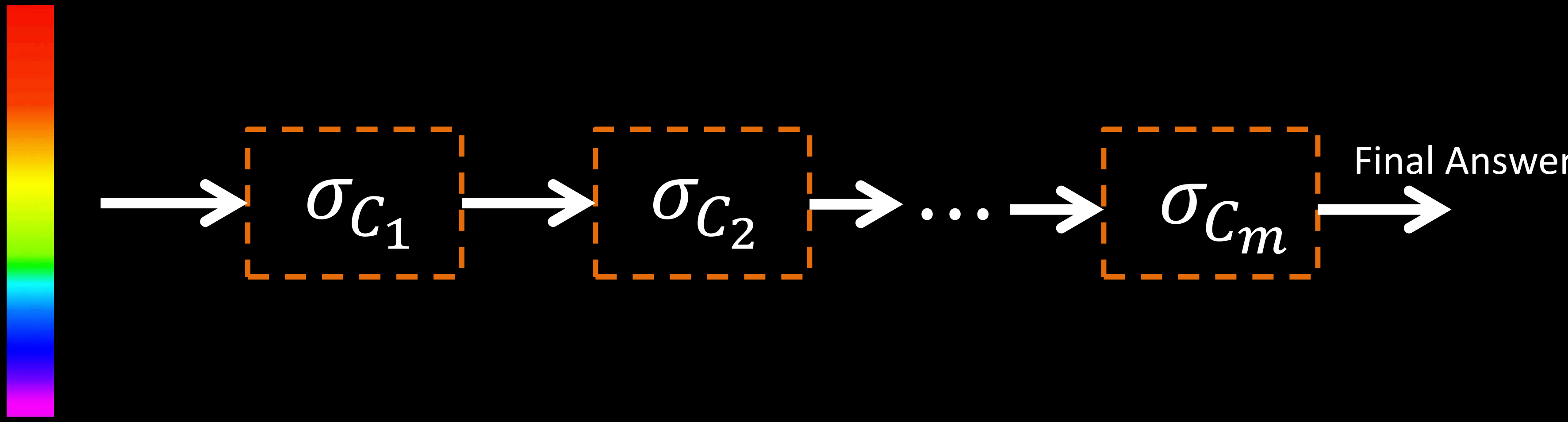
Combined Clause Selectors

$C = l_1 \vee l_2 \vee l_3$



CNF Formula Selector

$$\sigma_{C_1 \wedge C_2 \wedge \cdots \wedge C_m}$$



Drop wavelengths not satisfying clauses each after other.
At the end, remaining wavelengths indicate that the formula is satisfiable.

Complexity

- Preprocessing
  - Simple clause selectors
    - $O(n^3 2^n)$ time
    - $O(n^3)$ filters
  - Combined clause selectors
    - $O(mn2^n)$ time
    - $O(mn)$ filters

Complexity (cont’d)

- Each problem instance
  - $O(m)$ time
  - $O(m)$ optical devices

- $O(2^n)$ long filters
  - $1.3\ m$ long filters for $n = 15$

- $O(2^n)$ different wavelengths

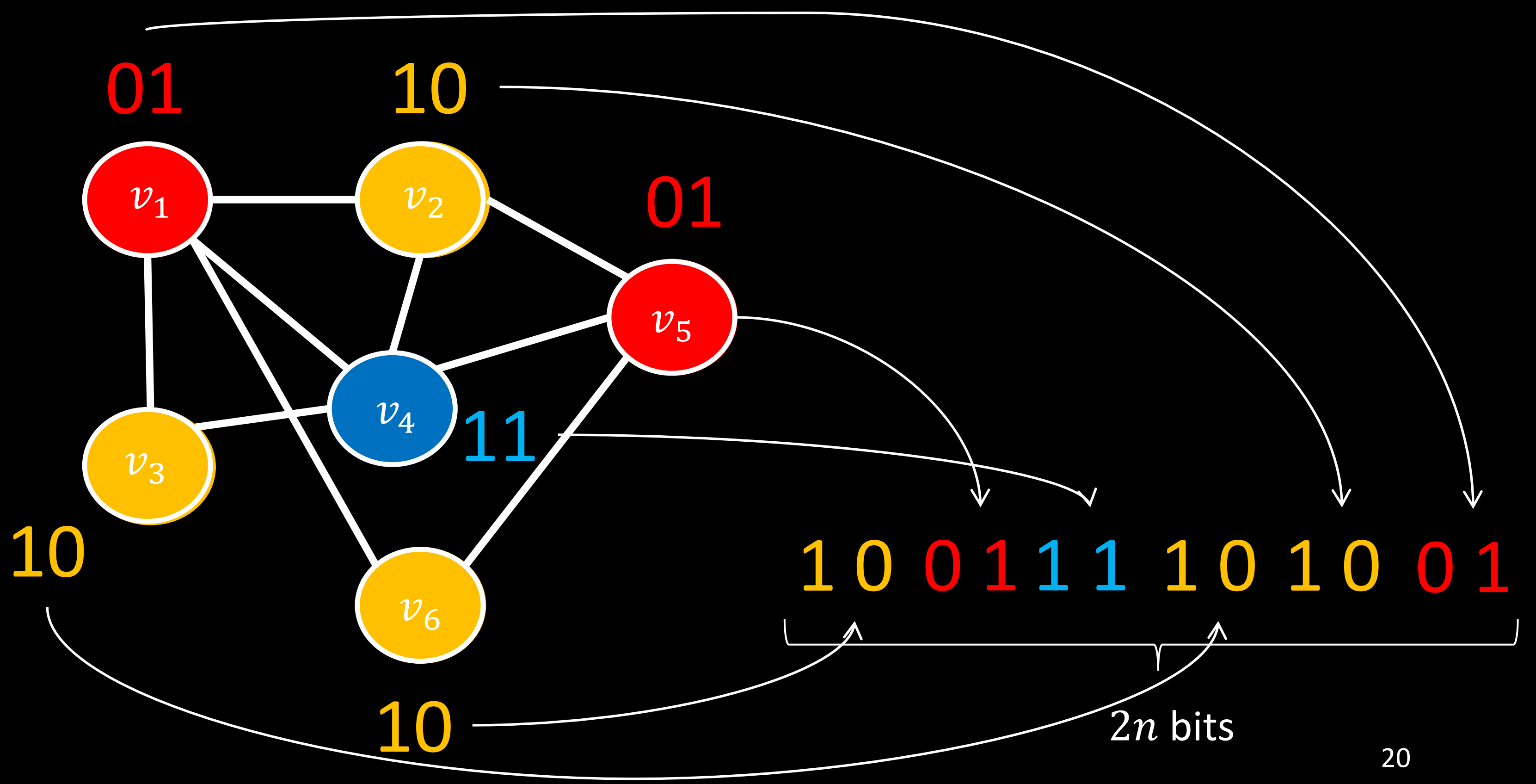
OPTICAL GRAPH 3-COLORABILITY

3-Colorability Problem

- Is the given graph 3-colorable?
  - Graph coloring
    - Assign a color to each vertex of a graph
  - Proper 3-coloring
    - Using at most 3 colors
    - Different colors for adjacent vertices

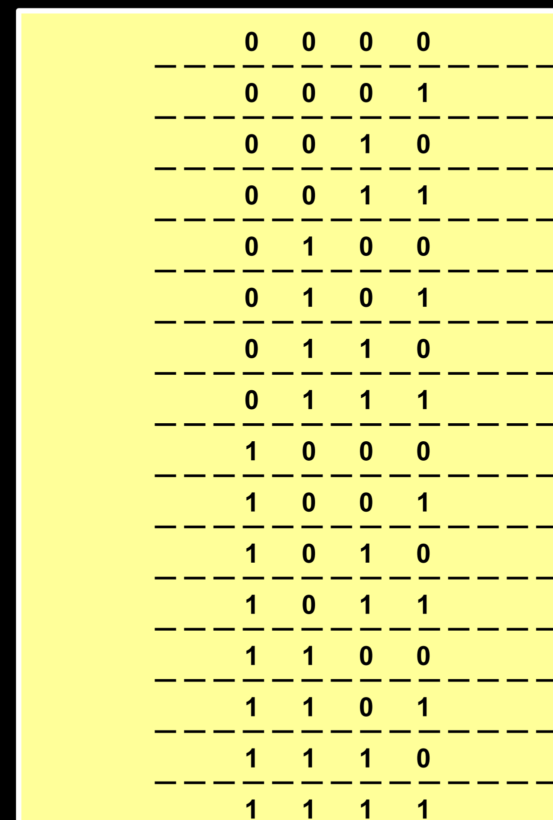
Graph Colorings as Binary Sequences

Write the color of vertices in binary format each after other



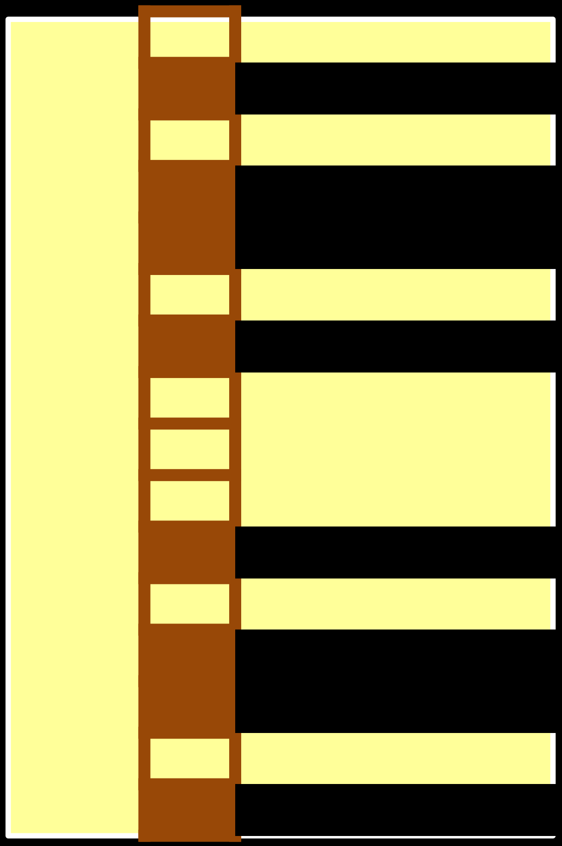
Light Rays as Binary Numbers

- Consider a wide ribbon of light
  - Divide the ribbon into 2^{2n} sections
  - Assign each section to binary number



Filter

- An opaque ribbon
  - Punched in some places
  - Use to drop some binary numbers

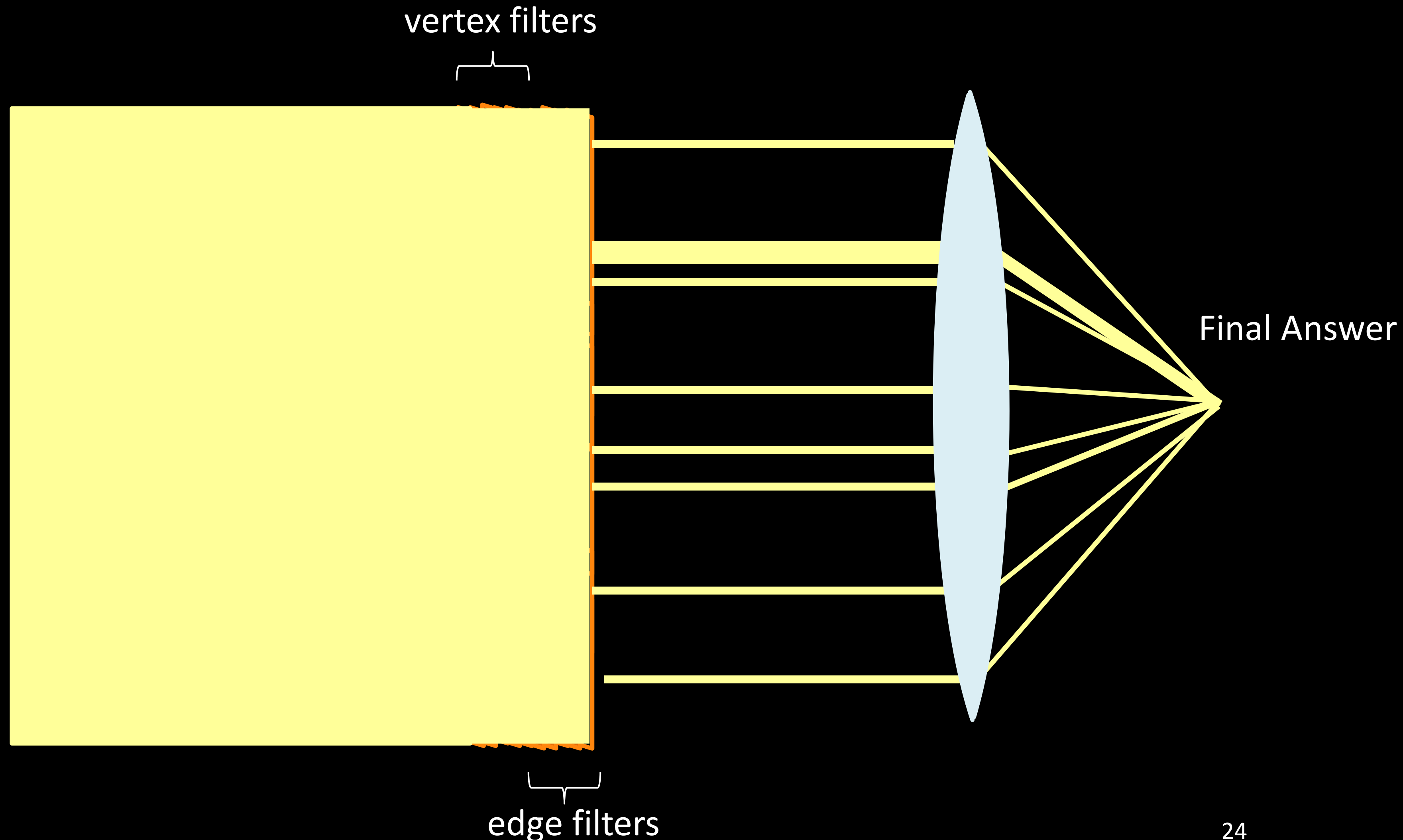


Filters to Drop Improper Colorings

- A valid color for each vertex
  - n vertex filters
    - f_i drops rays where 00 is assigned to v_i
- Different colors for adjacent vertices
  - e edge filters
    - $f_{(i,j)}$ drops light rays where the same color is assigned to v_i and v_j

Filters are created in preprocessing phase

Is the Graph 3-Colorable?



Complexity

- Preprocessing
  - n vertex filters
    - $O(n2^{2n})$ time
  - $\binom{n}{2}$ edge filters
    - $O(n^2 2^{2n})$ time
- Each problem instance
  - $O(n+e)$ time
- $O(2^{2n})$ long filters
  - $1.3\ m$ long filters for $n = 15$ (square shape filters)
- $O(2^{2n})$ number of photons

FUTURE WORKS

Future Works

- Computational power
- Polynomial time solutions for NPC problems
- Reduce filter sizes