



Vaguenes

Many-valued Logics
Supervaluationism

Fuzzy (Sub)se

Fuzzy Turin Machines

uzzy P Syster

Fuzzy Chemical Abstract Machin

Computing under Vagueness

Apostolos Syropoulos

Xanthi, Greece
asyropoulos@yahoo.com

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Outline

Computing under Vagueness



Vagueness

Many-valued Log

Supervaluationism

Fuzzy (Sub)set

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Fuzzy P System

- 1 Vaqueness
 - General Ideas
 - Many-valued Logics
 - Supervaluationism
 - Contextualism
 - Fuzzy (Sub)sets
- 2 Fuzzy Turing Machines
- 3 Fuzzy P Systems
- 4 Fuzzy Chemical Abstract Machine

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General Ide

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Fuzzy (Sub)

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Fuzzy Chemical Abstract Machin It is widely accepted that a term is vague to the extent that it has borderline cases.





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General Ideas

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- It is widely accepted that a term is vague to the extent that it has borderline cases.
- What is a borderline case?.



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- It is widely accepted that a term is vague to the extent that it has borderline cases.
- What is a borderline case?.
- A case in which it seems impossible either to apply or not to apply a vague term.



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- The adjective "tall" is a typical example of a vague term.





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1 uzzy 1 System

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 - Serena is 1,75 m tall, is she tall?



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 - Eubulides of Miletus: The Sorites Paradox.



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 - The old puzzle about bald people.

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Fuzzy Chemical Abstract Machin Bertrand Russell: Per contra, a representation is vague when the relation of the representing system to the represented system is not one-one, but one-many.





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- Example: A small-scale map is usually vaguer than a large-scale map.



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- Example: A small-scale map is usually vaguer than a large-scale map.
- Charles Sanders Peirce: A proposition is vague when there are possible states of things concerning which it is intrinsically uncertain whether, had they been contemplated by the speaker, he would have regarded them as excluded or allowed by the proposition. By intrinsically uncertain we mean not uncertain in consequence of any ignorance of the interpreter, but because the speaker's habits of language were indeterminate.



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- Max Black demonstrated this definition using the word chair.

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General Idea

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Fuzzy Chemical Abstract Machin Vagueness, ambibuity, and generality are entirely different notions.



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- A term of phrase is ambiguous if it has at least two specific meanings that make sense in context.



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- Example: He ate the cookies on the couch.



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- Example: Again consider the word chair.

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Fuzzy Chemical Abstract Machin Aristotle in Περὶ ἐρμηνεῖας (De Interpretatione) discussed the problem of future contingencies (i.e., what is the truth value of a proposition about a future event?).

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- Can someone tell us what is the truth value of the proposition "The Higgs boson exists?"



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- C.C. Chang proposed MV-algebras, that is, an algebraic model of many-valued logics.



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- In 1965, Lotfi A. Zadeh introduced his fuzzy set theory and its accompanying fuzzy logic.

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Fuzzy Chemical Abstract Machin Supervaluationism is a special kind of classical indeterminacy theory.

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Fuzzy P Syste

- Supervaluationism is a special kind of classical indeterminacy theory.
- Each predicate P is associated with two non-overlapping sets $E^+(P)$, the **determinate extension**, and $E^-(P)$, the **determinate anti-extension**.



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Fuzzy P Syste

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- Truth is defined relative to each choice of candidate extensions.
- A sentense is "super-true" if it comes out true realtive to each choice of candidate extensions for its predicates.

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Fuzzy Chemical Abstract Machin • The correct application of a vague predicate varies with context.





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- The correct application of a vague predicate varies with context.
- Example: A person may be tall relative to American men but short relative to NBA players.

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- The correct application of a vague predicate varies with context.
- Example: A person may be tall relative to American men but short relative to NBA players.
- Epistemological contextualism maintains the general idea that whether one knows is somehow relative to context.

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Fuzzy Chemical Abstract Machin Fuzzy set theory is based on the idea that elements may belong to sets to a degree

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- Fuzzy set theory is based on the idea that elements may belong to sets to a degree
- Given a universe X, a **fuzzy subset** A of X is characterized by a function $A: X \to I$, I = [0,1], where A(x) = i means that x belongs to A with degree equal to i.



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- When A(x) = 0 we say that x belongs to A with degree equal to 0 not that x does not belong to A!



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- When A(x) = 0 we say that x belongs to A with degree equal to 0 not that x does not belong to A!
- A consequence of this is that one cannot build a topos of all fuzzy subsets.

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Fuzzy Chemical Abstract Machine Assume that A and B are two fuzzy subsets of X. Then



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$$(A \cup B)(x) = \max\{A(x), B(x)\};$$



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 \circ the **scalar** cardinality of A is

$$|A| = \sum_{x \in X} A(X).$$

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Fuzzy Chemical Abstract Machin Let's **transform** the statement "Serena is tall to a degree of 0,70" into the "Serena is 70% tall!"



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- The statement "Serena is 70% tall" can be either true or false!
- The statement "Serena is tall to a degree of 0,70" says that Serena is tall and the truth value of this statement is 0,70!



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- It it possible to perform estimations that involve fuzzy probabilities expressed as likely, unlikely, not very likely, etc.?



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- How can we express in probability theory judgments like tomorrow will be a warm day or there will be a strong earthquake in the near future, etc.?
- It it possible to perform estimations that involve fuzzy probabilities expressed as likely, unlikely, not very likely, etc.?
- Bart Kosko advocates the idea that fuzziness is a basic characteristic of our cosmos! (I agree with Kosko!)

Computing ander Vaguenes



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Many-valued Lo

Supervaluationis

Fuzzy (Sub)se

Fuzzy Turin

Fuzzy P System





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Fuzzy Chemical Abstract Machin In 1968, Zadeh informally introduced the notions of fuzzy algorithm and fuzzy Turing machine.

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Zadeh's Fuzzy Algorithms and Turing Machines



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- Zadeh argued that everyday activities can be viewed as fuzzy algorithms, but Carol E. Cleland argued that recipes are algorithms!!!
- A fuzzy Turing machine is Turing machine where $q^{n+} = f(q^n, s^n)$ is associated with a feasability degree.

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- The formulations of Chang and Tanaka-Mizumoto are special cases of Santos's formulation.

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Fuzzy Chemical Abstract Machi A Santos type fuzzy Turing machine is a septuple $(S, Q, q_i, q_f, \delta, W, \delta_W)$ where:

 \circ S represents a finite non-empty set of input symbols;

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- $q_{i'}q_f \in Q$ are symbols designating the initial and final state, respectively;
- δ ⊂ (Q × S) × (Q × (S × {L, N, R})) is the next-move relation;



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- W is the semiring (W, \land, \lor) ;



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- W is the semiring (W, \land, \lor) ;
- $\delta_W: (Q \times S) \times (Q \times (S \times \{L, N, R\})) \to W$ is a W-function that assigns a degree of certainty to each machine transition.



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Fuzzy P Syste

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Nowadays, we use t-norms and t-conorms instead of semirings.

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Fuzzy Chemical Abstract Machin • There is no universal fuzzy Turing machine!

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- Benjamín Callejas Bedregal and Santiago Figueira argue that they have found flaws in Wiedermann proof though they did not refute his original argument.
- Later on Yongming Li showed that Wiedermann's results is true for *L*-fuzzy Turing machines.

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Fuzzy P System

Fuzzy Chemical Abstract Machin A model of computation mimicking the way cells function which was proposed by Gheorghe Păun.

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- A model of computation mimicking the way cells function which was proposed by Gheorghe Păun.
- A **compartment** is space that is surrounded by a porous membrane.

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- Nested compartments make a P system. Each compartment is a multiset.





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- The result of the computation is equal to the cardinality of the multiset contained in a special compartment called the **output** compartment.



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- Computation stops when no rule can be applied.
- The result of the computation is equal to the cardinality of the multiset contained in a special compartment called the **output** compartment.
- Processing takes place in discrete steps but rules are applied in parallel.

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Fuzzy Chemical Abstract Machin A P system with fuzzy data is a n-tuple

$$(O,\mu,w^{(1)},\dots,w^{(m)},R_1,\dots,R_m,i_0)$$

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where

• O is the alphabet of **objects**;

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- O is the alphabet of **objects**;
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- $w^{(i)}: O \to \mathbb{N} \times I$, $1 \le i \le m$, represent multi-fuzzy sets over O associated with the region surrounded by membrane i;

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- R_i , $1 \le i \le m$, are finite sets of multiset rewriting rules of the form $u \to v$, $u \in O^*$ and $v \in O^*_{TAR}$, where $O_{TAR} = O \times TAR$, and $TAR = \{\text{here, out}\} \cup \{\text{in}_i | 1 \le j \le m\}$.



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- $i_0 \in \{1, 2, ..., m\}$ is the label of a compartment called the **output** membrane.

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- A P system is fuzzy when it uses fuzzy multisets, fuzzy rewrite rules or both.
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- P systems that process fuzzy multisets can compute real numbers, therefore are more expressive than Turing machines.



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- Conjecture P systems with fuzzy processing rules that process fuzzy data should have at least the computational power of fuzzy Turing machines.



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- By replaceing fuzzy multisets with L-fuzzy multisets, we get L-fuzzy P systems.



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- P systems that process fuzzy multisets can compute real numbers, therefore are more expressive than Turing machines.
- **Conjecture** P systems with fuzzy processing rules that process fuzzy data should have at least the computational power of fuzzy Turing machines.
- \cdot By replaceing fuzzy multisets with L-fuzzy multisets, we get L-fuzzy P systems.
- Analogously, L-fuzzy P systems should be as powerful as L-fuzzy Turing machines. Maybe, they are more powerful.

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Fuzzy Chemical Abstract Machine • Typically, programs are viewed as types and processes as tokens.

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- Typically, programs are viewed as types and processes as tokens.
- But, for example, different people that use a particular web browser view different web pages and have different numbers of tabs open at any given moment.

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- But, for example, different people that use a particular web browser view different web pages and have different numbers of tabs open at any given moment.
- Thus, we cannot talk about processes that are identical, but we can surely talk about processes that are similar.

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- Assume that p is an executable program for some computational platform (operating system, CPU, etc.). Then an archetypal process π of p is the program p running with minimum resources.

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- How can we compare processes?
- Assume that p is an executable program for some computational platform (operating system, CPU, etc.). Then an archetypal process π of p is the program p running with minimum resources.
- The term **minimum resources** means that the archetypal process consumes the least possible resources (e.g., memory, CPU time, etc.).

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Fuzzy Chemical Abstract Machine Assume that \boldsymbol{p}_1 and \boldsymbol{p}_2 are two processes initiated from the same binary \boldsymbol{p} .

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- Assume that p_1 and p_2 are two processes initiated from the same binary p.
- Also assume that π is an archetypal process and that δ_π is a method that measures the similarity degree of some process p_i to π .

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- That is, $\delta_{\pi}(p) = i$ means that p is similar with πi to a degree equal to i.
- If $\Delta_\pi(p_1,p_2)$ denotes the degree to which the two processes p_1 and p_2 are similar, then

$$\Delta_{\pi}(p_1, p_2) = 1 - |\delta_{\pi}(p_1) - \delta_{\pi}(p_2)|.$$

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- A fuzzy labeled transition system over a crisp set of actions $\mathscr A$ is a pair $(\mathscr Q,\mathscr T)$ consisting of
- a set @ of states;
- a fuzzy relation $\mathcal{T}(\mathcal{Q},\mathcal{A},\mathcal{Q})$ called the **fuzzy transition relation**.

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Contextualism

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Fuzzy Turi Machines

Fuzzy P Syste

- A fuzzy labeled transition system over a crisp set of actions $\mathscr A$ is a pair $(\mathscr Q,\mathscr T)$ consisting of
- a set @ of states;
- o a fuzzy relation $\mathcal{T}(\mathcal{Q},\mathcal{A},\mathcal{Q})$ called the **fuzzy transition relation**.
- If the membership degree of (q, α, q') is d, $q \xrightarrow[d]{\alpha} q'$ denotes that the plausibility degree to go from state q to state q' by action α is d.

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Fuzzy Chemical Abstract Machine • Let $(\mathcal{Q}, \mathcal{T})$ be an FLTS.

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Fuzzy P System

- Let $(\mathcal{Q}, \mathcal{T})$ be an FLTS.
- Let $\mathcal S$ is a fuzzy binary relation over $\mathscr Q$.

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- Let $(\mathcal{Q}, \mathcal{T})$ be an FLTS.
- Let $\mathcal S$ is a fuzzy binary relation over $\mathscr Q$.
- $\mathcal S$ is a strong fuzzy simulation over $(\mathscr Q,\mathcal T)$ with simulation degree s if, whenever $\mathcal S(p,q)\geq s$ if $p\overset{\alpha}{\underset{d_1}{\longrightarrow}}p'$, then there exists

$$q' \in \mathcal{Q}$$
 such that $q \xrightarrow{\alpha}_{d_2} q'$, $d_2 \ge d_1$, and $\mathcal{S}(p', q') \ge \mathcal{S}(p, q)$.

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Fuzzy P System

Fuzzy Chemical Abstract Machine $(m_i)_{i=1,\dots,k}$ and $(m'_j)_{j=1,\dots,l}$ are archetypal molecules, then

$$m_1, \ldots, m_k \xrightarrow[\lambda]{} m_1', \ldots, m_l',$$

is an ideal fuzzy reaction rule;



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Fuzzy Chemical Abstract Machin $(m_i)_{i=1,\dots,k}$ and $(m'_j)_{j=1,\dots,l}$ are archetypal molecules, then

$$m_1, \ldots, m_k \xrightarrow[\lambda]{} m_1', \ldots, m_l',$$

is an ideal fuzzy reaction rule;

· Let M_i be an instance of m_i to degree $\lambda_\pi(M_i)$. Then

$$[M_1,\ldots,M_k] \underset{\lambda}{\to} [M_1',\ldots,M_l']$$

is **feasible** with feasibility degree equal to λ if $\min\{\delta_{\pi}(M_1), \dots, \delta_{\pi}(M_k)\} \geq \lambda$.



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Fuzzy Chemical Abstract Machin $(m_i)_{i=1,\dots,k}$ and $(m_j')_{j=1,\dots,l}$ are archetypal molecules, then

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is an ideal fuzzy reaction rule;

· Let M_i be an instance of m_i to degree $\lambda_\pi(M_i)$. Then

$$[M_1,\ldots,M_k]\underset{\lambda}{\to} [M_1',\ldots,M_l']$$

is **feasible** with feasibility degree equal to λ if $\min\{\delta_{\pi}(M_1), \dots, \delta_{\pi}(M_k)\} \geq \lambda$.

 If some reacting molecules may be able to yield different molecules, then the reaction rule with the highest feasibility degree is really applicable.

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$$\frac{\left(S_1 \underset{\lambda}{\rightarrow} S_2\right) \quad \left(\forall m_i \in S_3 : \delta_{\pi}(m_i) \geq \lambda\right)}{S_1 \uplus S_3 \underset{\lambda}{\rightarrow} S_2 \uplus S_3}$$

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$$\frac{\left(S_1 \underset{\lambda}{\rightarrow} S_2\right) \quad \left(\forall m_i \in S_3 : \delta_{\pi}(m_i) \geq \lambda\right)}{S_1 \uplus S_3 \underset{\lambda}{\rightarrow} S_2 \uplus S_3}$$

Let
$$\Delta_{\delta}(S) = \min \Big\{ \delta_{\pi}(p) \mid p \in S \Big\}$$
, then
$$\frac{\lambda \leq \min \Big\{ \Delta_{\delta}(S), \delta_{\pi}(m) \Big\}}{[m] \uplus S \underset{\lambda}{\leftrightarrow} [m \lhd S]}$$

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$$\frac{\left(S_1 \underset{\lambda}{\rightarrow} S_2\right) \quad \left(\forall m_i \in S_3 : \delta_{\pi}(m_i) \geq \lambda\right)}{S_1 \uplus S_3 \underset{\lambda}{\rightarrow} S_2 \uplus S_3}$$

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, then
$$\frac{\lambda \leq \min \left\{ \Delta_{\delta}(S), \delta_{\pi}(m) \right\}}{[m] \uplus S \leftrightarrow [m \lessdot S]}$$

$$\frac{\left(S \underset{\lambda}{\to} S'\right) \quad \left(\lambda \leq \min\left\{\Delta_{\delta}(S), \delta_{\pi}(C())\right\}\right)}{[C(S)] \underset{\lambda}{\to} [C(S')]}$$

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I have briefly presented

• The notion of vagueness.

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- The notion of vagueness.
- Fuzzy Turing Machines.

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More about fuzzy computability in my forthcoming book!

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More about fuzzy computability in my forthcoming book! Thank you so much for your attention!