Computing under Vagueness

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Second international workshop
New Worlds of Computation 2011
LIFO, University of Orléans (France)
Monday, May 23, 2011

Outline

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  - The old puzzle about bald people.

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- Max Black demonstrated this definition using the word **chair**.

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- In 1965, Lotfi A. Zadeh introduced his **fuzzy set theory** and its accompanying **fuzzy logic**.

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- A sentense is "super-true" if it comes out true realtive to each choice of candidate extensions for its predicates.

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- Epistemological contextualism maintains the general idea that whether one knows is somehow relative to context.

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- When $A(x) = 0$ we say that x belongs to A with degree equal to 0 not that x does not belong to A!
- A consequence of this is that one cannot build a topos of all fuzzy subsets.

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$$|A| = \sum_{x \in X} A(X).$$

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- Bart Kosko advocates the idea that fuzziness is a basic characteristic of our cosmos! (I agree with Kosko!)

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- A fuzzy Turing machine is Turing machine where $q^{n+} = f(q^n, s^n)$ is associated with a feasability degree.

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- The formulations of Chang and Tanaka-Mizumoto are special cases of Santos's formulation.

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Nowadays, we use t-norms and t-conorms instead of semirings.

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- Later on Yongming Li showed that Wiedermann's results is true for L-fuzzy Turing machines.

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- A **compartment** is space that is surrounded by a porous **membrane**.
- Nested compartments make a P system. Each compartment is a multiset.
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- Processing takes place in discrete steps but rules are applied in parallel.

P Systems with Fuzzy Data

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- R_i, $1 \leq i \leq m$, are finite sets of multiset rewriting rules of the form $u \to v$, $u \in O^*$ and $v \in O_{\mathrm{TAR}}^*$, where $O_{\mathrm{TAR}} = O \times \mathrm{TAR}$, and $\mathrm{TAR} = \{\mathrm{here}, \mathrm{out}\} \cup \{\mathrm{in}_j | 1 \leq j \leq m\}$.

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- $i_0 \in \{1, 2, \ldots, m\}$ is the label of a compartment called the **output** membrane.

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- **Conjecture** P systems with fuzzy processing rules that process fuzzy data should have at least the computational power of fuzzy Turing machines.
- By replaceing fuzzy multisets with L-fuzzy multisets, we get L-fuzzy P systems.
- Analogously, L-fuzzy P systems should be as powerful as L-fuzzy Turing machines. Maybe, they are more powerful.

Similar Processes

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- The term **minimum resources** means that the archetypal process consumes the least possible resources (e.g., memory, CPU time, etc.).

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- That is, $\delta_\pi(p) = i$ means that p is similar with πi to a degree equal to i.
- If $\Delta_\pi(p_1, p_2)$ denotes the degree to which the two processes p_1 and p_2 are similar, then

$$\Delta_\pi(p_1, p_2) = 1 - \left| \delta_\pi(p_1) - \delta_\pi(p_2) \right| .$$

Fuzzy Labeled Transition Systems

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- a set $\mathscr{Q}$ of states;
- a fuzzy relation $\mathscr{T}(\mathscr{Q},\mathscr{A},\mathscr{Q})$ called the **fuzzy transition relation**.
- If the membership degree of (q,α,q') is d, $q \xrightarrow[d]{\alpha} q'$ denotes that the plausibility degree to go from state q to state q' by action α is d.

Strong Fuzzy Simulation

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- Let $\mathscr{S}$ is a fuzzy binary relation over $\mathscr{Q}$.
- $\mathscr{S}$ is a **strong fuzzy simulation over** $(\mathscr{Q},\mathscr{T})$ **with simulation degree** s if, whenever $\mathscr{S}(p,q) \geq s$ if $p \xrightarrow[d_1]{\alpha} p'$, then there exists $q' \in \mathscr{Q}$ such that $q \xrightarrow[d_2]{\alpha} q'$, $d_2 \geq d_1$, and $\mathscr{S}(p',q') \geq \mathscr{S}(p,q)$.

The Fuzzy Chemical Abstract machine

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- $(m_i)_{i=1,\ldots,k}$ and $(m'_j)_{j=1,\ldots,l}$ are archetypal molecules, then

$$m_1, \ldots, m_k \underset{\lambda}{\rightarrow} m'_1, \ldots, m'_l,$$

is an **ideal** fuzzy reaction rule;

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  is an **ideal** fuzzy reaction rule;

- Let M_i be an instance of m_i to degree $\lambda_\pi(M_i)$. Then

$$[M_1,\ldots,M_k] \underset{\lambda}{\rightarrow} [M'_1,\ldots,M'_l]$$

  is **feasible** with feasibility degree equal to λ if $\min\{\delta_\pi(M_1),\ldots,\delta_\pi(M_k)\} \geq \lambda$.

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  is **feasible** with feasibility degree equal to λ if $\min\{\delta_\pi(M_1),\ldots,\delta_\pi(M_k)\} \geq \lambda$.
- If some reacting molecules may be able to yield different molecules, then the reaction rule with the highest feasibility degree is **really applicable**.

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- $$\frac{\left(S_1 \underset{\lambda}{\rightarrow} S_2\right) \quad \left(\forall m_i \in S_3 : \delta_\pi(m_i) \geq \lambda\right)}{S_1 \uplus S_3 \underset{\lambda}{\rightarrow} S_2 \uplus S_3}$$

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- Let $\Delta_\delta(S) = \min\left\{\delta_\pi(p) \,\middle|\, p \in S\right\}$, then

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- $$\frac{\left(S \underset{\lambda}{\longrightarrow} S'\right) \quad \left(\lambda \leq \min\left\{\Delta_\delta(S), \delta_\pi(C())\right\}\right)}{[C(S)] \underset{\lambda}{\longrightarrow} [C(S')]}$$

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Thank you so much for your attention!