``Non-Gaussian effects in quantum magnets (and black holes)"

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Introduction:

Extremal black holes are the simplest examples where certain quantum properties of gravity can be consistently computed. The reason is that extremal black holes do not radiate as Hawking, since their Hawking temperature vanishes. However they do have non-zero entropy;

Therefore particle probes of the geometry possess a space of states of fixed dimensionality that is an integer. We can take this integer to be a prime number.

This space of states of a single particle probe of the \bulk" has the geometry of a deformation of the single{sheeted hyperboloid, i.e. AdS2, whose points{therefore the states of the probe{canbe labelled the following way:

Each point corresponds to a state of the single particle probe. The evolution operator of the quantum particle is an N/N, unitary matrix, whose construction is very similar to that for a magnet, of spin S, whose space of states is 2S+1dimensional and whose evolution operator is a unitary matrix, in the 2S +1dimensionalm the quantum particle explores this deformation of the one {sheeted hyperboloid and the way a spin explores SU (2).

A spin in a flux and SU(2): A quantum particle in a flux on the sphere:

Quantum mechanically A spin-1/2 particle has space of states 2{dimensional, spanned by the linear combinations of the states up $|+\rangle$ and down $|-\rangle$ that are 2{component vectors:

$$|\psi\rangle = a\,|+\rangle + b\,|-\rangle$$

a; b are complex numbers. In the Heisenberg representation The evolution operator is given by the expression

$$U(\boldsymbol{\theta}) = e^{i\boldsymbol{\theta}.\boldsymbol{\sigma}} = 1_{2\times2}\cos|\boldsymbol{\theta}| + i\boldsymbol{\theta}.\boldsymbol{\sigma}\sin|\boldsymbol{\theta}|$$

$$U(\theta_3)^n = 1_{2\times2}\cos n\theta_3 + i\sigma_3 \sin n\theta_3$$

The nth state

$$|\psi\rangle_n = ae^{in\theta_3}\,|+\rangle + be^{in\theta_3}\,|-\rangle$$

$\theta_3 = \frac{2\pi}{N},\ n = 0,1.2,\dots,N-1,$ N, prime number.

Density of N dimensional space of states

$$\rho_n = \frac{|\langle\psi|\,\psi\rangle_n|^2}{\sum_{n=0}^{N-1}|\langle\psi|\,\psi\rangle_n|^2}$$

$$\rho_n = \frac{1}{N} + \frac{\mathbf{a}}{N}\cos\frac{2\pi}{N}n$$

$$\mathbf{a} = \frac{1-(|a|^2-|b|^2)^2}{1+(|a|^2-|b|^2)^2}$$

$$|a|^2+|b|^2 = 1,\ 0 \le \mathbf{a} \le 1$$

Properties of density of states ρ(x) :

- The density ρ*n* and its moments,
- $$\langle n^k\rangle = \sum_{n=0}^{N-1} n^k\rho_n = \frac{1}{N}\sum_{n=0}^{N-1} n^k + \frac{\mathbf{a}}{N}\sum_{n=0}^{N-1} n^k\cos\frac{2\pi n}{N}$$
- Clearly that the N states are visited periodically, with period N
- Introducing the scaling expressions Nρ(*n*=*xN*) =ρ(x) we're interested in the properties of ρ(x) and its moments and, in particular the fourth order cumulant.

$$Q_4(\mathrm{a},\mathrm{x},\mathrm{N}) \equiv \frac{1}{N^4}\left(\langle(n-\langle n\rangle)^4\rangle - \langle(n-\langle n\rangle)^2\rangle^2\right)$$

- Once the corresponding expressions for it are non-zero then it retains the memory of the initial condition through the parameter a-which can be expressed in terms of the moments.

The fourth order cumulant:

$$using\ variable, \qquad x = \frac{n}{N}$$

The fourth order cumulant

$Q_4(\mathbf{a},\mathrm{x},\mathrm{N}) = \langle x^4\rangle - 4\langle x^3\rangle\langle x\rangle + 12\langle x^2\rangle\langle x\rangle^2 - 6\langle x\rangle^4 - 3\langle x^2\rangle^2$

The N independent fourth order cumulant

$\lim_{N\to\infty} N\rho_{n=xN} = 1 + \mathbf{a}\cos 2\pi x \equiv \rho_\infty(x)$; Defined on the interval , $x \in [0,1]$

$$\lim_{N\to\infty} Q_4(\mathbf{a},\mathrm{x},\mathrm{N}) = -\left(\frac{\pi^4 + 180\mathbf{a} + 90\mathbf{a}^2}{120\pi^4}\right) \neq 0$$

NON-GAUSSIAN

$$\mathbf{a} = \frac{8\pi^2}{3}\langle x^3\rangle - 2\pi^2\langle x^2\rangle$$

Correspondence of AdS2 near Horizon geometry to the Spin charge in a flux of spin 1/2:

AdS_2 Near Horizon geometry ${x_0}^2 + {x_1}^2 - {x_2}^2 = 1$, 2D Space of Hyperboloid sheet

$$X = \begin{pmatrix} x_0 & x_1+x_2 \\ x_1-x_2 & -x_0 \end{pmatrix} = [T_-(\mu_-)T_+(\mu_+)]\sigma_3[T_-(\mu_-)T_+(\mu_+)]^{-1}$$

$$\begin{cases} detX = -1 \\ T_-(\mu_-) = \begin{pmatrix} 1 & 0 \\ \mu_- & 1 \end{pmatrix},\quad T_+(\mu_+) = \begin{pmatrix} 1 & \mu_+ \\ 0 & 1 \end{pmatrix} \\ \mu_+ = \frac{x_1+x_2}{2},\ \mu_- = \frac{x_1-x_2}{1+x_0} \end{cases}$$

A point $X(x_0, x_1, x_2) \in AdS_2$ ↔corresponds to a spin state

Time evolution of $'X'$ ⟹ $X' = AXA^{-1}$ ⟺ the Spin charge in a flux of spin ½

Arnold's cat map of modulo N. The description of the dynamics of the particle on the deformation of AdS_2, that is described by the mod N operation.

The evolution operator A= $\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$, chosen by the AdS_2, N=number of states of quantum probe particle.

Remarks and conclusions:

- The Extremal Blackholes do not emit Hawking radiation so single particle quantum probe with certain quantum properties of gravity can be consistently computed. The space of states of a single particle(spin 1/2) probe of the bulk has the deformed geometry of a single sheeted hyperboloid, as AdS2
- Results of our computation shows independent on number of states N, the fourth order cumulant is non vanishing and globally negative, implies Non Gaussian distribution of states.
- If the cumulant had vanished, this would be an indication of Gaussian correlations; while these only make sense for continuous distributions, what is interesting is that the limit from the discrete distribution to the continuous one is smooth.