Geometric computation on the plane

– signal machines –

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Outline

1. Origins

2. Model

3. Computational aspects

4. Singularities

Origins

- Geometrical mosaic, frieze

- Particle trajectories

- Cellular automata algorithms based on geometrical constructions

$$\text{regular movements} \rightsquigarrow \text{lines}$$
$$\text{encouters} \rightsquigarrow \text{rules}$$

Model

- Simple

- Minimal

- Deterministic

Meta-signal $\equiv$ state and velocity
Rule $\qquad\equiv$ Meta-signals $\rightarrow$ Meta-signals
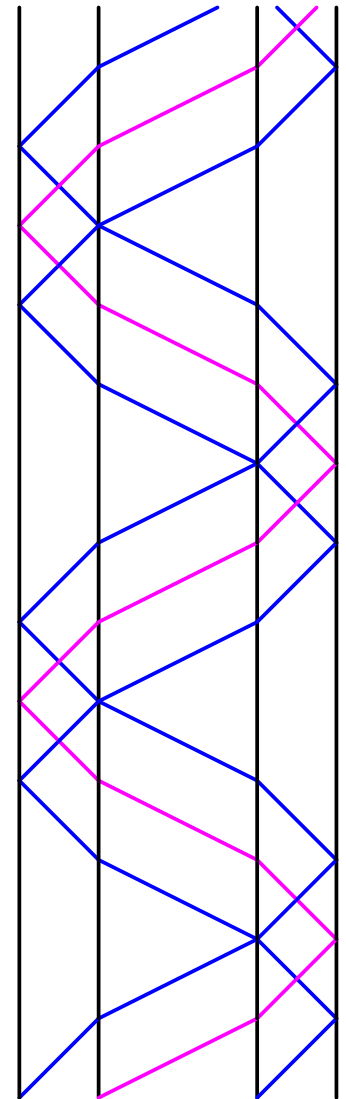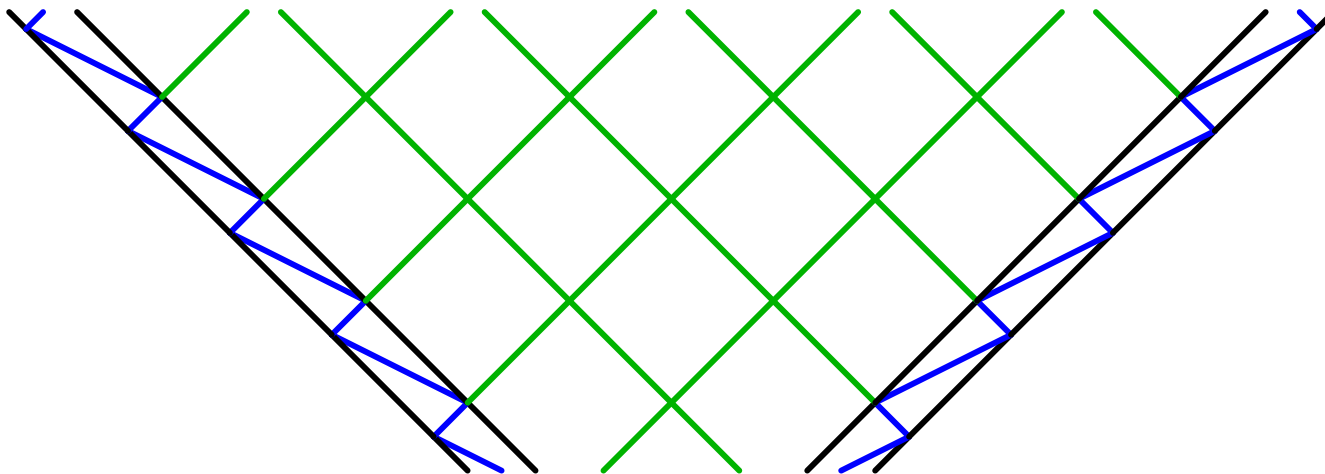
Model

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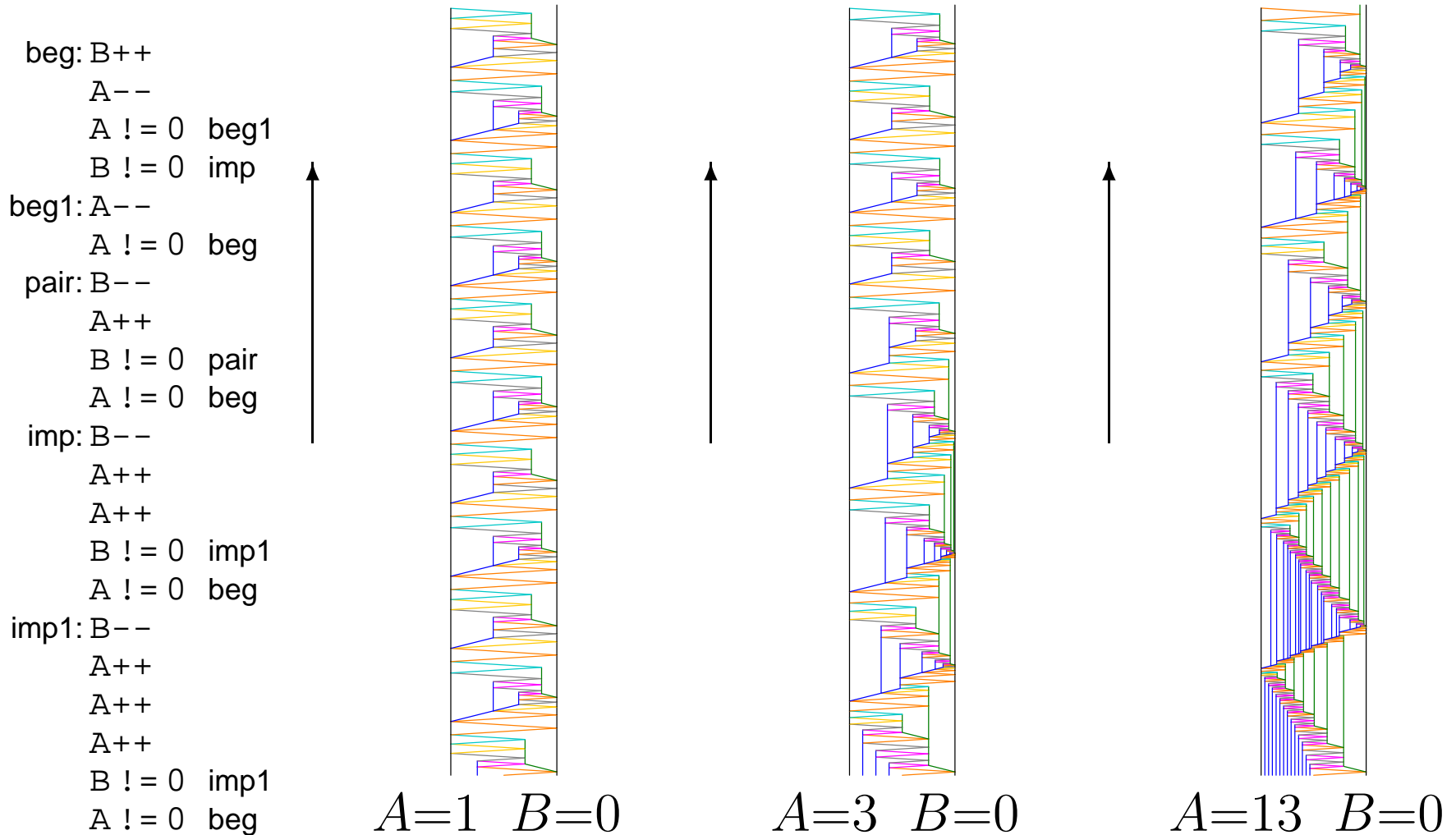
Space-time diagram & configuration

Properties

- local

- light cone

- uniform in space and time

- finite number of states

- continuous space

- continuous time

⤳ Zeno paradox?

Computation universal (Turing)

Simulate any 2-counter automaton

```
beg:  B++
      A--
      A != 0   beg1
      B != 0   imp
beg1: A--
      A != 0   beg
pair: B--
      A++
      B != 0   pair
      A != 0   beg
imp:  B--
      A++
      A++
      B != 0   imp1
      A != 0   beg
imp1: B--
      A++
      A++
      A++
      B != 0   imp1
      A != 0   beg
```
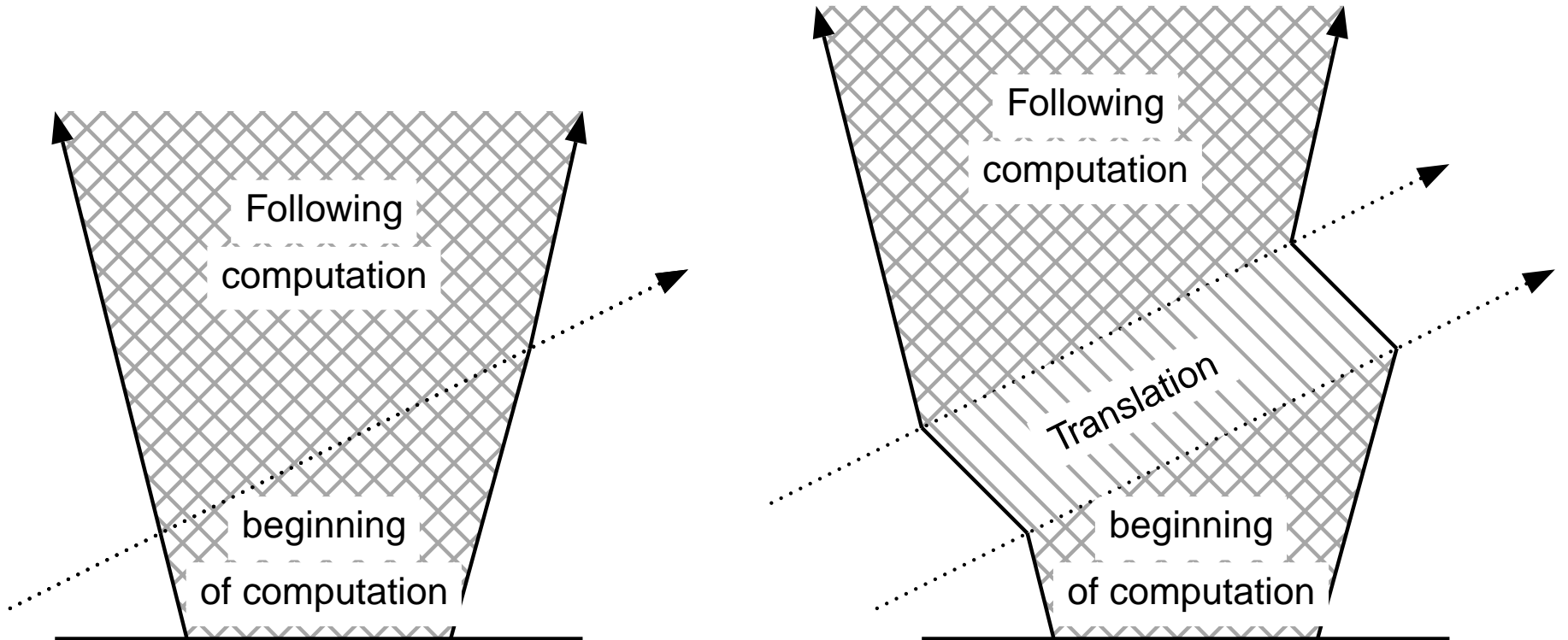
$A=1 \quad B=0$ $A=3 \quad B=0$ $A=13 \quad B=0$

⤳ Highly unpredictable

Geometrical constructions

- Static
  - Translations
  - Homotecy

- Dynamic
  - Freezing
  - Translations
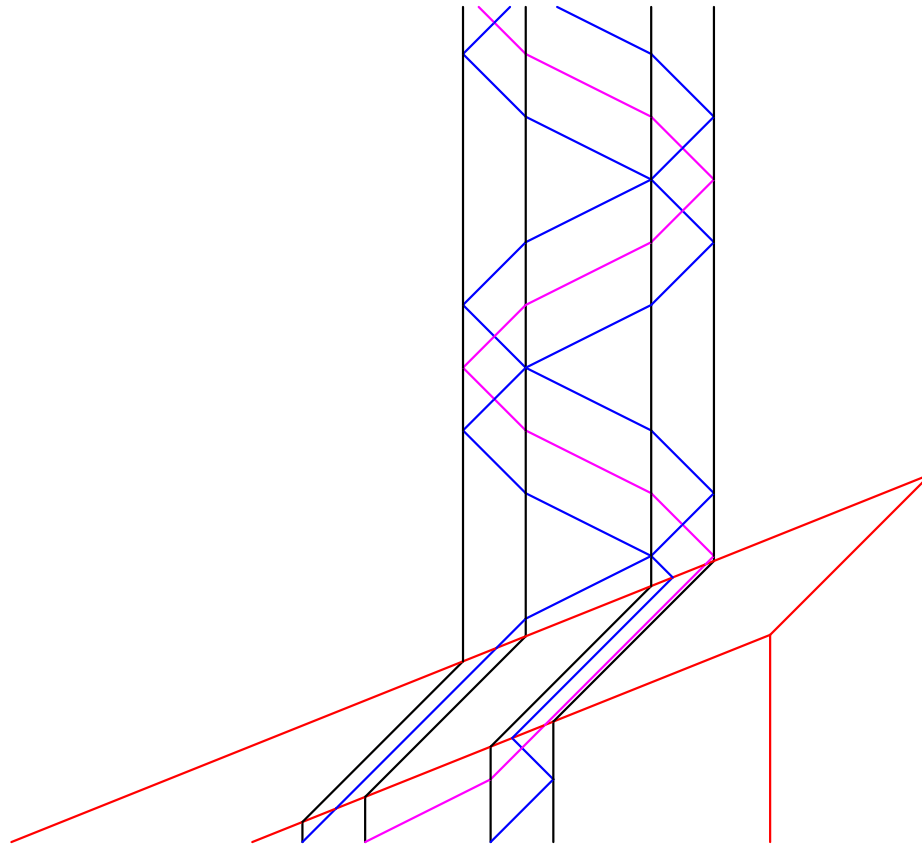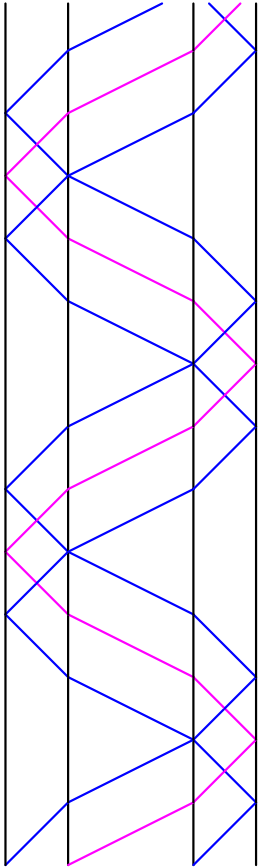 - Homotecy

⤳ Construction with these "operators"

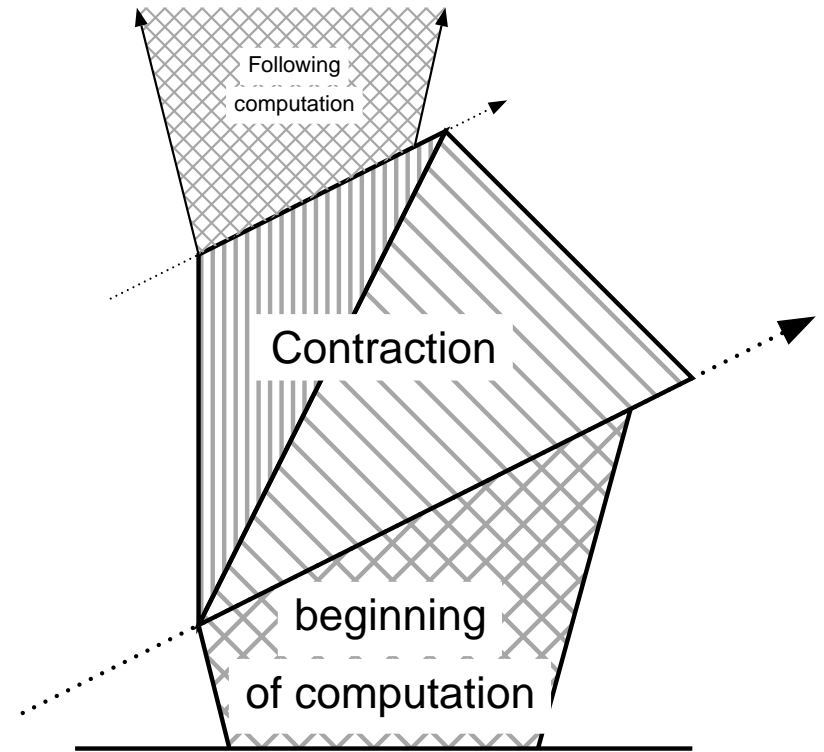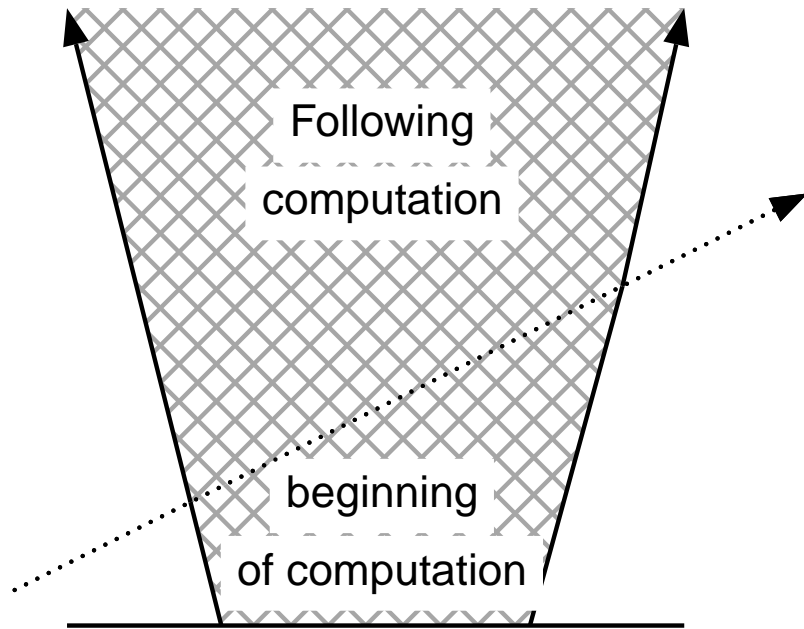# Freezing the computation

Freeze then restore

Freezing the computation

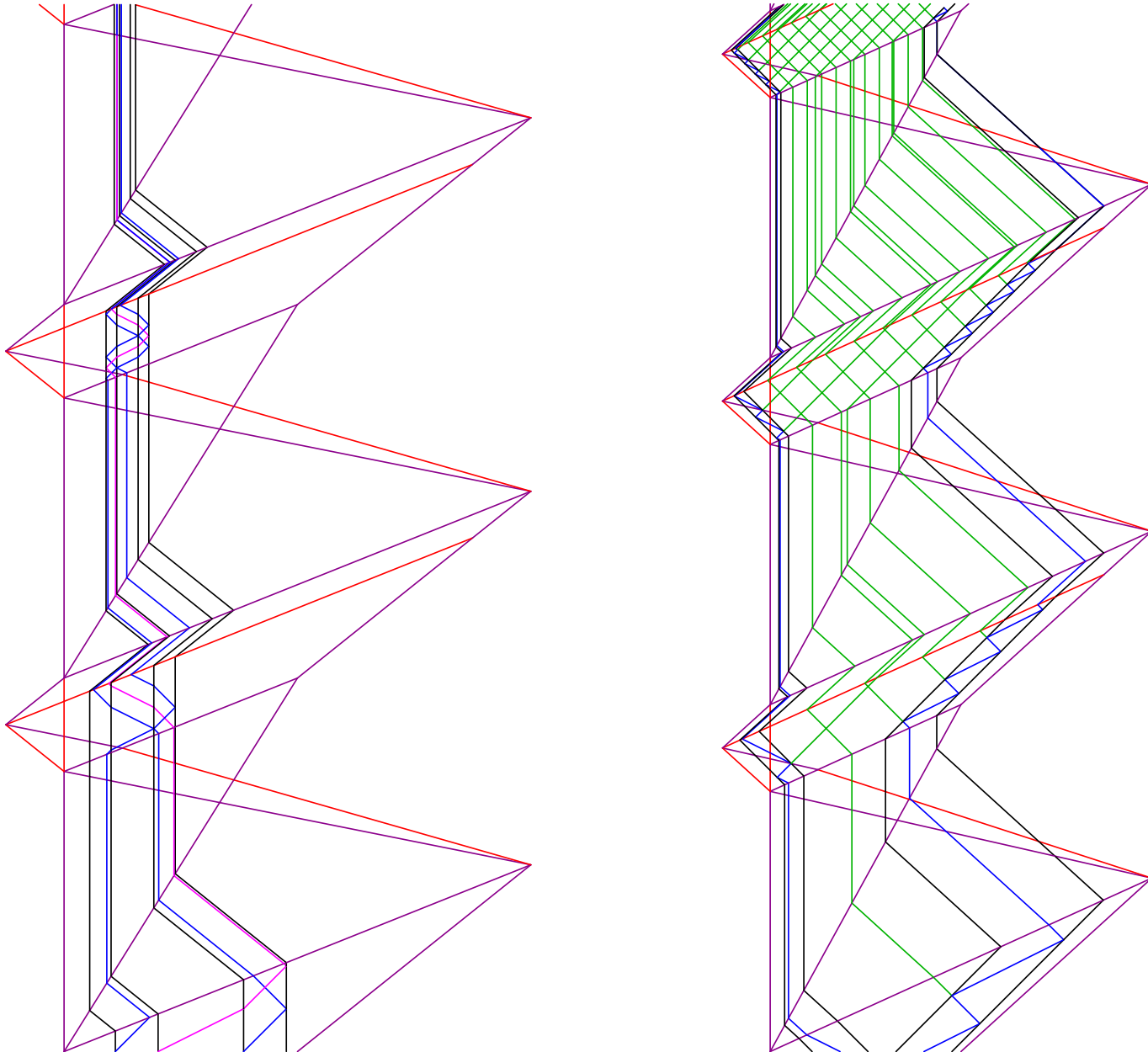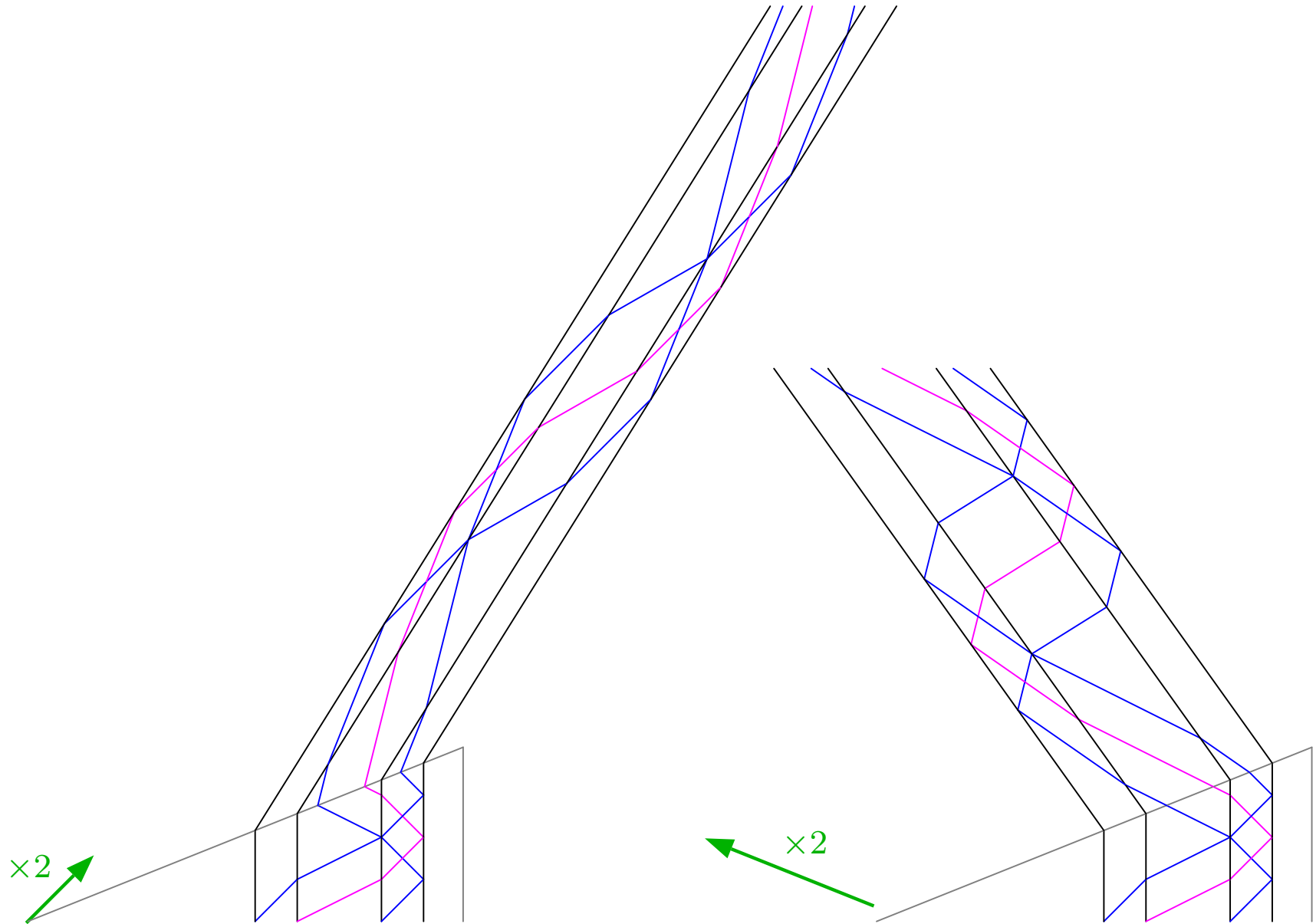Freeze then restore

Contraction to a ribbon
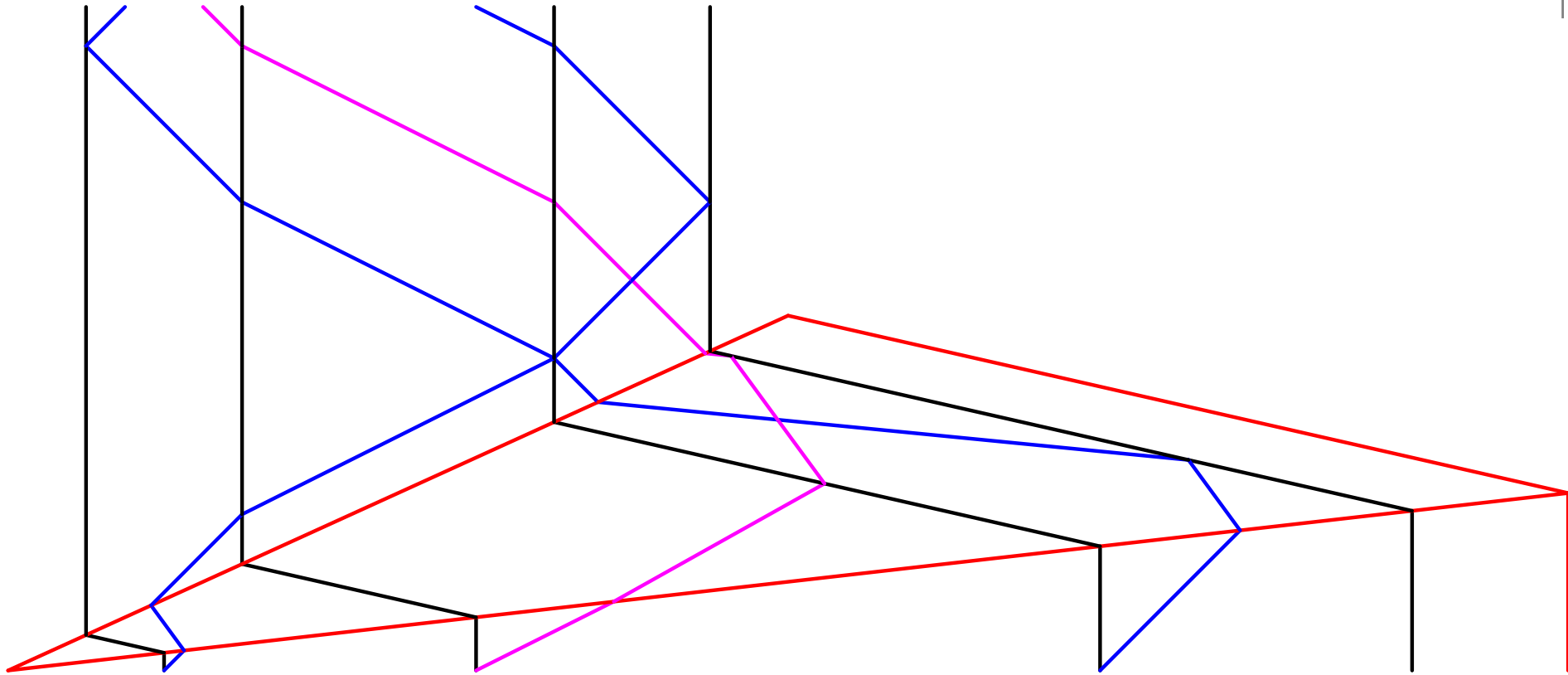

Following
computation

beginning
of computation

Following
computation
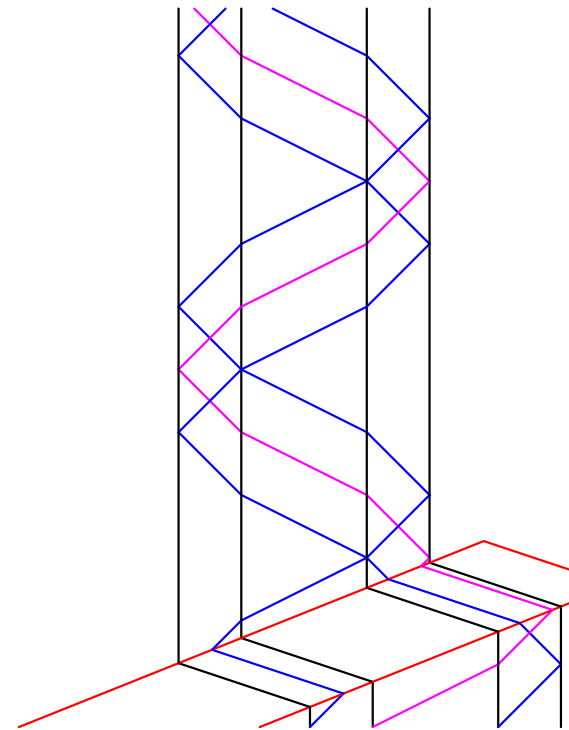
Contraction

beginning
of computation

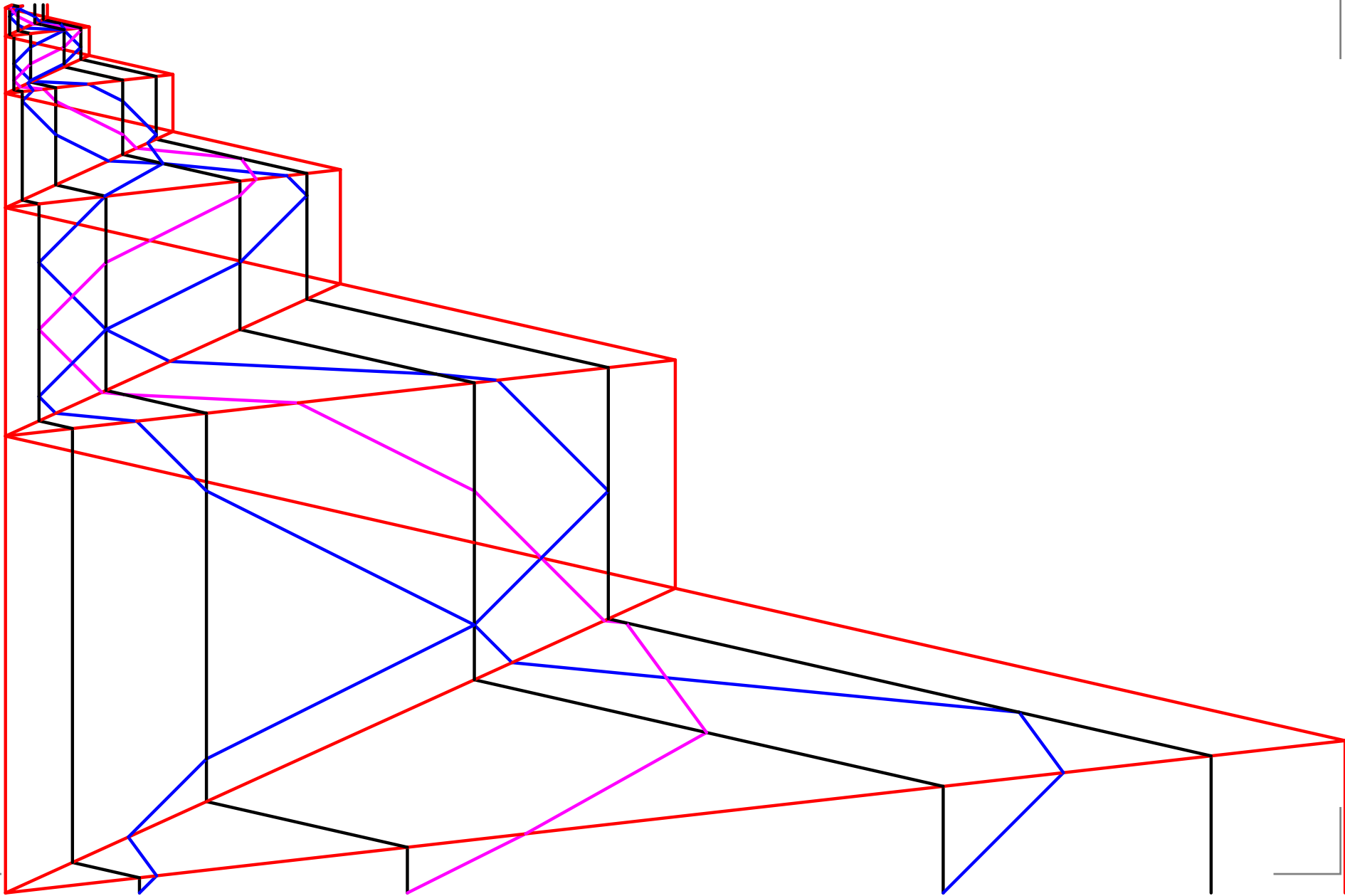# Contraction to a ribbon

Distortion

$\times 2$

$\times 2$

Continuous contraction

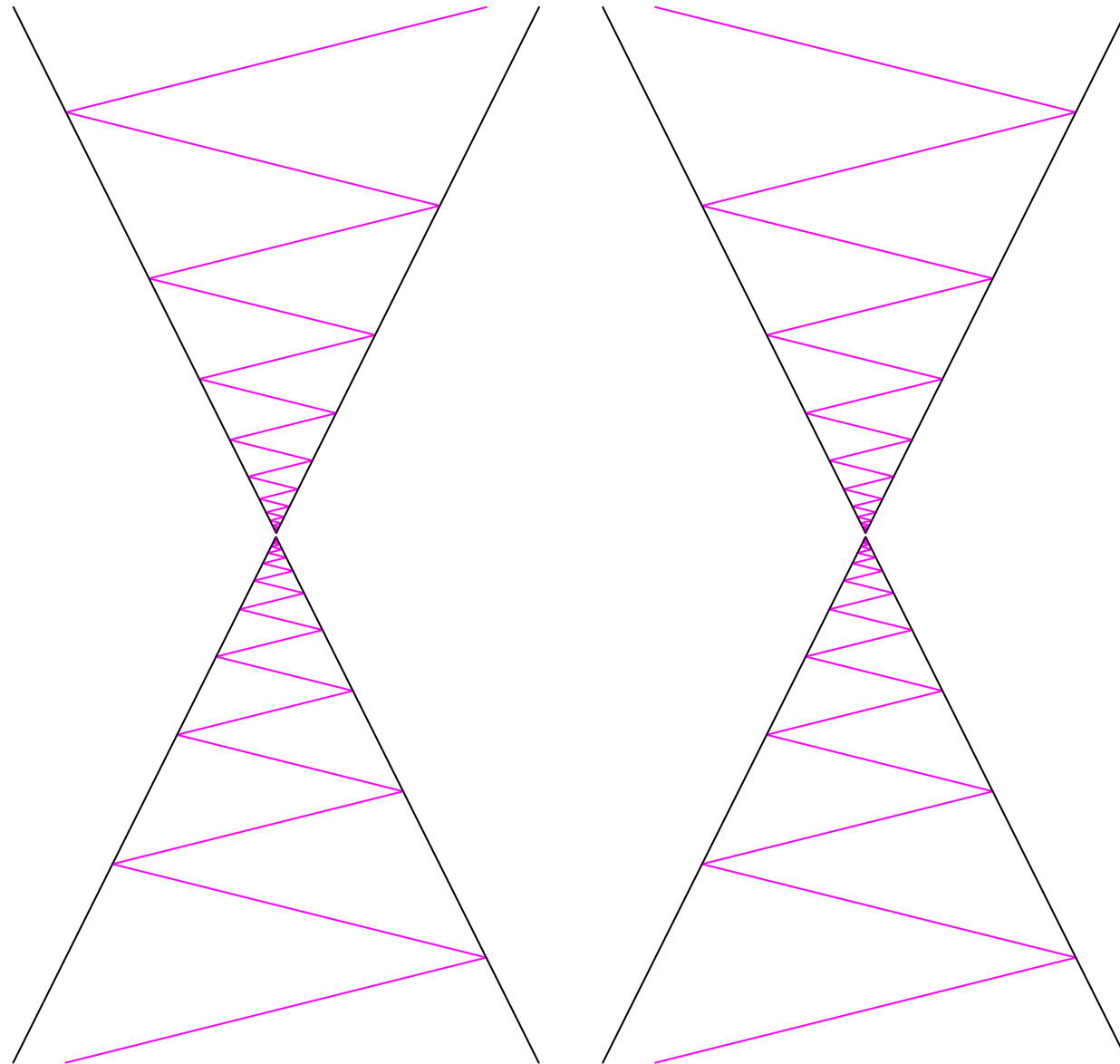# Contraction to a square

Isolated singularities

Topological definition of space-time diagrams

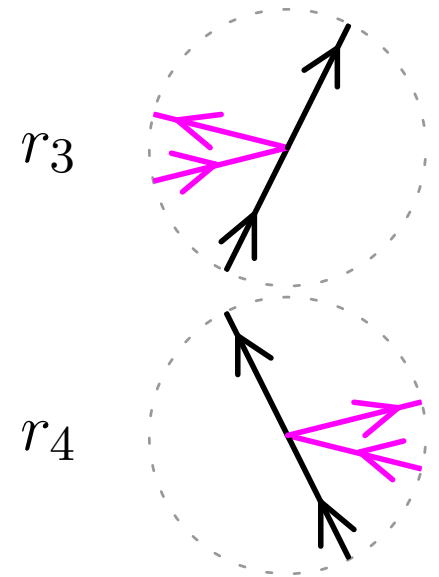It is not a trace of an execution anymore

Rather an extension

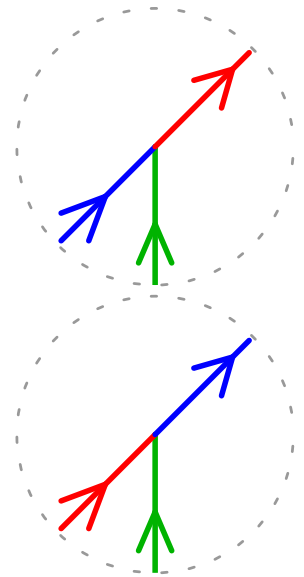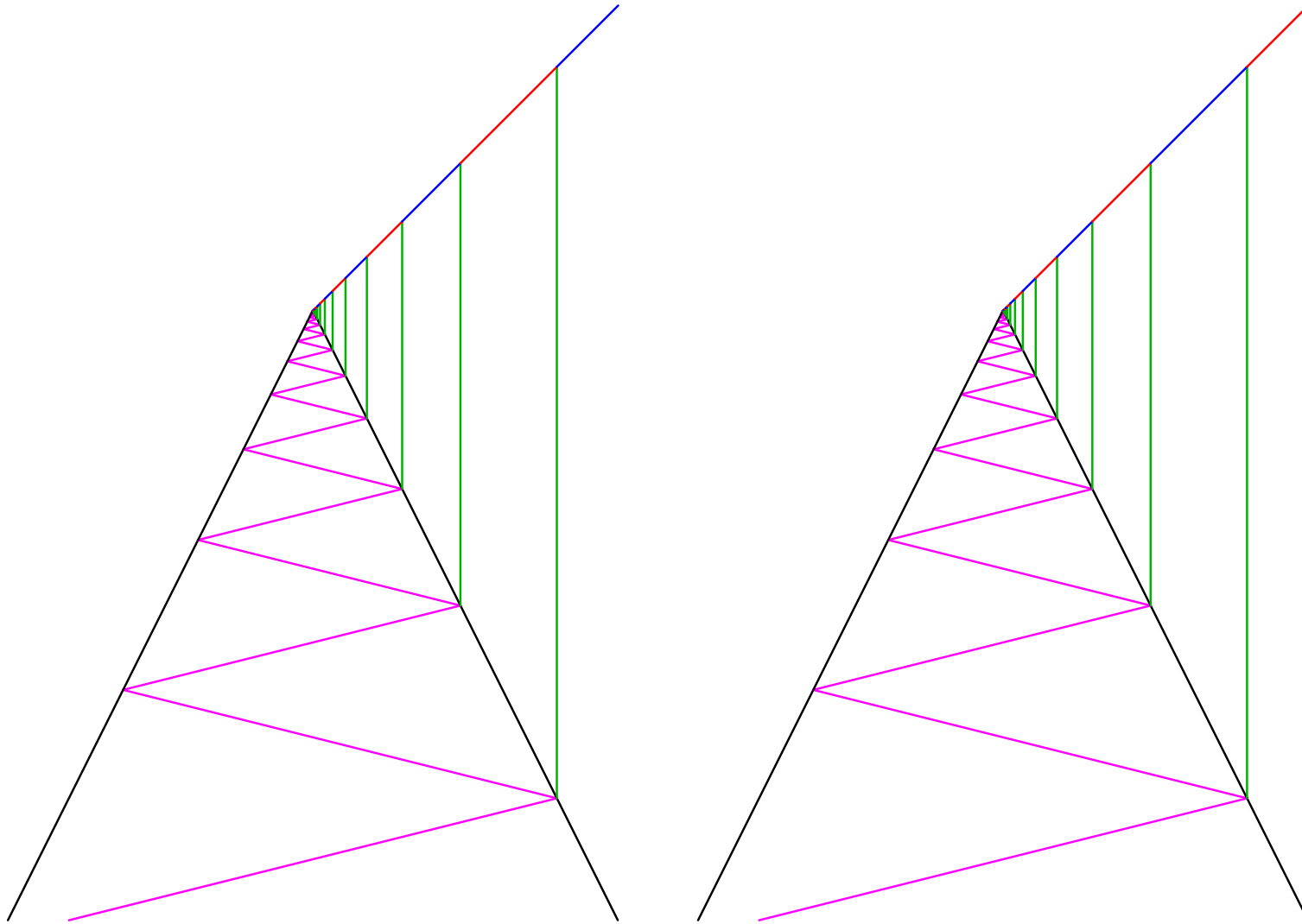Topological criteria of accumulated values around singularities

Non determinism

Infinitely many possibilities
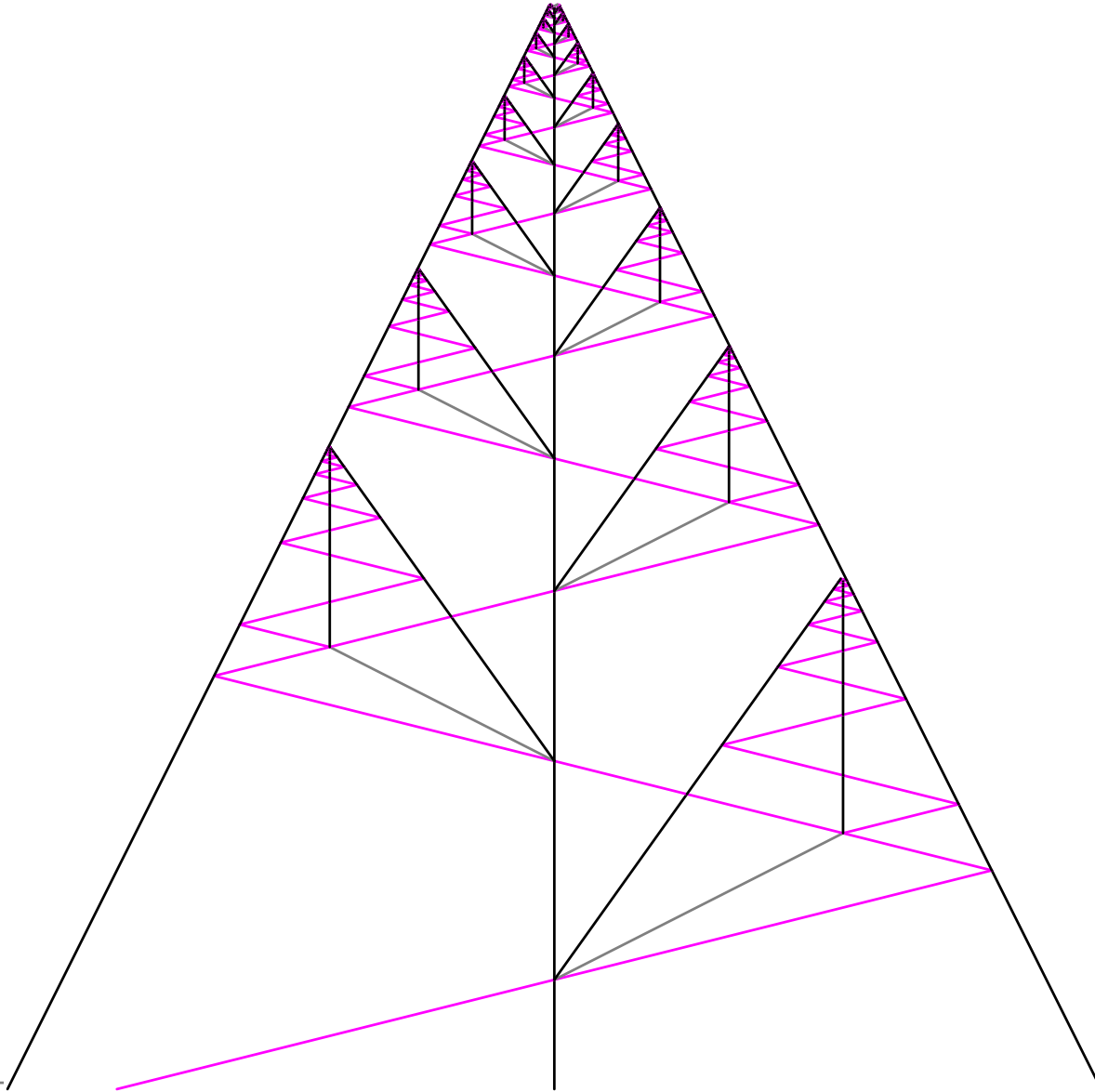
r_3

r_4

Non determinism

Finitely many possibilities

Second order isolated singularities

Decidability

Instance

    M: rational geometrical machine,

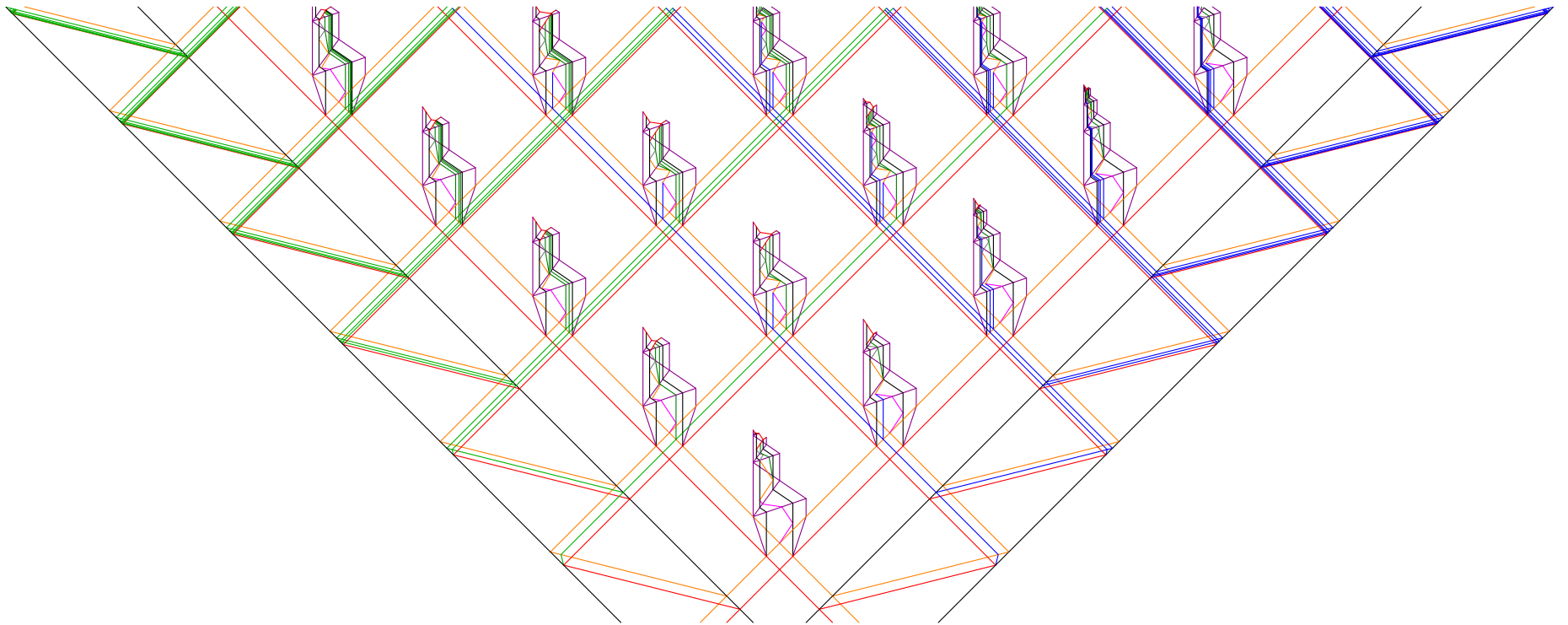 c_0: finite initial configuration,

Question

    Will there be any singularity?

Highly undecidable (Σ_2^0 in the arithmetical hierarchy)

Reduction: Accumulation iff there is an initial value for which the 2 counter automaton does not stop

Reduction

Un-isolated singularities