La perspective du signal: des automates cellulaires aux machines à signaux

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Cellular Automata

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1001202020010000202100020021
0123303030322222202232121230
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0001231230222200002022123000
0000100100022000000200210000
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0000001003002120030021000000
0000000123101022221230000000
0000000010012022002100000000
0000000001233030123000000000
0000000000100320210000000000
0000000000012312300000000000
```

$$Q = \{0, 1, 2, 3\}$$

$$f(x, y, z) = 3x + 2y + z + xy \mod 4$$

Definition

- Q: finite set of *states*
- $f\colon Q^k \to Q$ *local function*

Dynamical system

Global function, $\mathcal{G} : Q^{\mathbb{Z}} \to Q^{\mathbb{Z}}$

Orbit and space-time diagram

Value in $Q^{\mathbb{Z}\times\mathbb{N}}$
Image with big pixels

Background and Signals

Background

(2-d) Pattern that may form
a valid space-time diagram by bi-periodic repetition.

Signal

- Pattern that (legally) repeats 1-periodically on a background
- Pattern repeating 1-periodically and separating two backgrounds

Illustration by examples

Understanding the dynamics

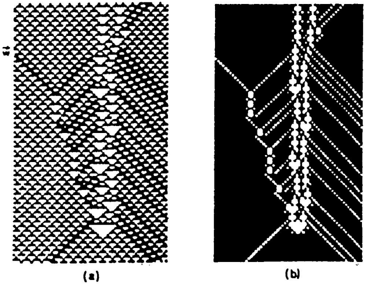


FIG. 7. Rule 54. (a) Annihilation of the radiating particle. (b) The same as (a) with the mapping defined in Fig. 6.

[Boccara et al., 1991, Fig. 7]

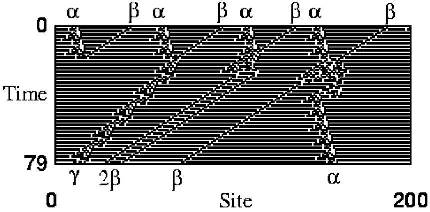


FIG. 7. The four different (out of 14 possible) interaction products for the $\alpha + \beta$ interaction.

[Hordijk et al., 2001, Fig. 7]

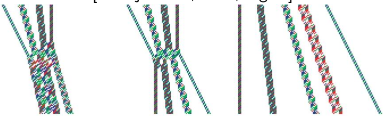


Figure 5. Two collisions of filtrons, and five free filtrons supported by the FPS model; ST diagram applies $q = 1$.

[Siwak, 2001, Fig. 5]

Generating prime numbers

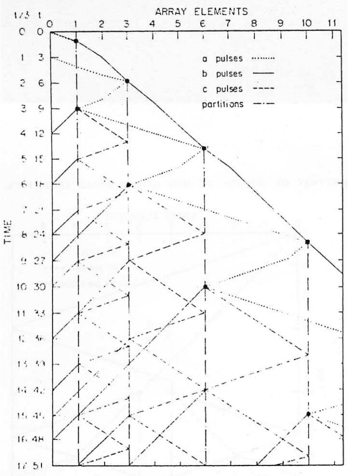


FIG. 2. Solution to the prime problem

[Fischer, 1965, Fig. 2]

Computing by simulating a Turing machine

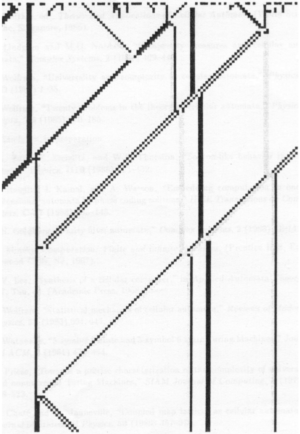


Figure 4: The $k = 4$, $r = 2$ universal cellular automaton of table 4 simulated starting from a random initial state. The symbols 0, 1, ␣, and + are represented by

[Lindgren and Nordahl, 1990, Fig. 4]

Firing Squad Synchronization

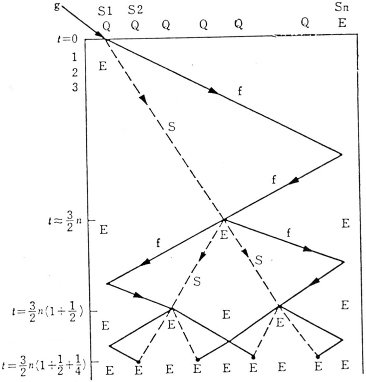


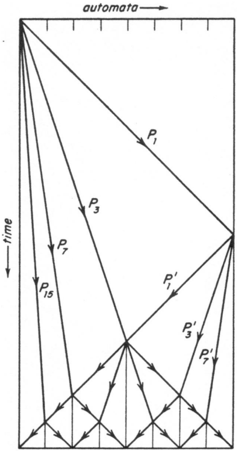
図 3·5 一斉射撃の問題（連続近似）

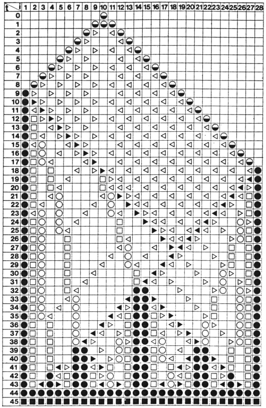
G	s_1	s_2	s_3	s_4	s_5	s_6
g	Q	Q	Q	Q	Q	E
$t=0$	f's'Efs	Q	Q	Q	Q	E
1	E	Q2f	Q	Q	Q	E
2	E	Q1	Qf	Q	Q	E
3	E	Q&	Q	Qf	Q	E
4	E	Q	Q2	Q	Qf	E
5	E	Q	Q1	Q	Q	f'Ef
6	E	Q	QS	Q	f'Q	E
7	E	Q	Q	a'Q*	Q	E
8	E	Q	f'S'ESf	f's'Esf	Q	E
9	E	f'2Q	E	E	Q2f	E
10	f'Ef	1Q	E	E	Q1	f'Ef
11	E	f'S'ESf	E	E	f's'Esf	E
12	a'Ea	E	a'Ea	a'Ea	E	a'Ea
13	F	F	F	F	F	F

図 3·6 一斉射撃解（$n=6$）

[Goto, 1966, Fig. 3+6]

Firing Squad Synchronization (again)





Notation

A	B	C	C	D	D	E_1	E_2	R	F
	◒	▶	◀	▷	◁	○	□	●	■

[Varshavsky et al., 1970, Fig 1 and 3]

Multiplication

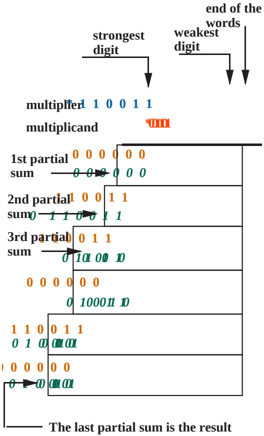


Figure 1: A human multiplication.

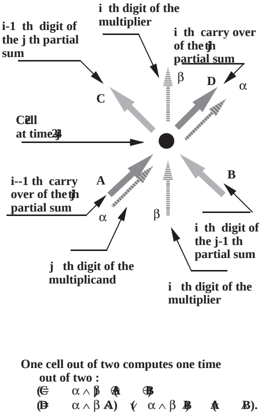


Figure 3: Computations done on one cell out of two, one unit of time out of two.

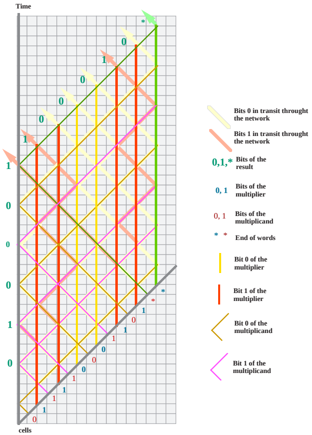


Figure 4: Multiplying 110011 by 10110.

[Mazoyer, 1996, Fig. 1, 3 andx 4]

A whole programming system

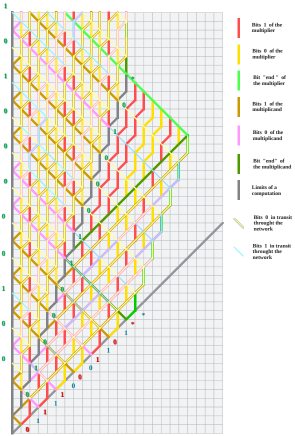


Figure 8: Computing $(ab)^2$.

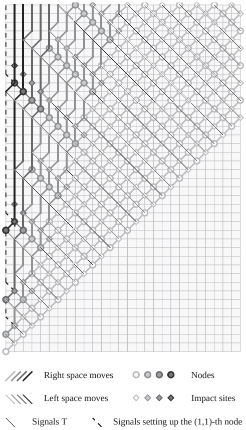


Figure 9: Setting up an infinite family of regular safe grids (the darkness of the grid indicates its rank).

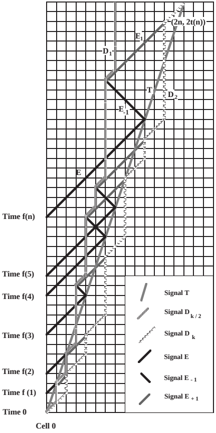


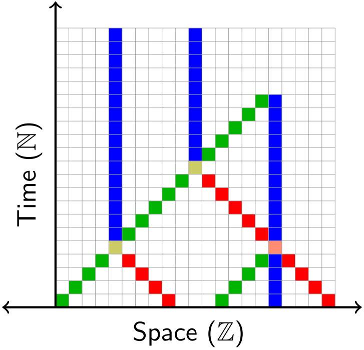
Figure 18: Characterization of the sites $(n, f(n))$.

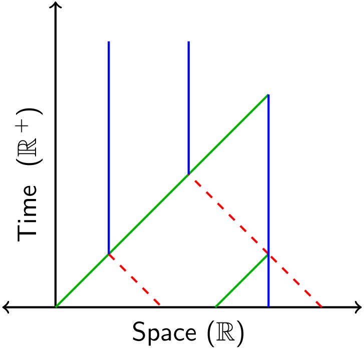
[Mazoyer, 1996, Fig. 8 and 19] and [Mazoyer and Terrier, 1999, Fig. 18]

Moving to the continuum

Forget about discreteness

$\rightsquigarrow$ continuous

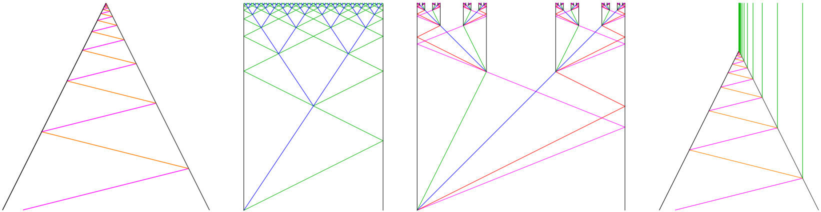




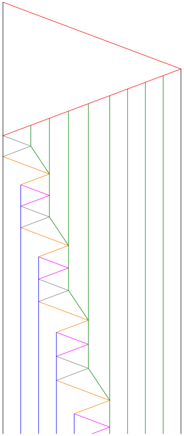
Vocabulary

- Signal (meta-signal)
- Collision (rule)

New kinds of *monsters*



Computability and undecidability [Durand-Lose, 2005]



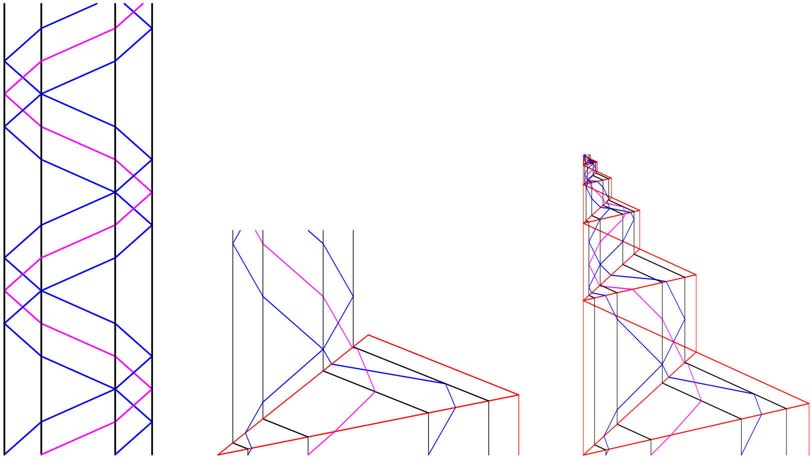
Two-counter simulation

Turing-machine can also
be simulated directly

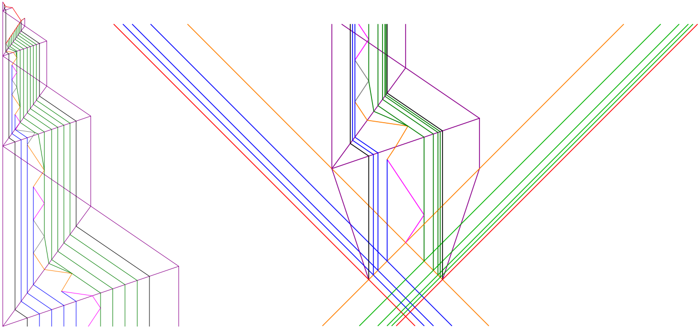
Undecidable

- total erasing
- finite number of signal
- signal/collision apparition

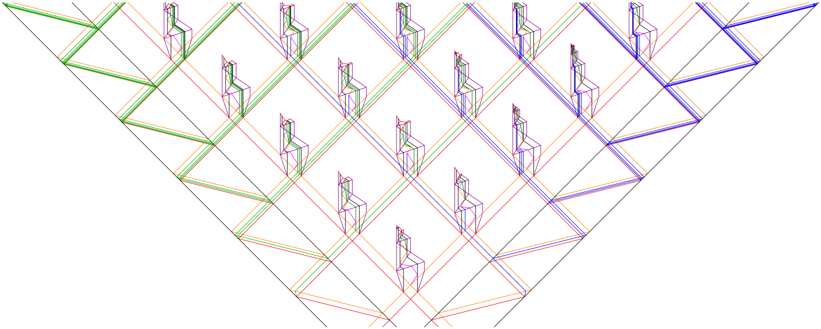
Scaling down and bounding the duration



Computing inside bounded room



Accumulation forecasting is Σ^2_0-complete [Durand-Lose, 2006b]



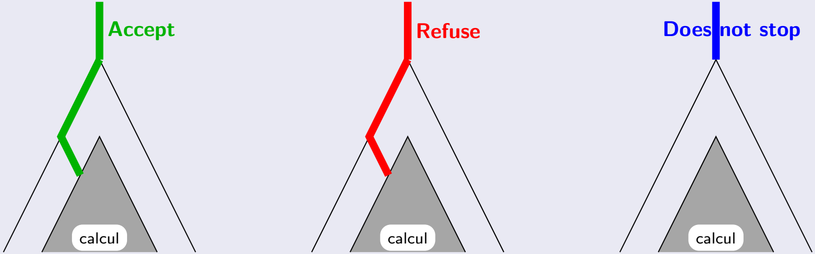
Link with the Black hole model [Durand-Lose, 2006a]

Principe

Two different timelike half-curves such that

- they have a point in common (used to set things and start)
- one is upward-infinite and fully contained in the casual past of a point of the other

Solving recursively enumerable problems



Links with the Blum, Shub and Smale model

Classical BSS model

Variables holds real numbers in exact precision

- input / output
- test $0 < x$
- shift (to access other variables)
- compute a polynomial function

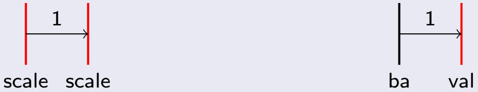
Linear BSS [Durand-Lose, 2007]

Restriction

- only linear function
- *i.e.* no inner multiplication

Encoding real numbers

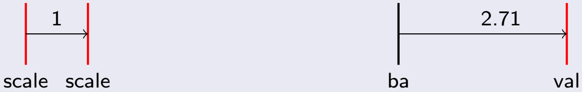
Scale + distance



- Common scale for all variables
- Sign test trivial

Encoding real numbers

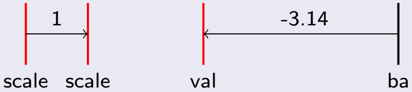
Scale + distance



- Common scale for all variables
- Sign test trivial

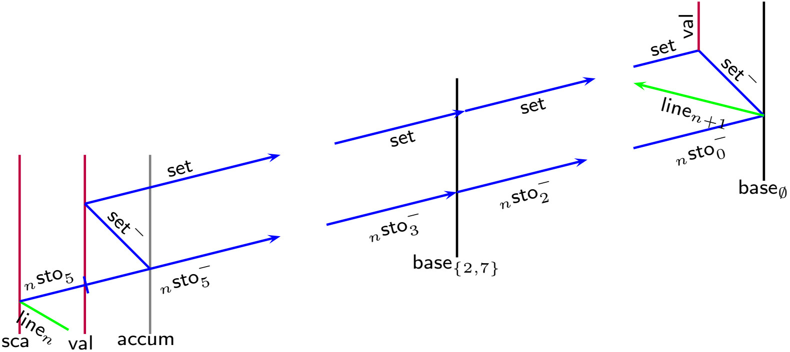
Encoding real numbers

Scale + distance

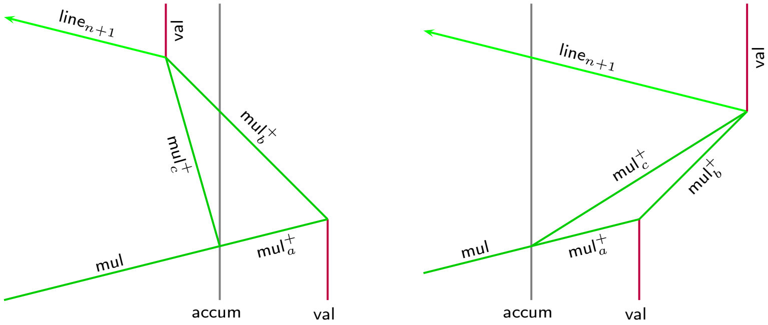


- Common scale for all variables
- Sign test trivial

Copy and Addition



External multiplication



Internal multiplication [Durand-Lose, 2008]

Computation

- Pre-treatment to ensure $0 < y < 1$
- Binary extension of y:

$$y = \mathtt{y}_0.\mathtt{y}_1\mathtt{y}_2\mathtt{y}_3\ldots$$

- Computation

$$xy = \sum_{0 \leq i} \mathtt{y}_i \left(\frac{x}{2^i}\right)$$

Principe

Computation on the margin
the margin is scaling down geometrically

Square rooting is also possible!

- Natural filiation with CA
- Continuous time
  - Zeno effect

Links with other models

- Black hole model
- Blum, Shub and Smale model

Future work

- Relate with CA
- Characterize the analog computing power