# Signal machines : localization of isolated accumulation

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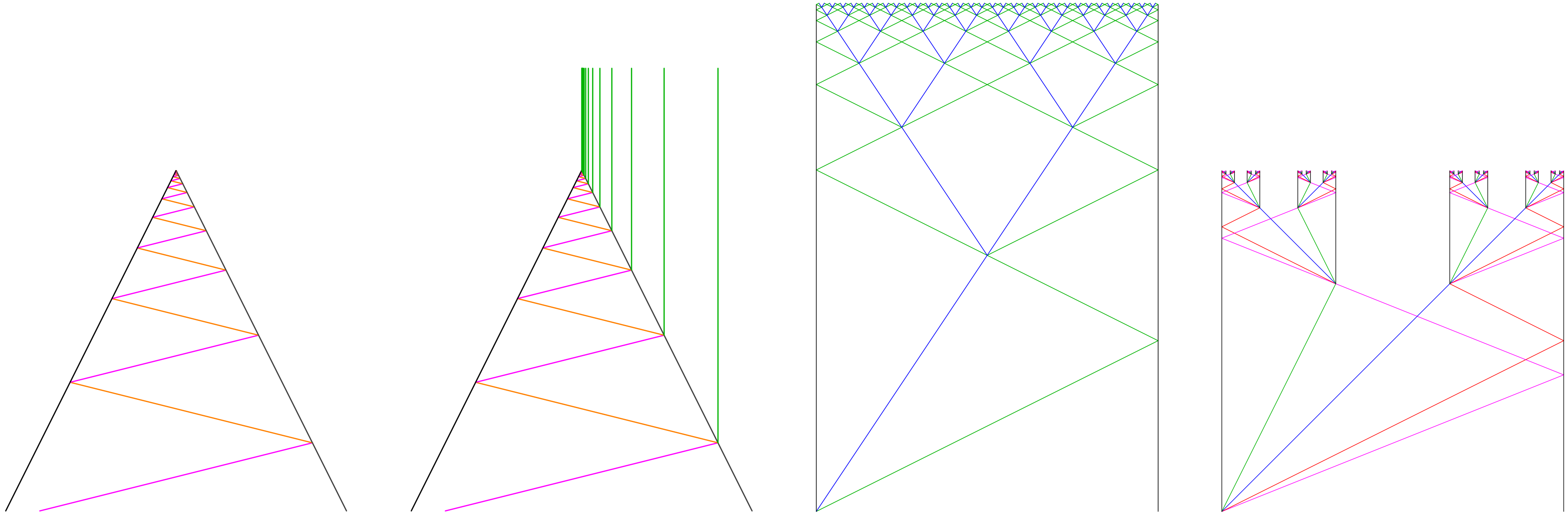
# “Nice regular drawings”



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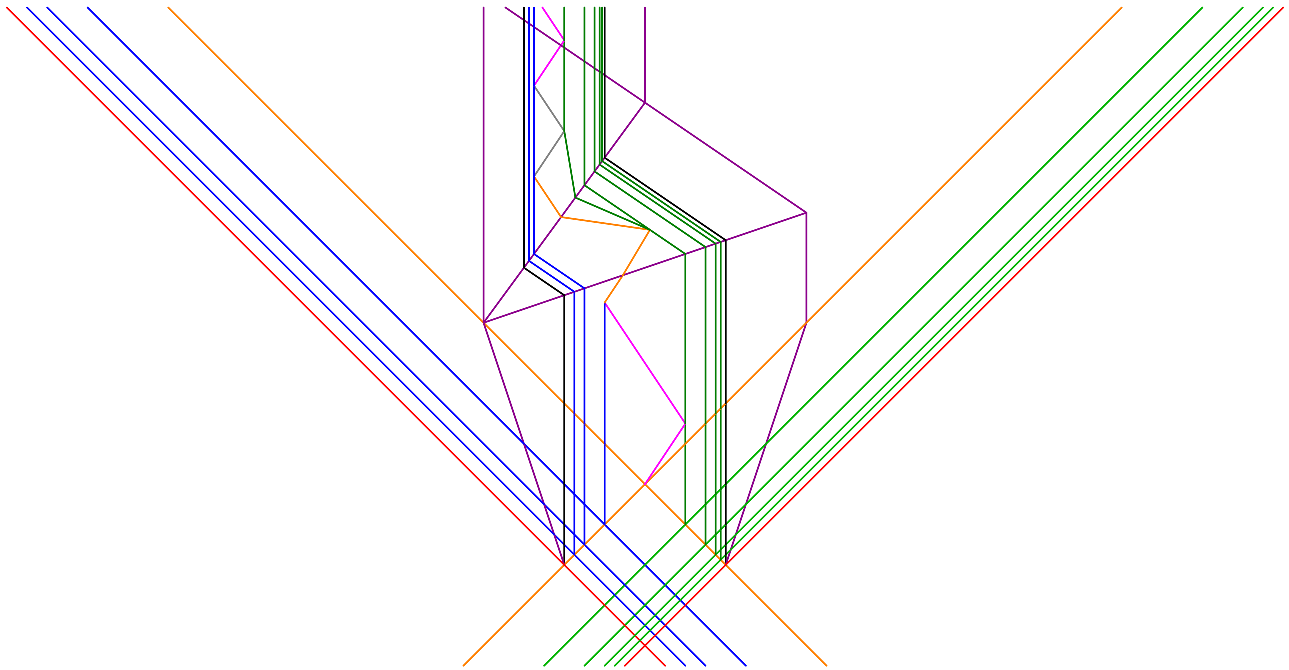

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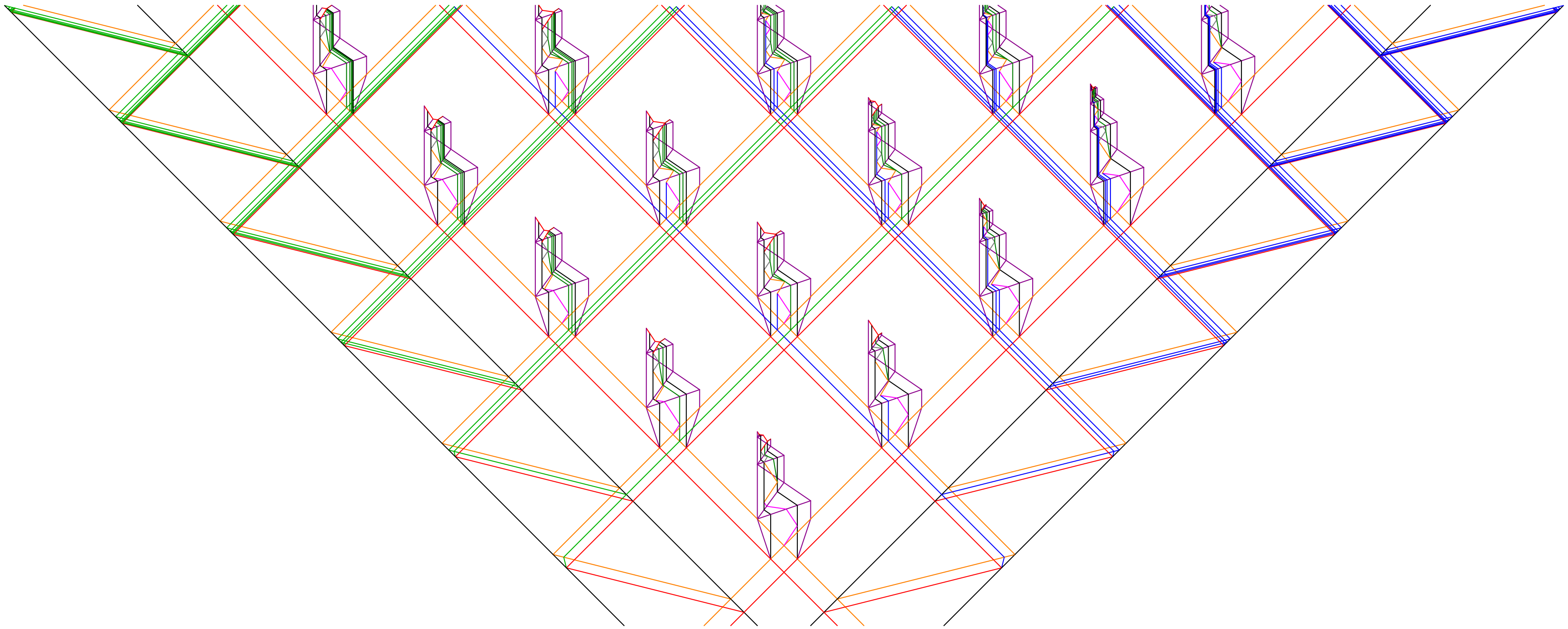

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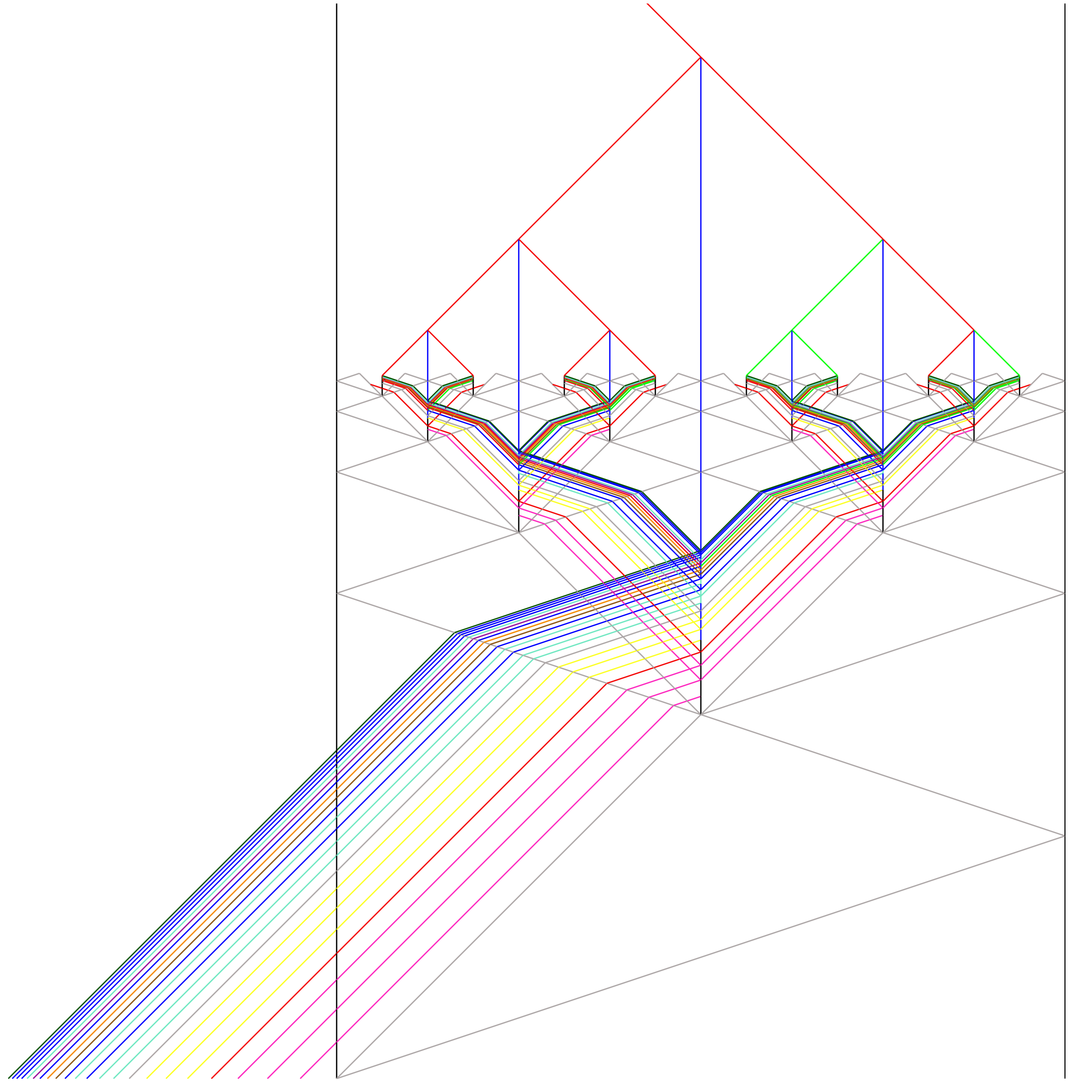
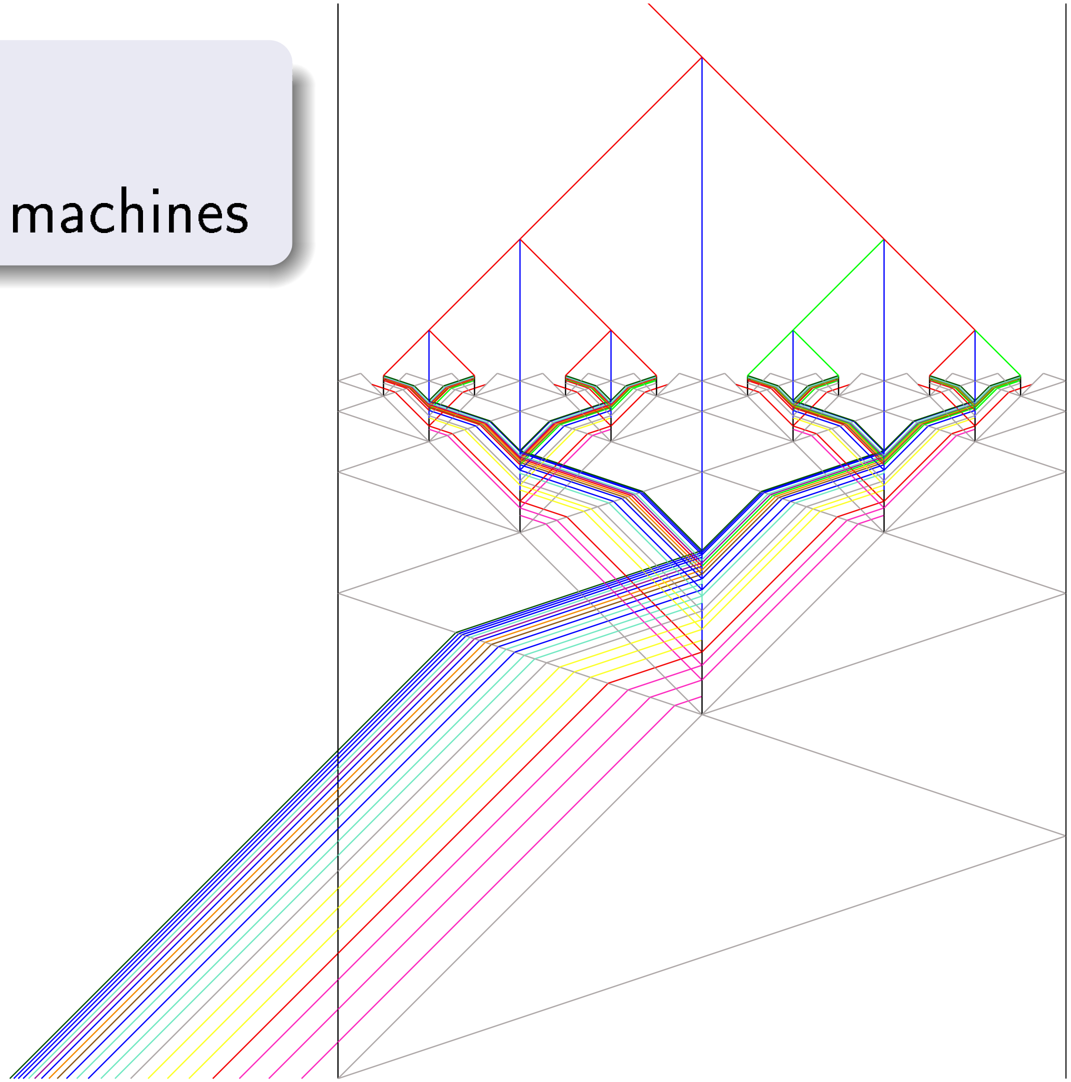


# "Nice regular drawings"

Lines: traces of signals

Space-time diagrams of signal machines



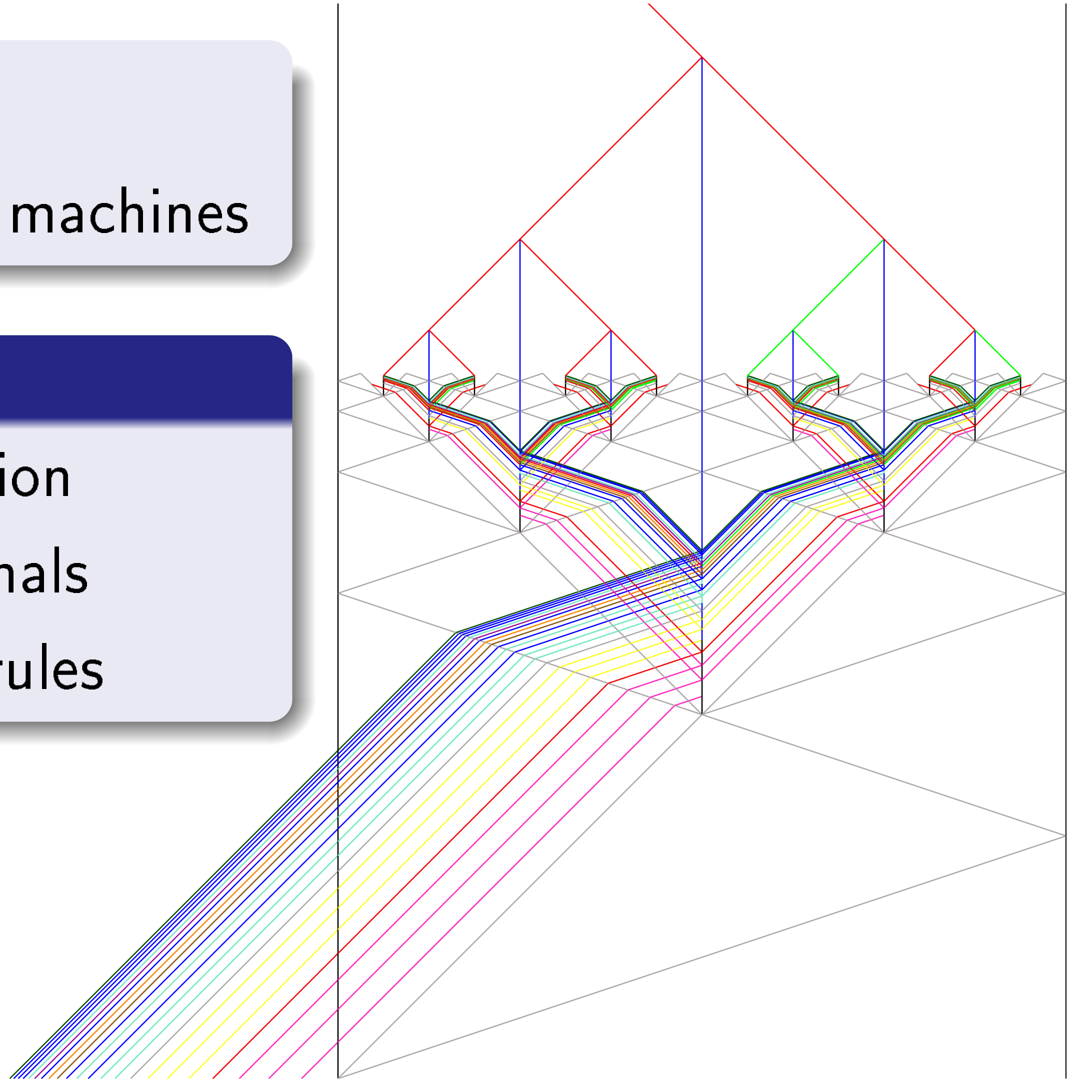
# "Nice regular drawings"

Lines: traces of signals

Space-time diagrams of signal machines

## Defined by

- bottom: initial configuration
- lines: signals ⇝ meta-signals
- end-points: collisions ⇝ rules



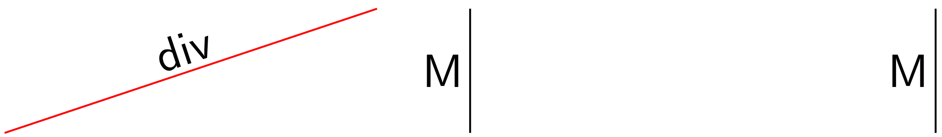
# Example: find the middle


## Meta-signals (speed)

M (0)

## Collision rules

# Example: find the middle

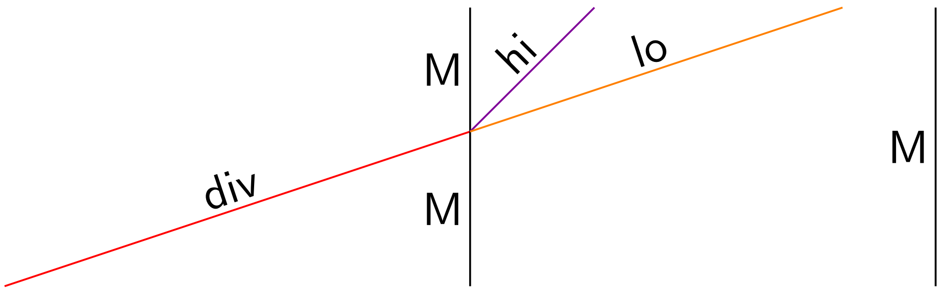



### Meta-signals (speed)

M (0)
div (3)

### Collision rules

# Example: find the middle



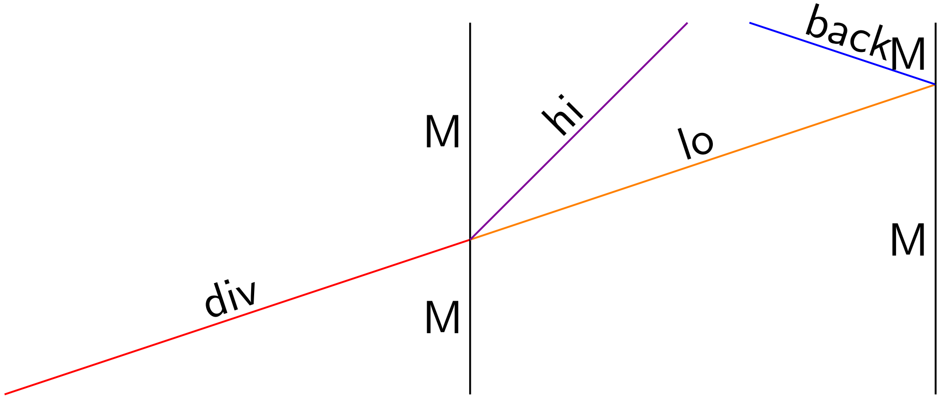

### Meta-signals (speed)

M (0)
div (3)
hi (1)
lo (3)

### Collision rules

{ div, M } $\rightarrow$ { M, hi, lo }

# Example: find the middle




## Meta-signals (speed)

- M (0)
- div (3)
- hi (1)
- lo (3)
- back (-3)

## Collision rules

{ div, M } → { M, hi, lo }
{ lo, M } → { back, M }

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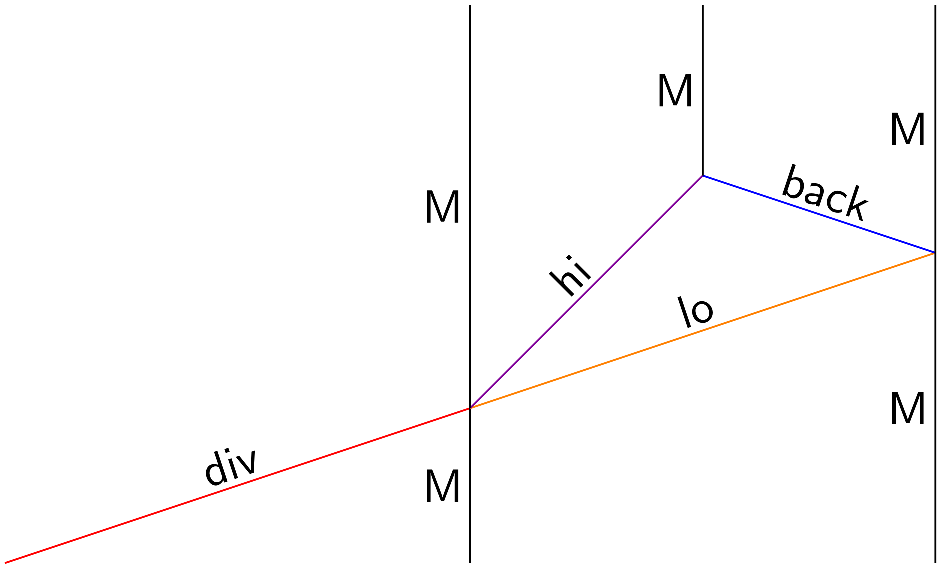


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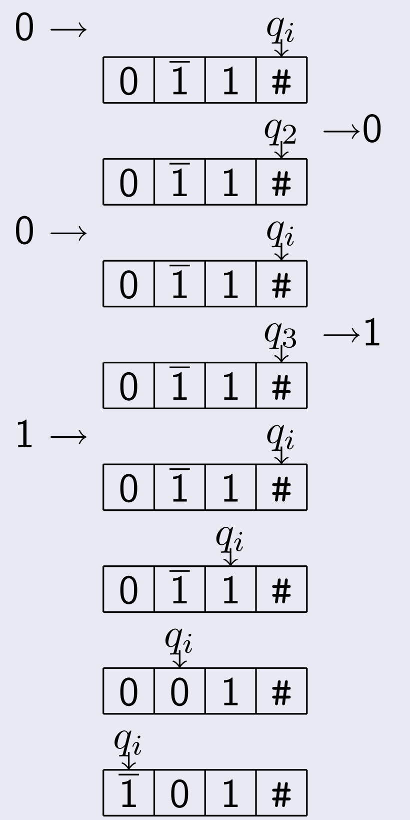
{ lo, M } $\rightarrow$ { back, M }
{ hi, back } $\rightarrow$ { M }

# Known results

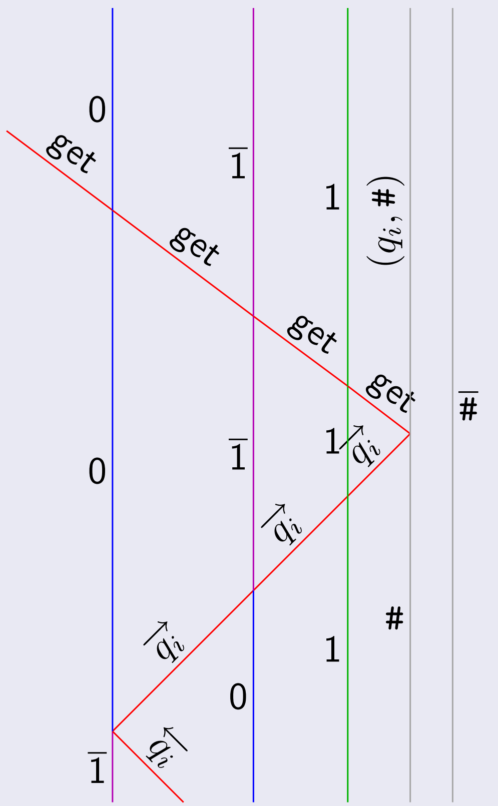
## Turing computations

- [Durand-Lose, 2011]

## TM run




## Simulation



# Known results

## Turing computations

- [Durand-Lose, 2011]

## Analog computations

- Computable analysis [Weihrauch, 2000] [Durand-Lose, 2010a]
- Blum, Shub and Smale model [Blum et al., 1989] [Durand-Lose, 2008]

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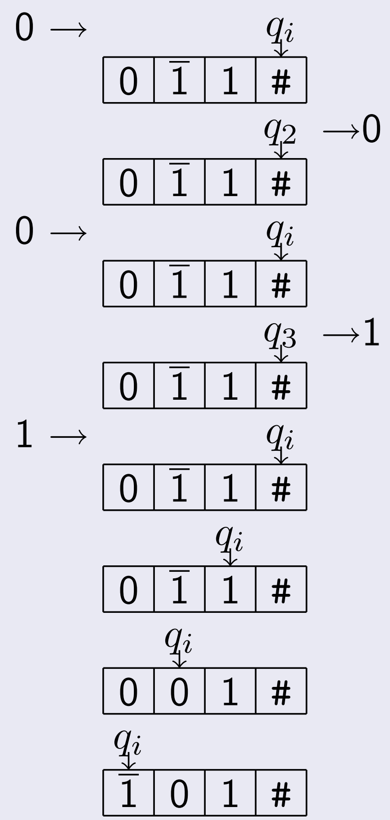


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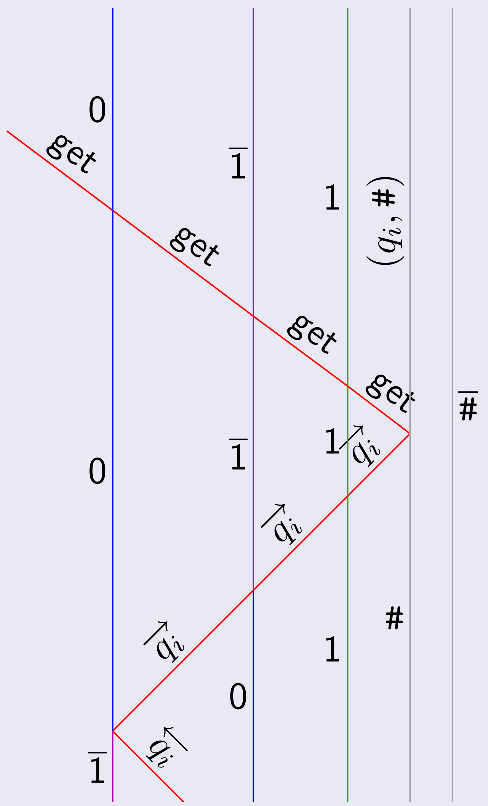

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## Analog computations

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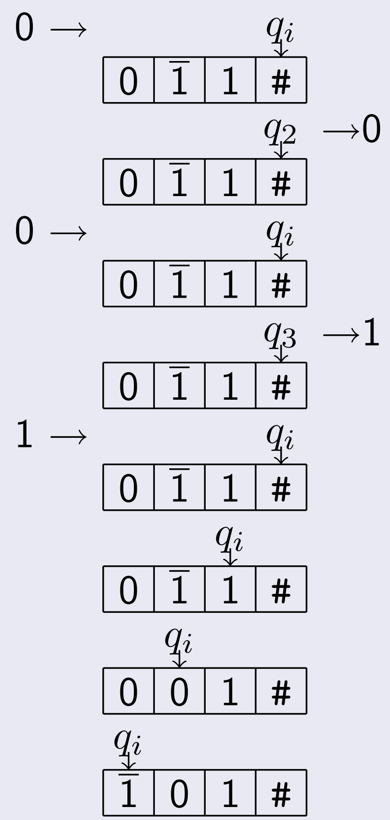
## "Black hole" implementation

- [Durand-Lose, 2009]

## TM run

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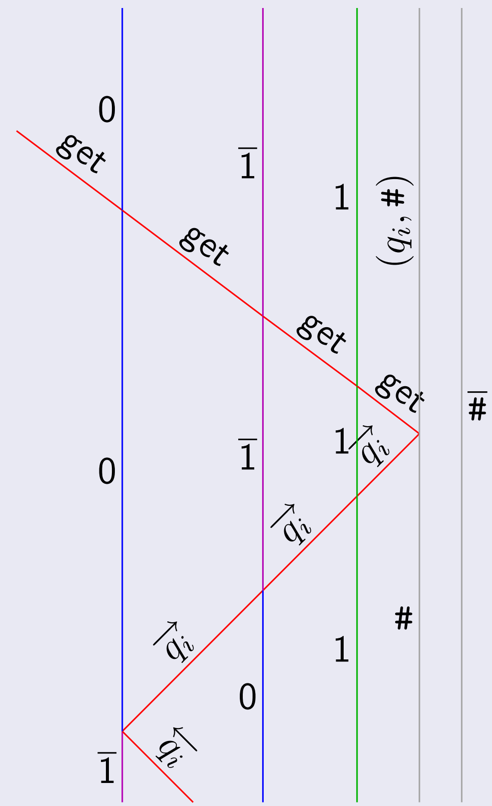
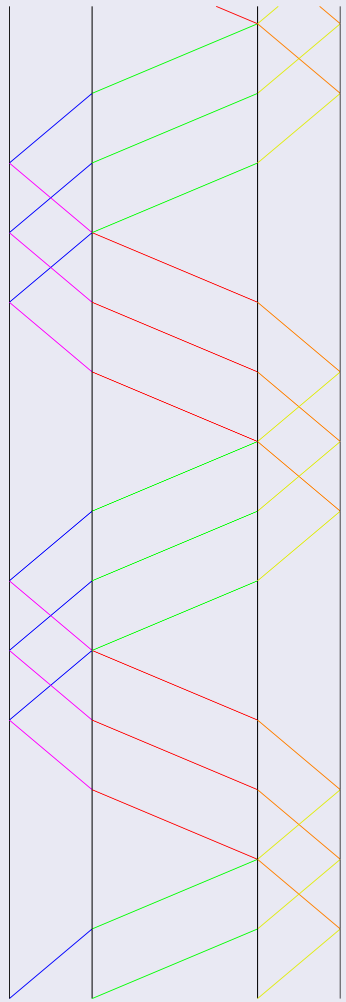
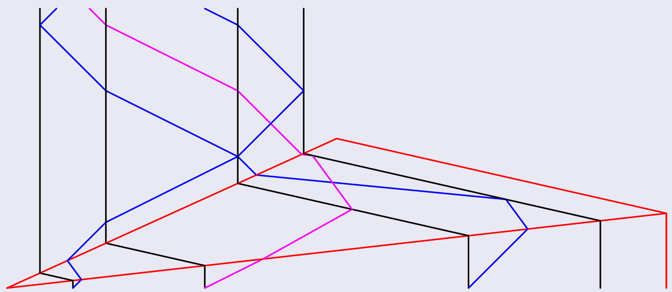


# Geometric primitives: accelerating and bounding time

## Normal



## Shrunk



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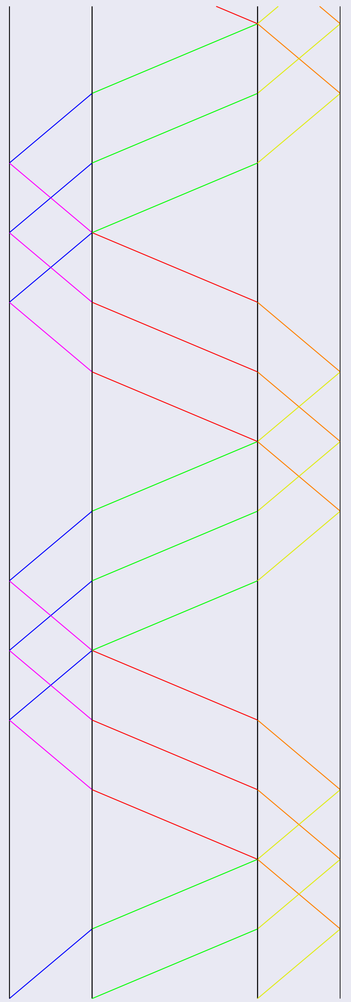

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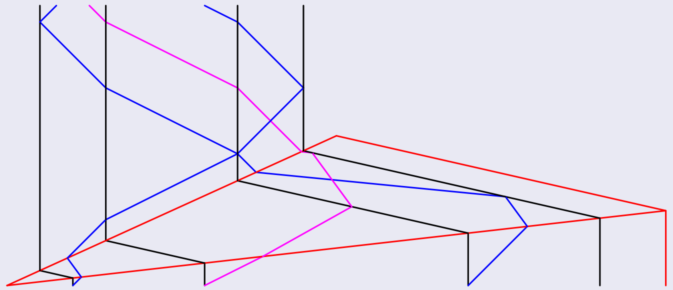
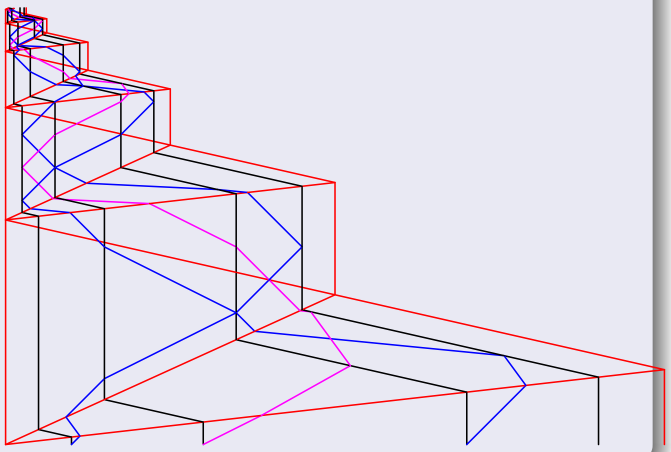


## Iterated



# Rational signal machines and isolated accumulations

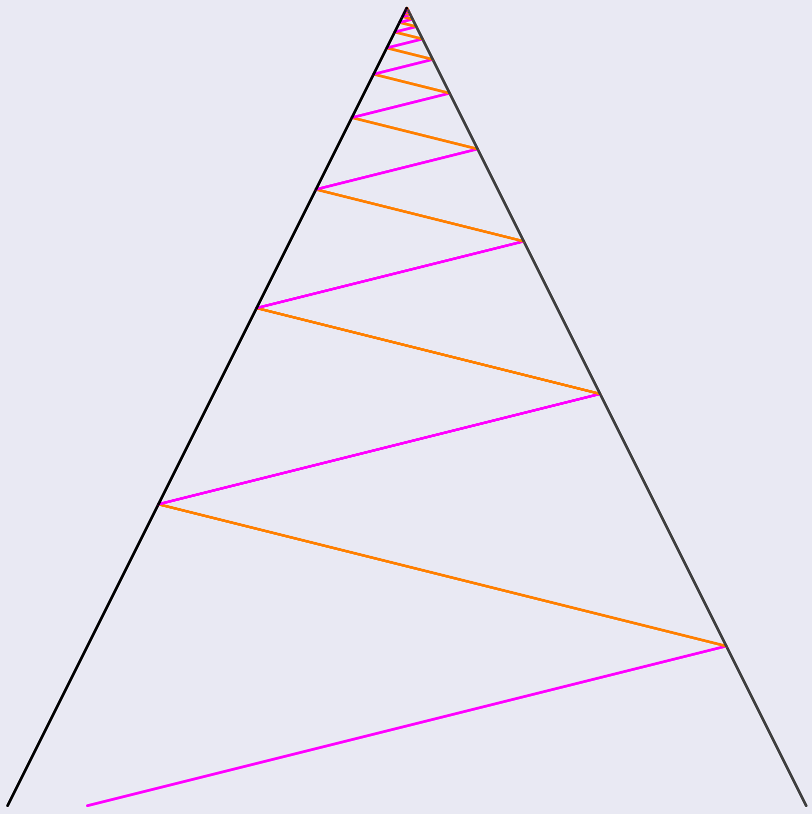
## $\mathbb{Q}$ signal machine

- all speed are in $\mathbb{Q}$
- all initial positions are in $\mathbb{Q}$
- $\Rightarrow$ all location remains in $\mathbb{Q}$

## Space and time location

- Easy to compute

## Simplest accumulation



# Rational signal machines and isolated accumulations

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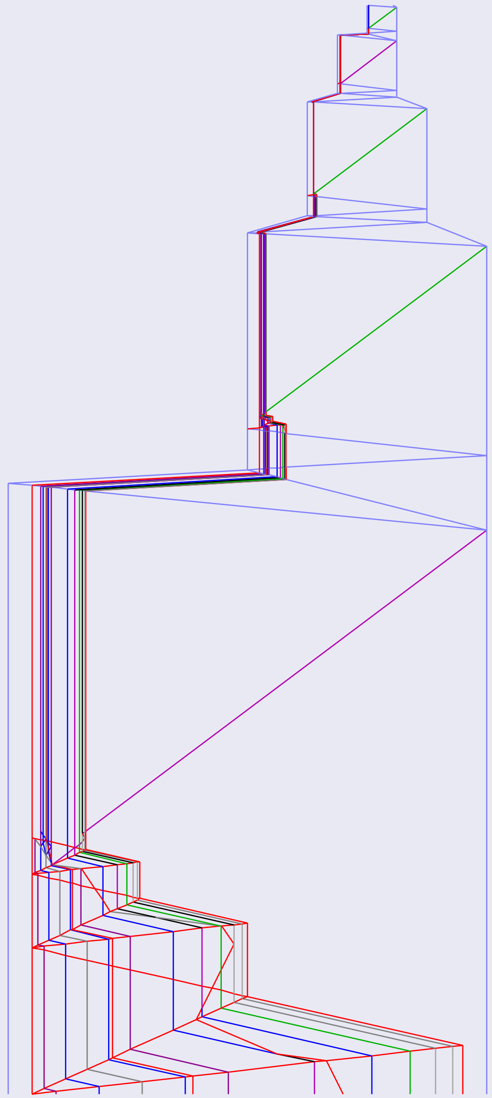
## Space and time location

- Easy to compute
- Not so easy to guess

## Accumulation?



# Rational signal machines and isolated accumulations

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## Space and time location

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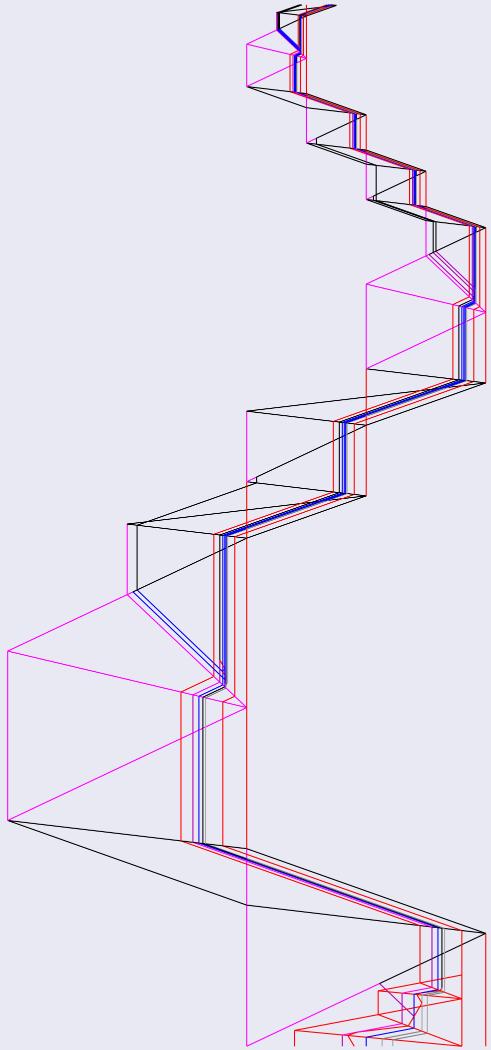
## Forecasting any accumulation

Highly undecidable
($\Sigma_2^0$ in the arithmetic hierarchy)
[Durand-Lose, 2006]

## Accumulation?



# Temporal coordinate

## $\mathbb{Q}$-signal machine

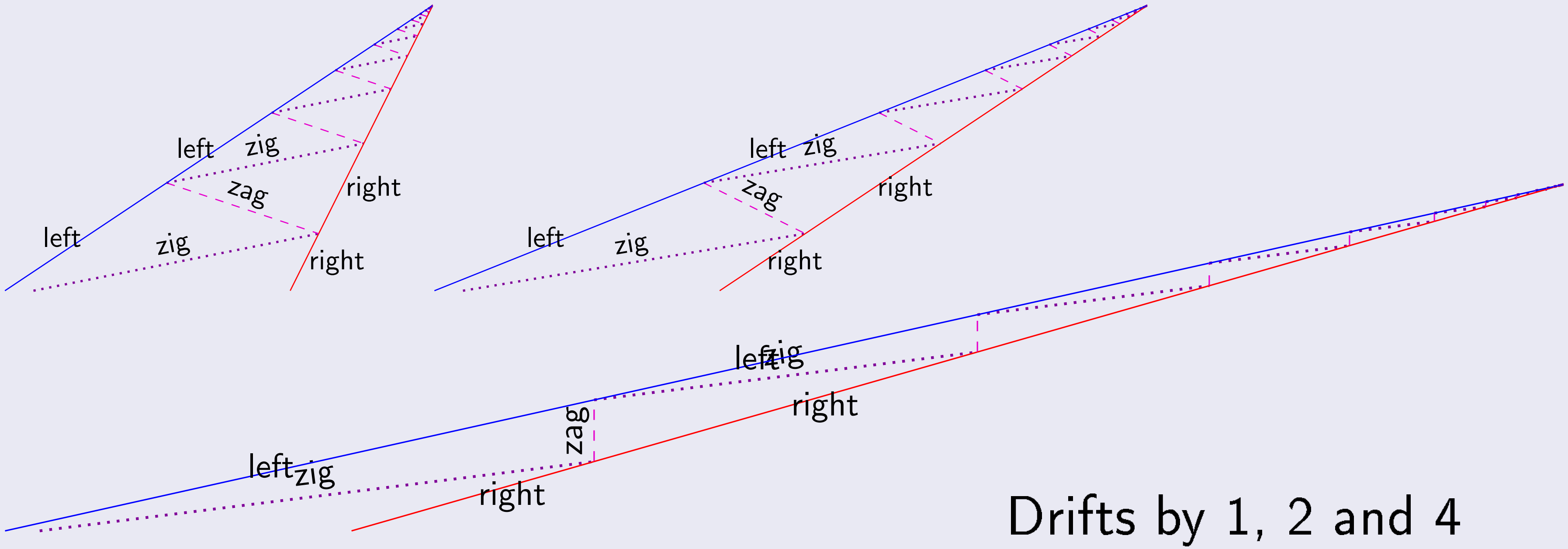
- $\mathbb{Q}$ on computers/Turing machine
  - exact representation
  - exact operations
- exact computations by TM (and implanted in Java)

## Simulation near an isolated accumulation

- on each collision, print the date
- $\rightsquigarrow$ increasing computable sequence of rational numbers (converges iff there is an accumulation)

# Spacial coordinate

## Static deformation by adding a constant to each speed




## With all speeds positive

- the left most coordinate is increasing (and computable) converges iff there is an accumulation
- correction by subtracting the date times the drift

## *c.e.* real number

- limit of a convergent increasing computable sequence of rational numbers
- no bound on the convergence rate
- represents a *c.e.* set (of natural numbers)
- stable by positive integer multiplication but not by subtraction

## *d-c.e.* real number

- difference of two *c.e.* real number
- form a field
- [Ambos-Spies et al., 2000]
  these are exactly the limits of a computable sequence of rational numbers that converges *weakly effectively*, *i.e.*,
  $$\sum_{n \in N} \left| x_{n+1} - x_n \right| \text{ converges}$$

# Encoding

## For *d-c.e.* real numbers

$$x = \sum_{i\in\mathbb{N}} \frac{z_i}{2^i} \ , z_i \in \mathbb{Z}$$

the sequence $i \to z_i$ is computable and

$$\sum_{i\in\mathbb{N}} \left|\frac{z_i}{2^i}\right| \text{ converges}$$

## For *c.e.* real numbers

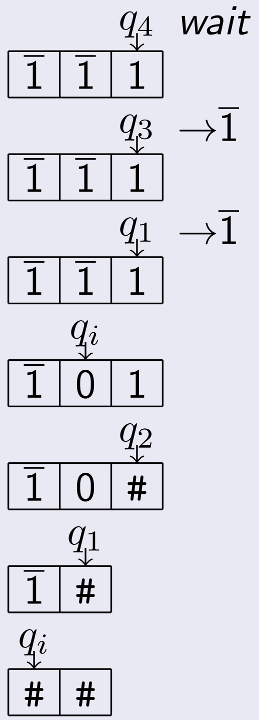
- identical but $z_i \in \mathbb{N}$

- $z_i$ in signed unary representation

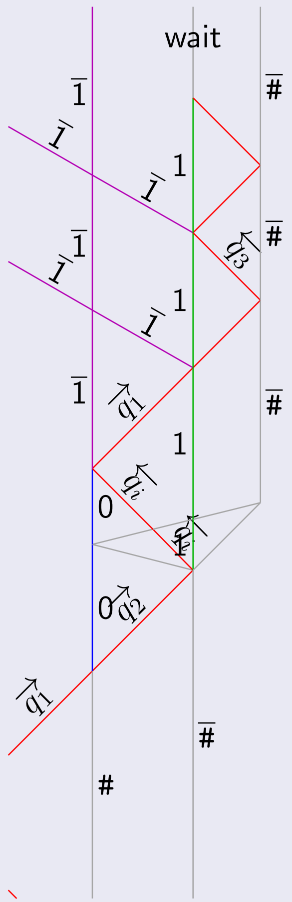
# TM outputting the infinite sequence

## Run

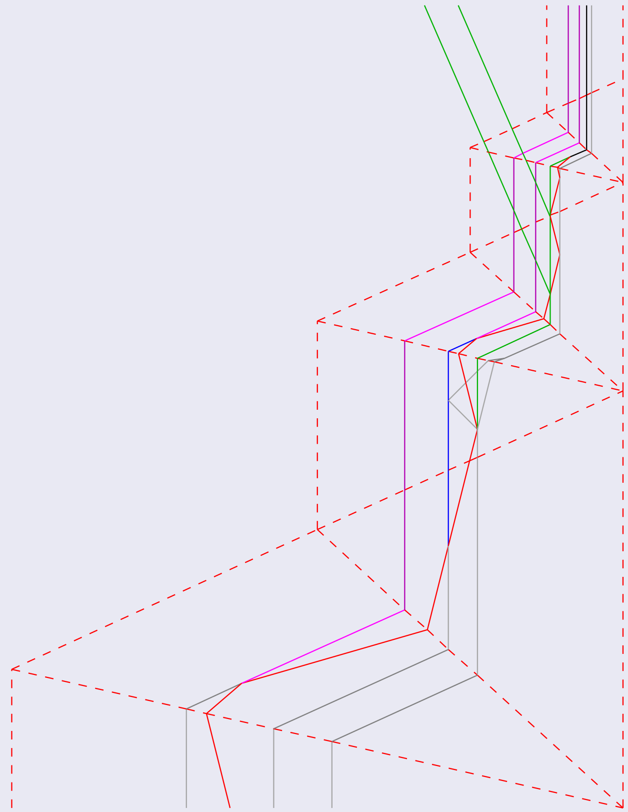
wait between each $z_i$




## Simulation




## Shrunk to output in bounded time

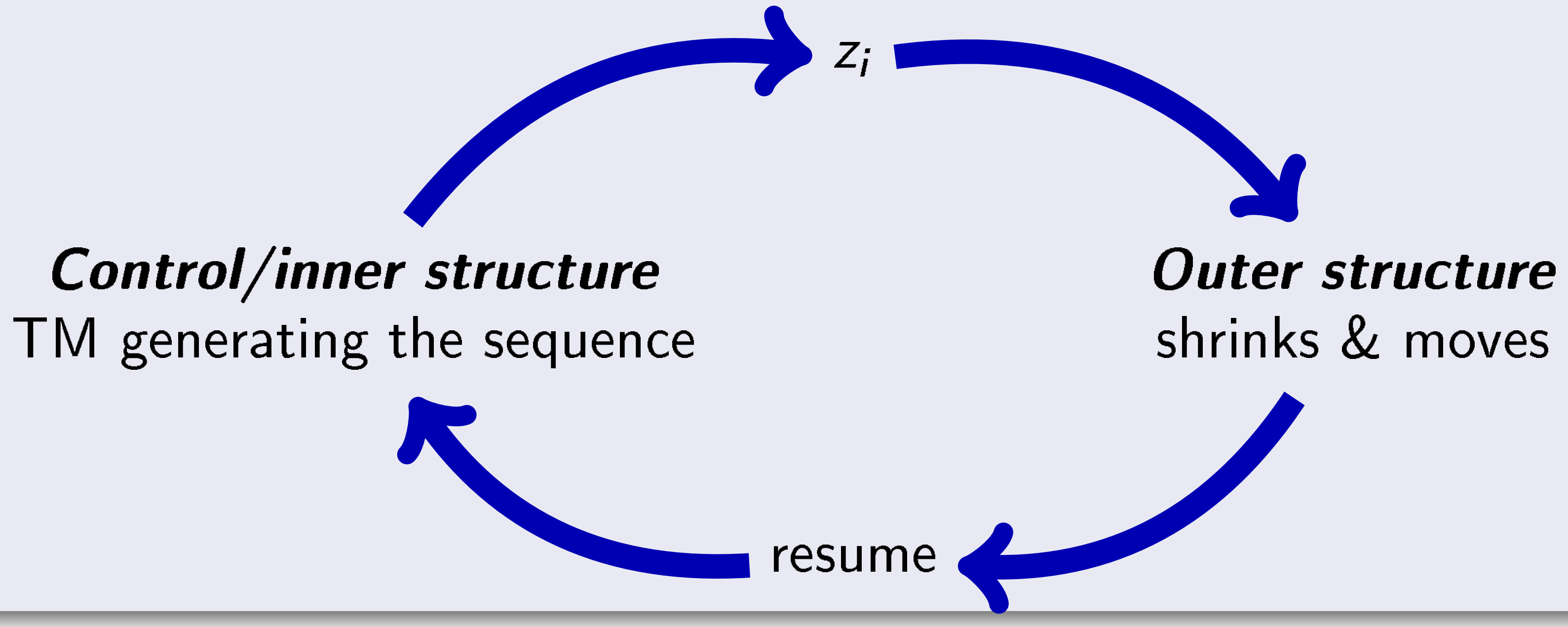


Simulation and shrinking structure stop after each value

# Two-level scheme

## Control/inner structure

- Provide the data for accumulating

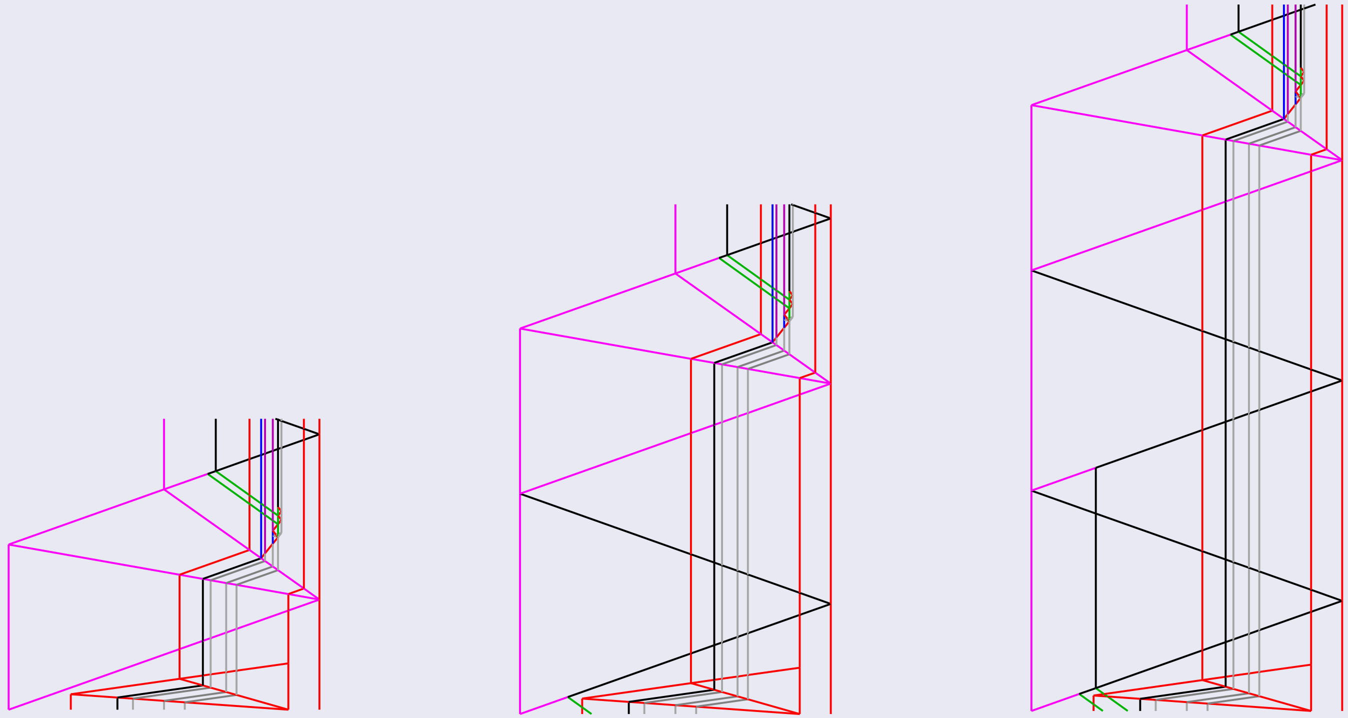



## Outer structure

- shrink and move the whole structure $\rightsquigarrow$ accumulation

# Temporal coordinate

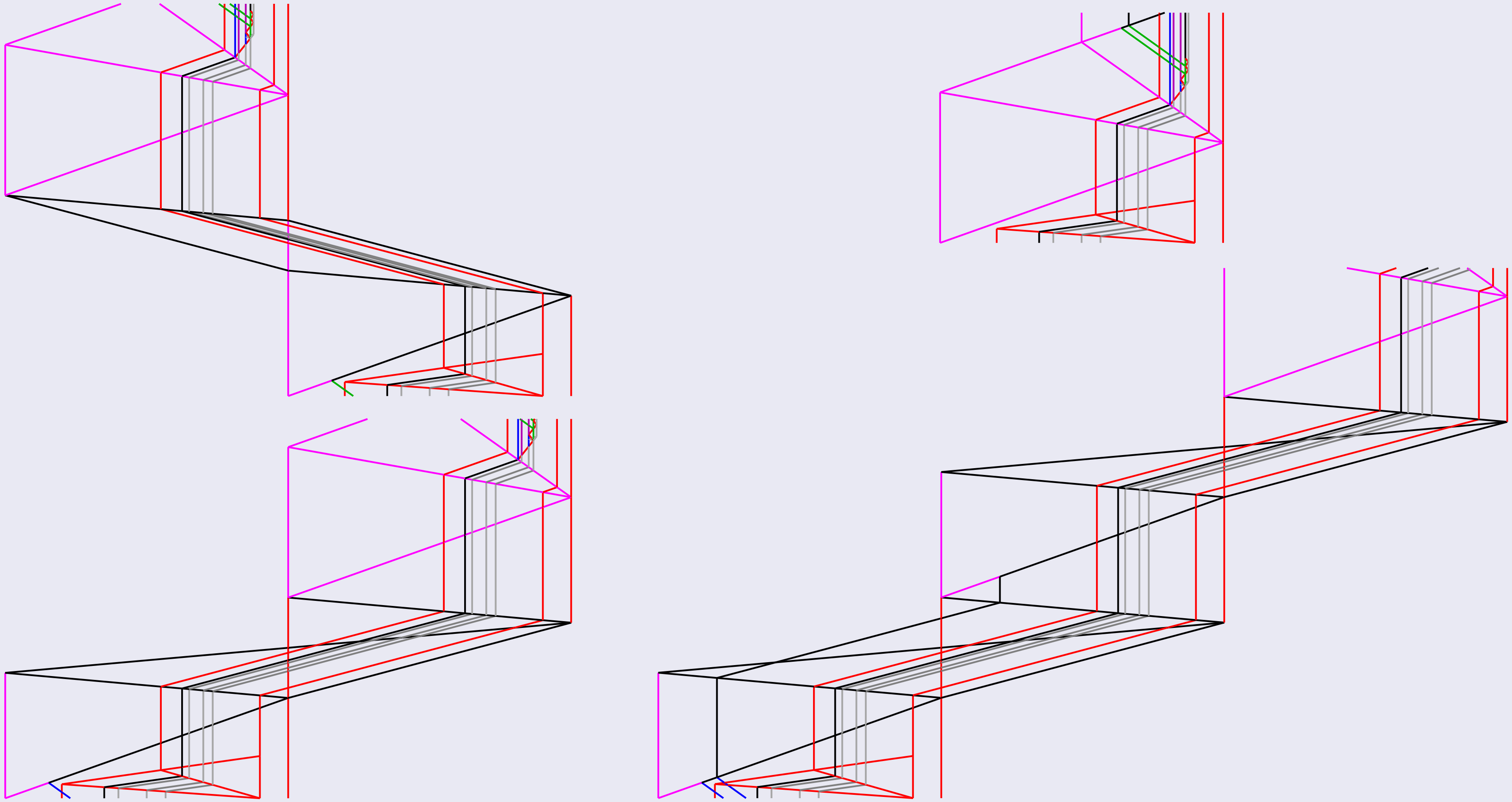
## Wait the corresponding time



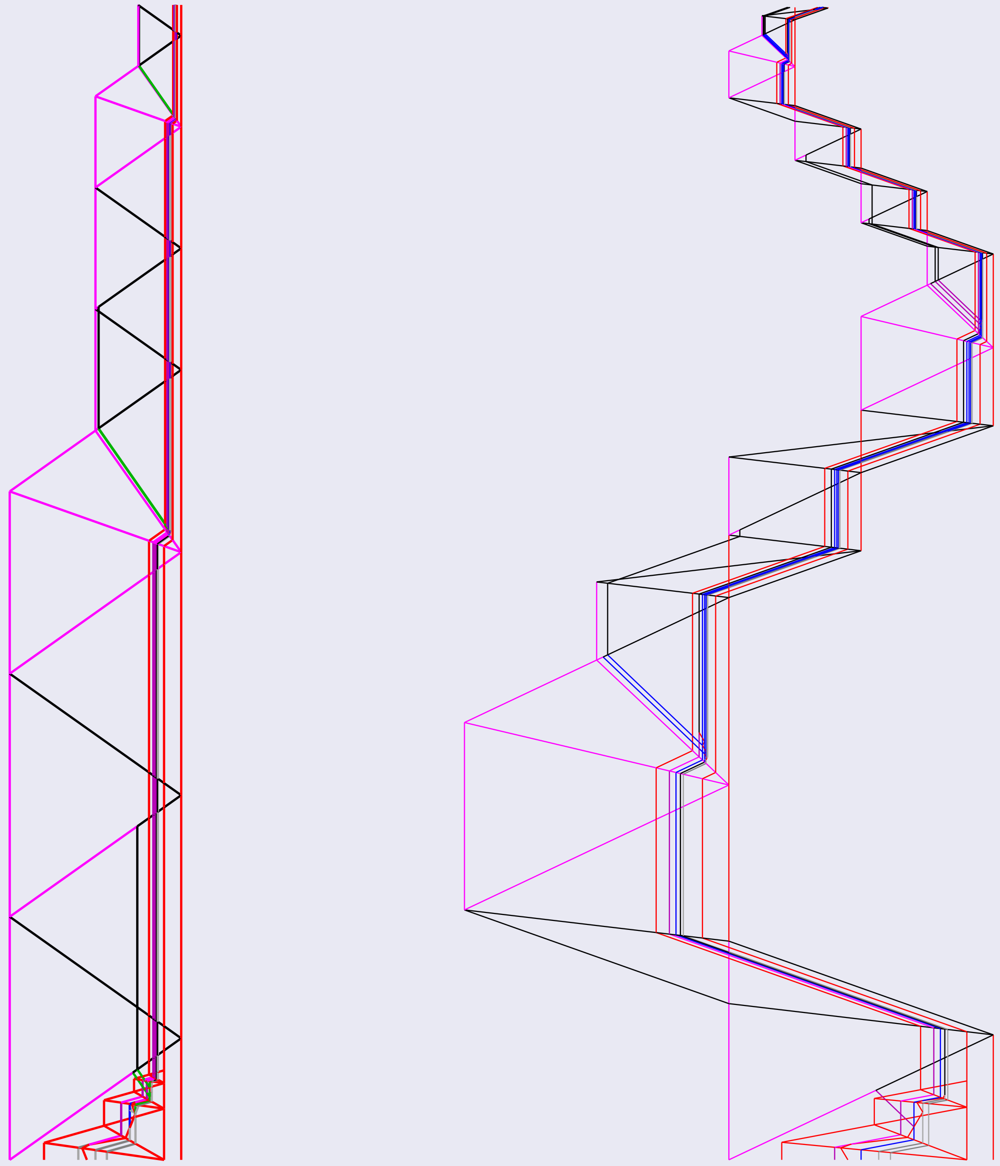
- Constant (up to scale) delay before outer structure action
- total delay is rational and should be previously subtracted

# Spatial coordinate

## Move left or right, more or less



# Examples



## Results

- Isolated accumulations happen at *d-c.e.* spacial and *c.e.* temporal coordinates
- Accumulation at any *c.e.* temporal coordinate is possible
- Accumulation at any *d-c.e.* spacial coordinate is possible

## Perspectives

- Uncorrelate space and time coordinate
  it is possible for *computable* coordinates [Durand-Lose, 2010b]
- Higher order isolated accumulations
- Non isolated accumulations

Ambos-Spies, K., Weihrauch, K., and Zheng, X. (2000).
Weakly computable real numbers.
*J. Complexity*, 16(4):676–690.

Blum, L., Shub, M., and Smale, S. (1989).
On a theory of computation and complexity over the real numbers: NP-completeness, recursive functions and universal machines.
*Bull. Amer. Math. Soc.*, 21(1):1–46.

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*Nat. Comput.*, 8(3):455–472.

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The coordinates of isolated accumulations [includes] computable real numbers.

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