Simulation between signal machines

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Algorithmic Questions in Dynamical Systems
March 2018 — Toulouse

Outline

One model/dynamical system

- signal machines

Relations to “usual” computational models

Intrinsic universality

Non determinism

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Relations to "usual" computational models

- Turing machines
- Linear Blum, Shub and Smale model

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- a family only

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One model/dynamical system

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Intrinsic universality

- a family only

Non determinism

- same power

Cellular Automata: signal use

Firing Squad Synchronization (Goto, 1966)

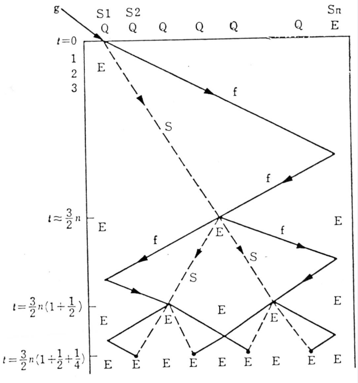
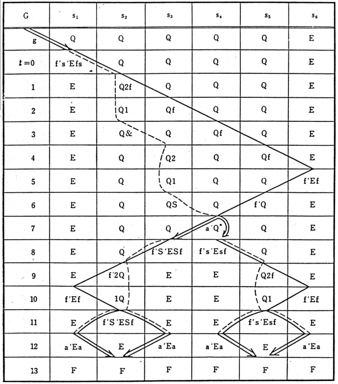


図 3·5　一斉射撃の問題（連続近似）



G	s_1	s_2	s_3	s_4	s_5	s_6
g	Q	Q	Q	Q	Q	E
t=0	f's'Efs	Q	Q	Q	Q	E
1	E	Q2f	Q	Q	Q	E
2	E	Q1	Qf	Q	Q	E
3	E	Q&	Q	Qf	Q	E
4	E	Q	Q2	Q	Qf	E
5	E	Q	Q1	Q	Q	f'Ef
6	E	Q	QS	Q	f'Q	E
7	E	Q	Q	a'Q*	Q	E
8	E	Q	f'S'ESf	f's'Esf	Q	E
9	E	f'2Q	E	E	Q2f	E
10	f'Ef	1Q	E	E	Q1	f'Ef
11	E	f'S'ESf	E	E	f's'Esf	E
12	a'Ea	E	a'Ea	a'Ea	E	a'Ea
13	F	F	F	F	F	F

図 3·6　一斉射撃解（n=6）

CA: Conception with signals

Fischer (1965)

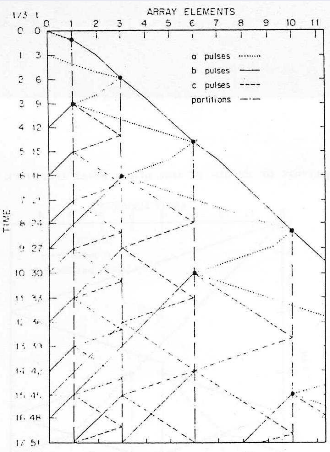
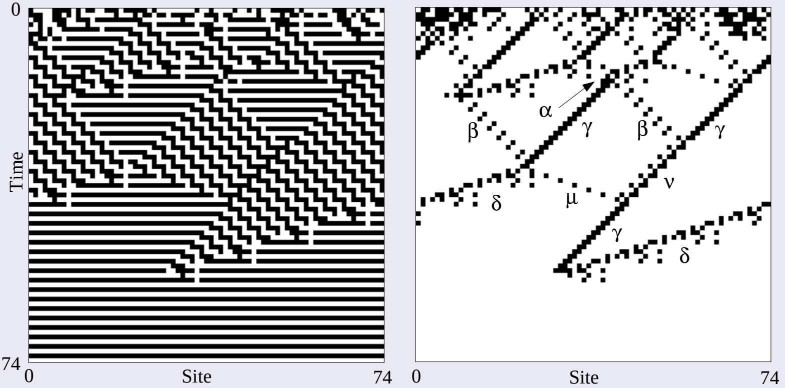


Fig. 2. Solution to the prime problem

CA: Analyzing with Signals

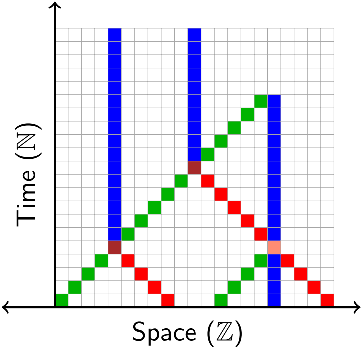
Das et al. (1995)

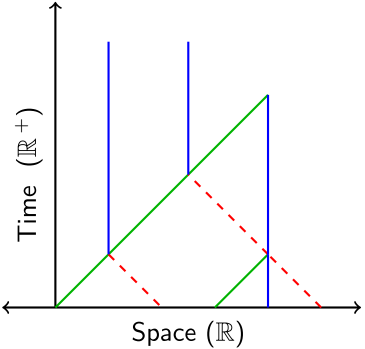


(a) Space-time diagram.

(b) Filtered space-time diagram.

Signals





- Signal (meta-signal)
- Collision (rule)

Vocabulary and Example: Find the Middle

Meta-signals (speed)

M (0)

Collision rules

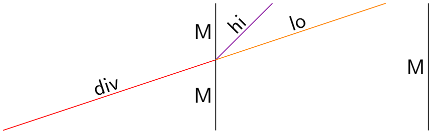
Vocabulary and Example: Find the Middle

Meta-signals (speed)

M	(0)
div	(3)

Collision rules

Vocabulary and Example: Find the Middle



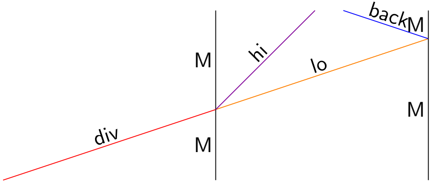
Meta-signals (speed)

M	(0)
div	(3)
hi	(1)
lo	(3)

Collision rules

{ div, M } → { M, hi, lo }

Vocabulary and Example: Find the Middle



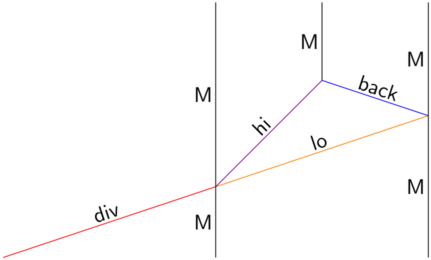
Meta-signals (speed)

M	(0)
div	(3)
hi	(1)
lo	(3)
back	(-3)

Collision rules

{ div, M } → { M, hi, lo }
{ lo, M } → { back, M }

Vocabulary and Example: Find the Middle



Meta-signals (speed)

M	(0)
div	(3)
hi	(1)
lo	(3)
back	(-3)

Collision rules

{ div, M } → { M, hi, lo }
{ lo, M } → { back, M }
{ hi, back } → { M }

Stack Implantation

Name	Speed
Add, Rem	1/3
A, E	1
U, M	0
$\overrightarrow{R}$	3
$\overleftarrow{R}$	-3

Collision rules

$\{\text{Add}, \text{M}\} \rightarrow \{\text{M}, \text{A}, \overrightarrow{R}\}$

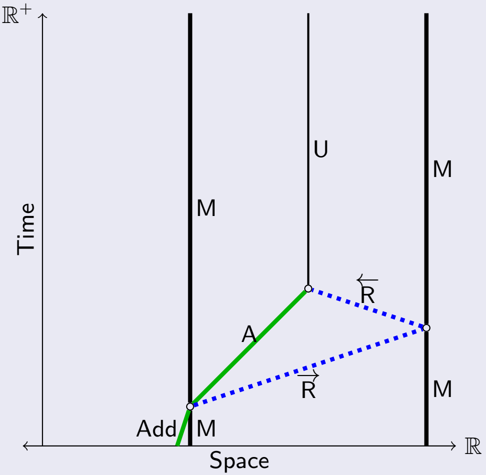
$\{\overrightarrow{R}, \text{M}\} \rightarrow \{\overleftarrow{R}, \text{M}\}$

$\{\text{A}, \overleftarrow{R}\} \rightarrow \{\text{U}\}$

$\{\overrightarrow{R}, \text{U}\} \rightarrow \{\overleftarrow{R}, \text{U}\}$

$\{\text{Rem}, \text{M}\} \rightarrow \{\text{M}, \text{E}\}$

$\{\text{E}, \text{U}\} \rightarrow \{\}$

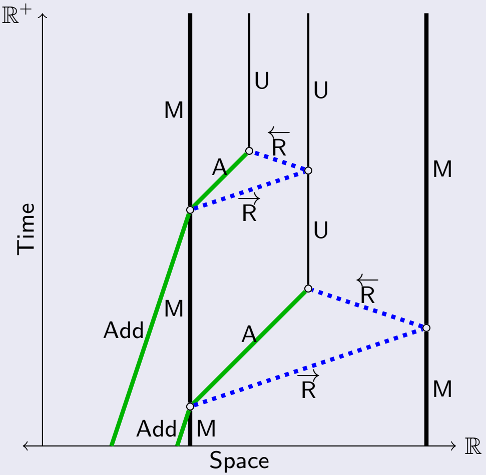


Stack Implantation

Name	Speed
Add, Rem	$1/3$
A, E	1
U, M	0
$\overrightarrow{R}$	3
$\overleftarrow{R}$	-3

Collision rules

$$\{\,\text{Add}, \text{M}\,\} \to \{\,\text{M}, \text{A}, \overrightarrow{\text{R}}\,\}$$
$$\{\,\overrightarrow{\text{R}}, \text{M}\,\} \to \{\,\overleftarrow{\text{R}}, \text{M}\,\}$$
$$\{\,\text{A}, \overleftarrow{\text{R}}\,\} \to \{\,\text{U}\,\}$$
$$\{\,\overrightarrow{\text{R}}, \text{U}\,\} \to \{\,\overleftarrow{\text{R}}, \text{U}\,\}$$
$$\{\,\text{Rem}, \text{M}\,\} \to \{\,\text{M}, \text{E}\,\}$$
$$\{\,\text{E}, \text{U}\,\} \to \{\,\}$$

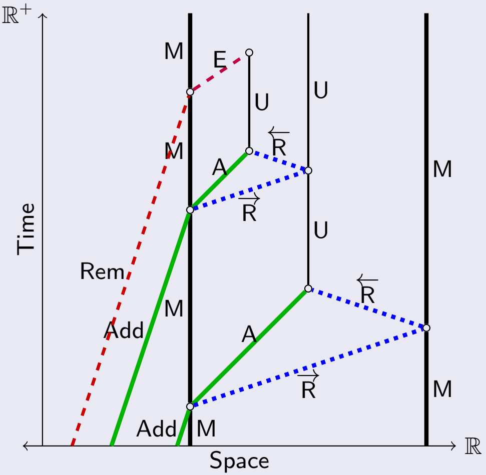


Stack Implantation

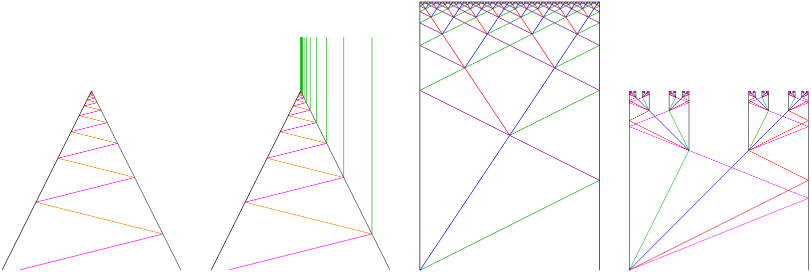
Name	Speed
Add, Rem	1/3
A, E	1
U, M	0
$\overrightarrow{R}$	3
$\overleftarrow{R}$	−3

Collision rules

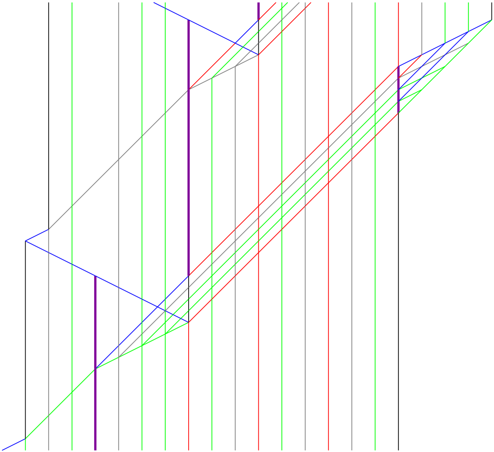
$\{ \text{Add}, \text{M} \} \rightarrow \{ \text{M}, \text{A}, \overrightarrow{\text{R}} \}$
$\{ \overrightarrow{\text{R}}, \text{M} \} \rightarrow \{ \overleftarrow{\text{R}}, \text{M} \}$
$\{ \text{A}, \overleftarrow{\text{R}} \} \rightarrow \{ \text{U} \}$
$\{ \overrightarrow{\text{R}}, \text{U} \} \rightarrow \{ \overleftarrow{\text{R}}, \text{U} \}$
$\{ \text{Rem}, \text{M} \} \rightarrow \{ \text{M}, \text{E} \}$
$\{ \text{E}, \text{U} \} \rightarrow \{ \}$



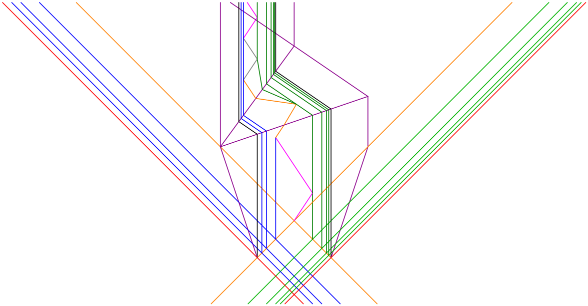
Fractal Generation



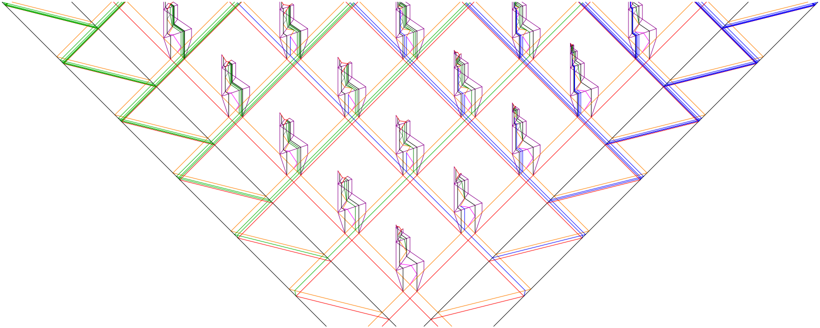
Complex Dynamics



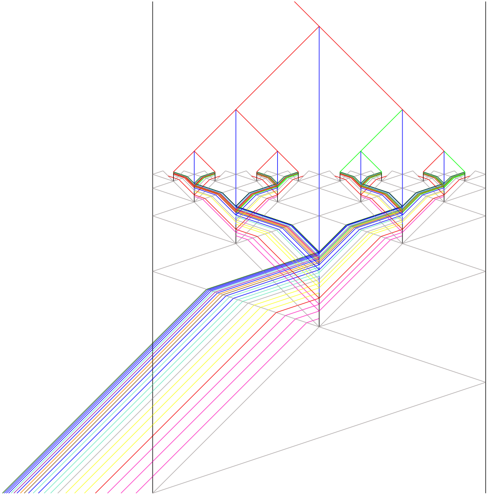
Complex Dynamics



Complex Dynamics

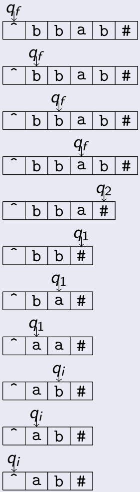


Complex Dynamics

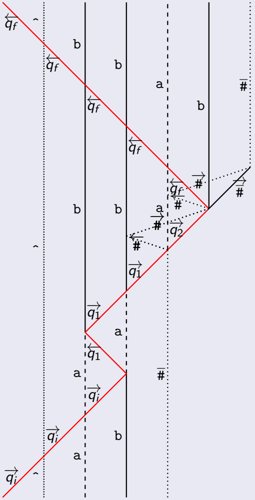


Turing-computation

Turing Machine



Simulation



Turing-computation

Also with restrictions

- all different speed
- only $2 \rightarrow 2$ rules (conservative)
- one-to-one rules (reversible)

Any above and

- rational ($\mathbb{Q}$)

Rational machines

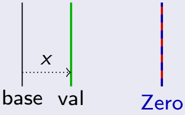
- speeds $\in \mathbb{Q}$
- initial positions $\in \mathbb{Q}$
- $\Rightarrow$ collision coordinates $\in \mathbb{Q}$
- exact simulation on computer/TM

Undecidability

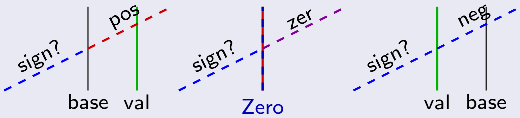
- finite number de collisions
- meta-signal appereance
- use of a rule
- disappearing of all signals
- involvement of a signal in any collision
- extension on the side, etc.

Computing with Real Numbers

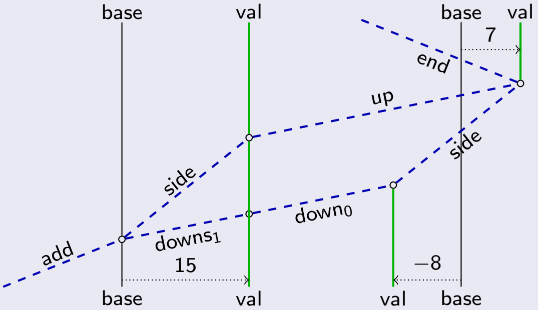
Encoding



Sign test

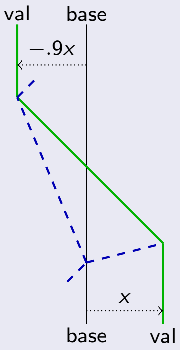


Addition

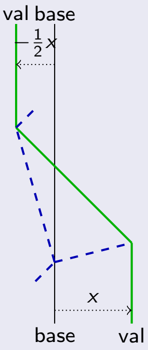


Multiplication by a constant

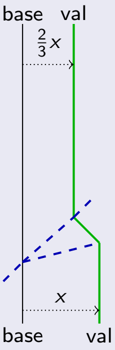
By $-.9$



By $-\frac{1}{2}$

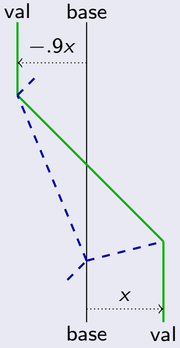


By $\frac{2}{3}$

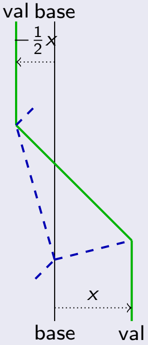


Multiplication by a constant

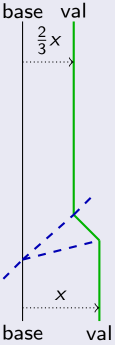
By $-.9$



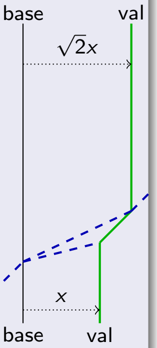
By $-\frac{1}{2}$



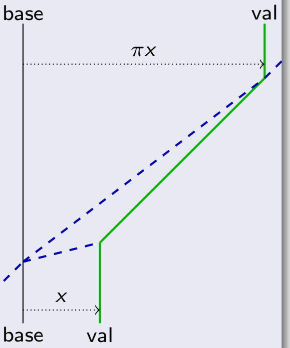
By $\frac{2}{3}$



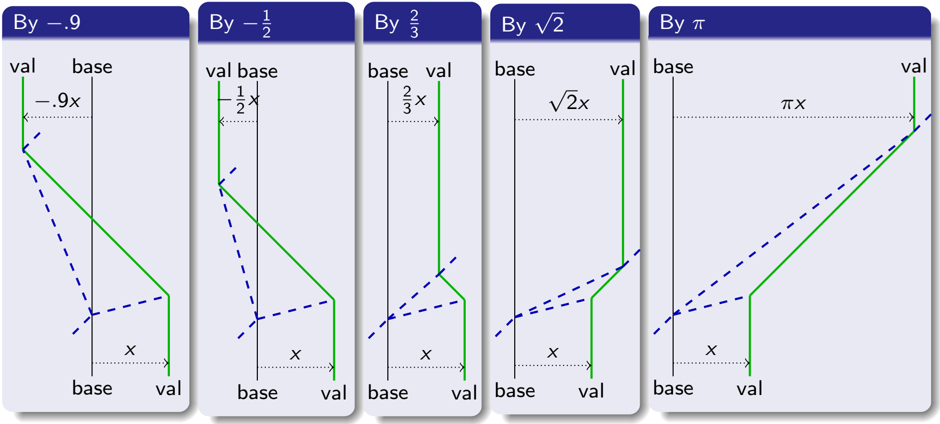
By $\sqrt{2}$



By π



Multiplication by a constant

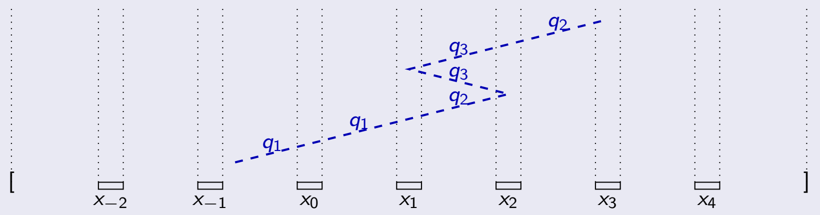


- Signal speeds are constants of the machine
- If $x \leq 0$ then val is meet before base

Zooming out

Finite sequence of real numbers

Zooming out



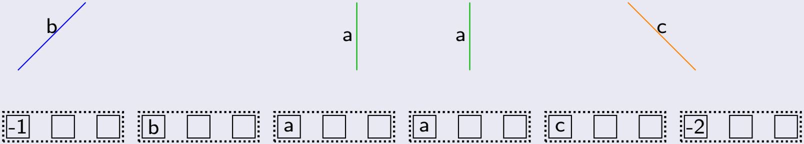
Finite sequence of real numbers + Dynamics

- finite state automata
- sign test
- addition, multiplication by constant
- (set constant value)
- (enlarge the array)

Like a Turing machine with real numbers on the tape

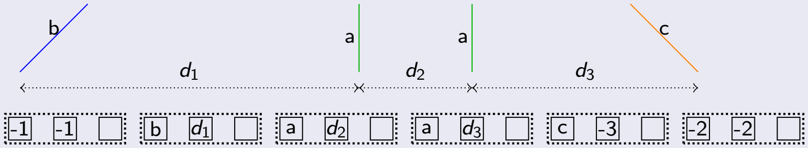
Linear Blum, Shub and Smale with shift

Encoding a configuration



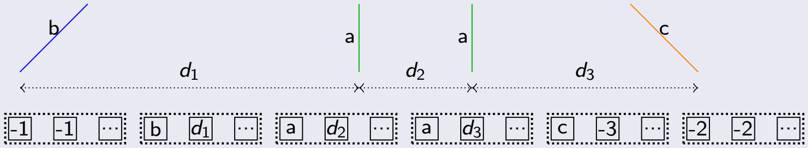
Linear Blum, Shub and Smale with shift

Encoding a configuration



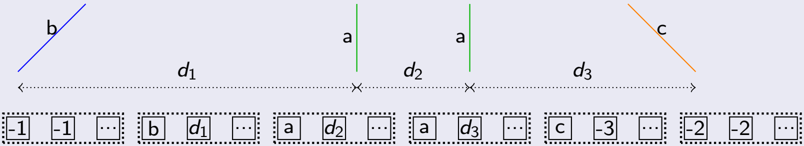
Linear Blum, Shub and Smale with shift

Encoding a configuration



Linear Blum, Shub and Smale with shift

Encoding a configuration



Simulating a signal machine: loop

1. Compute the minimum time to a collision, δ
2. Advance time by δ (update all distances)
3. Process collision(s)

Intrinsic Universality

Being able to *simulate* any other dynamical system of the its *class*.

Cellular Automata

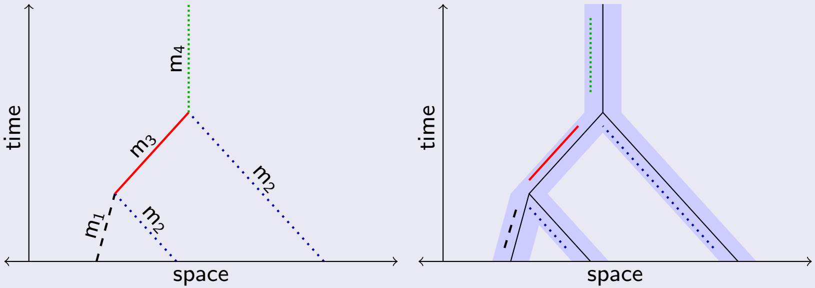
- regular (Albert and Čulik II, 1987; Mazoyer and Rapaport, 1998; Ollinger, 2001)
- reversible (Durand-Lose, 1997)
- freezing [Theyssier et Al.]

Tile Assembly Systems

- possible at T=2 and above (Woods, 2013)
- impossible at T=1 (Meunier et al., 2014)

Simulation for Signal Machines

Space-Time Diagram Mimicking



Signal Machine Simulation

$\mathcal{U}_\mathcal{S}$ simulates $\mathcal{A}$ if there is function from the configurations of $\mathcal{A}$ to the ones of $\mathcal{U}_\mathcal{S}$ s.t. the space-time issued from the image always mimics the original one.

Theorem

- For any finite set of real numbers $\mathcal{S}$, there is a signal machine $\mathcal{U}_{\mathcal{S}}$, that can simulate any machine whose speeds belong to $\mathcal{S}$.
- The set of $\mathcal{U}_{\mathcal{S}}$ where $\mathcal{S}$ ranges over finite sets of real numbers is an intrinsically universal family of signal machines.

Rest of this section

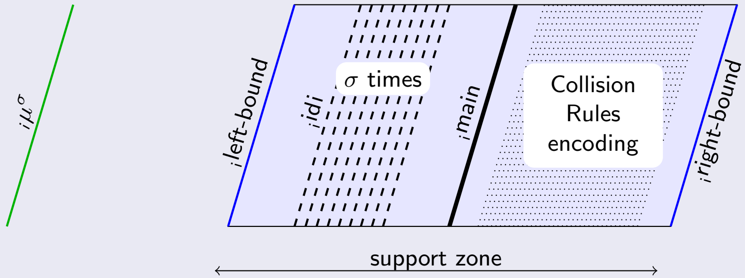
Let $\mathcal{S}$ be any finite set of real numbers,
let $\mathcal{A}$ be any signal machine whose speeds belongs to $\mathcal{S}$,
$\mathcal{U}_{\mathcal{S}}$ is progressively constructed as simulation is presented.

Macro-Signal

- Meta-signal of $\mathcal{A}$ identified with numbers
- Unary encoding of numbers

Structure

${}_i\mu^\sigma$: σth signal, ith speed



Global scheme

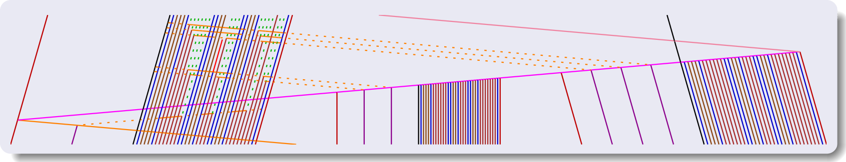
When Support Zones Meet

1. provide a delay
2. test if macro-collision is appropriate and what macro-signals are involved
3. if OK
    - start the macro-collision

Hypotheses for macro-collision

- no other macro-signal nor macro-collision will interfere
- speed of involved macro-signals ranged $[j, \ldots, i]$ (included)
- their main$^{\emptyset}$ signals intersect at a unique point

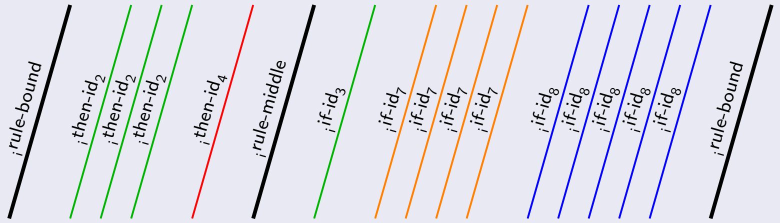
Removing Unused Tables and Sending ids to Table



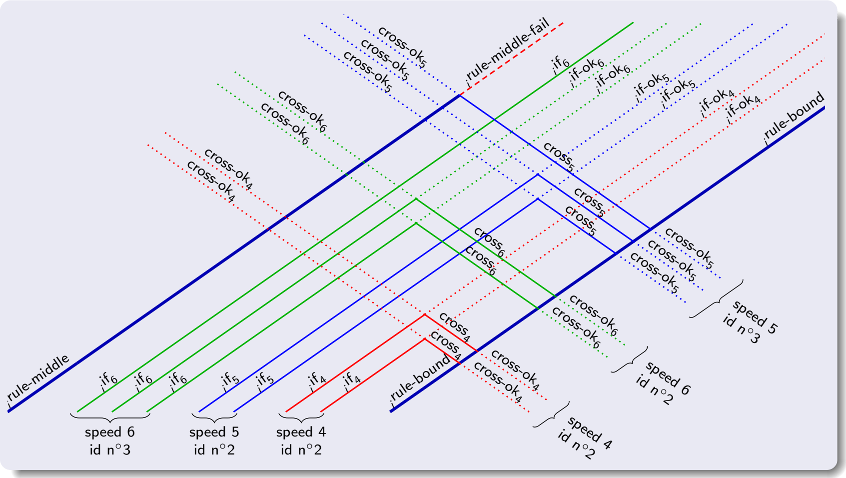
Collision Rules Encoding

One rule after the other

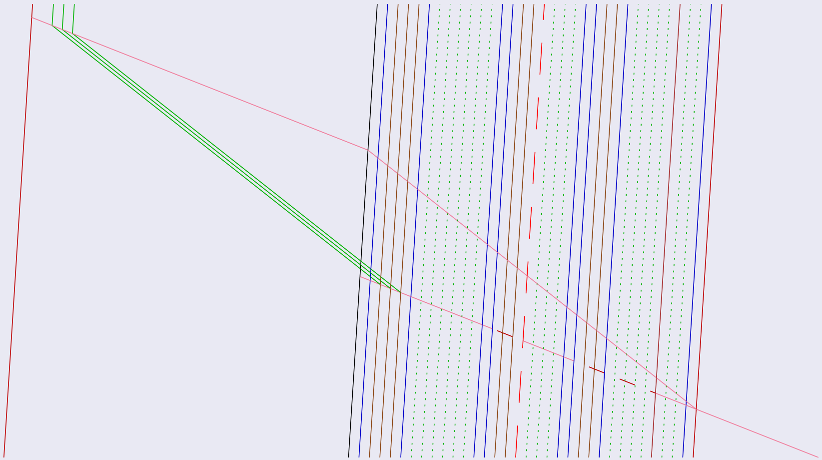
Encoding of $\{\, {}_3\mu^1,\ {}_7\mu^4,\ {}_8\mu^5 \,\} \to \{\, {}_2\mu^3,\ {}_4\mu^1 \,\}$ in the direction i.



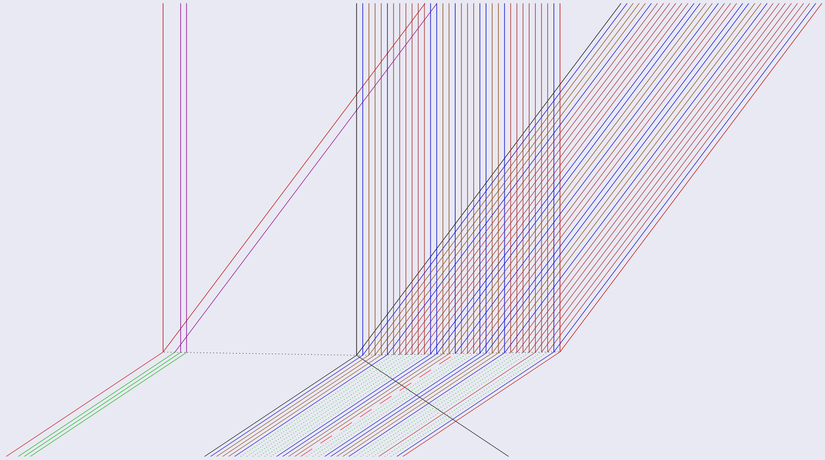
Comparison of id's in the if-part of a Rule



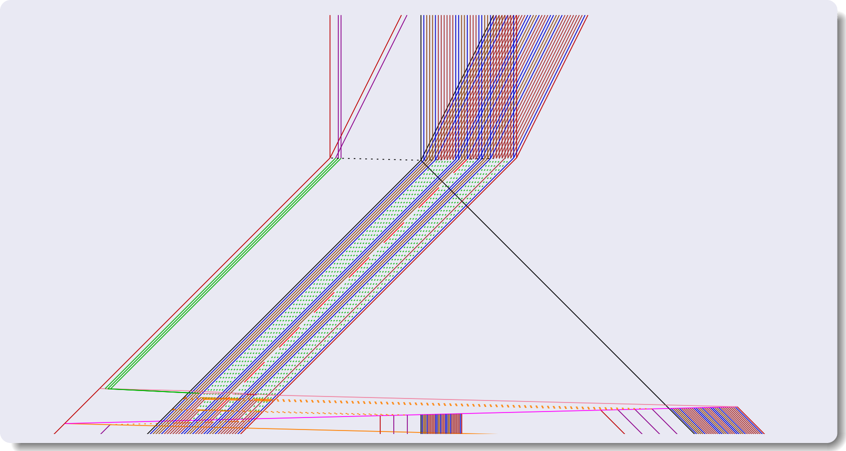
Rule Selection



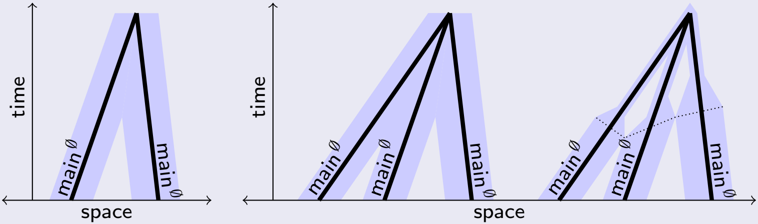
Generating the Output



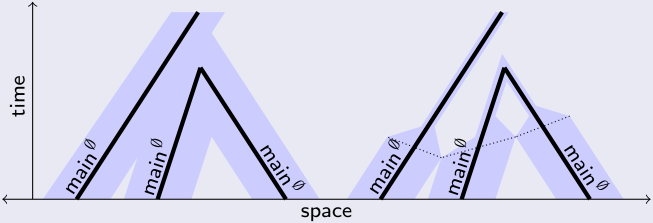
Whole resolution



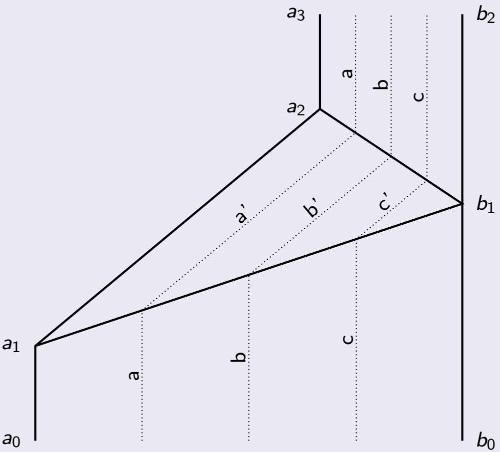
Good Cases



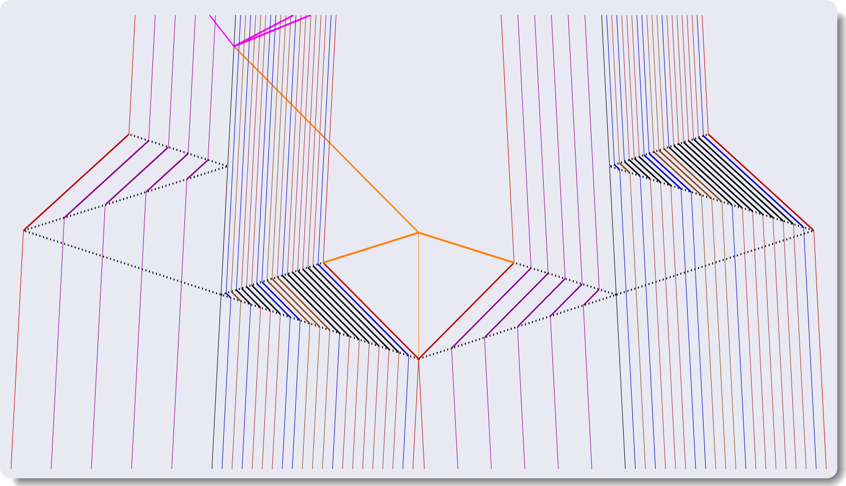
Bad Case



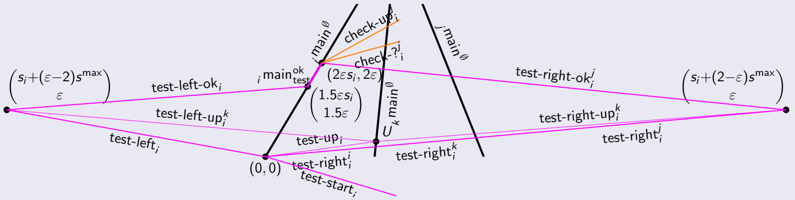
Shrinking Unit

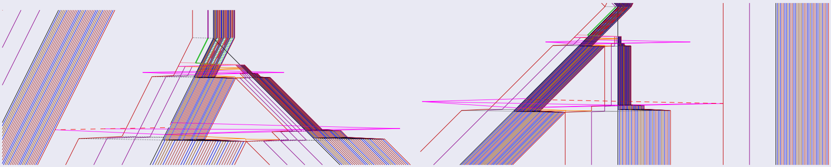


Shrink

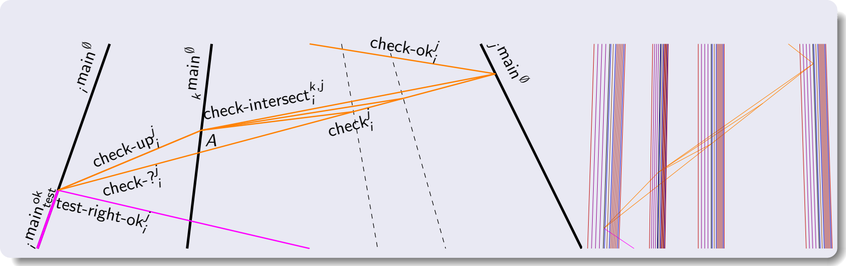


Testing for Other main$^{\emptyset}$ Signals

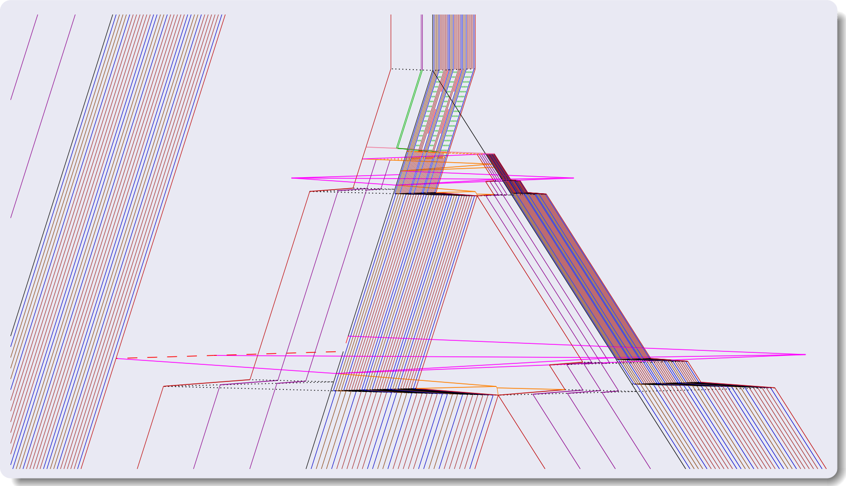




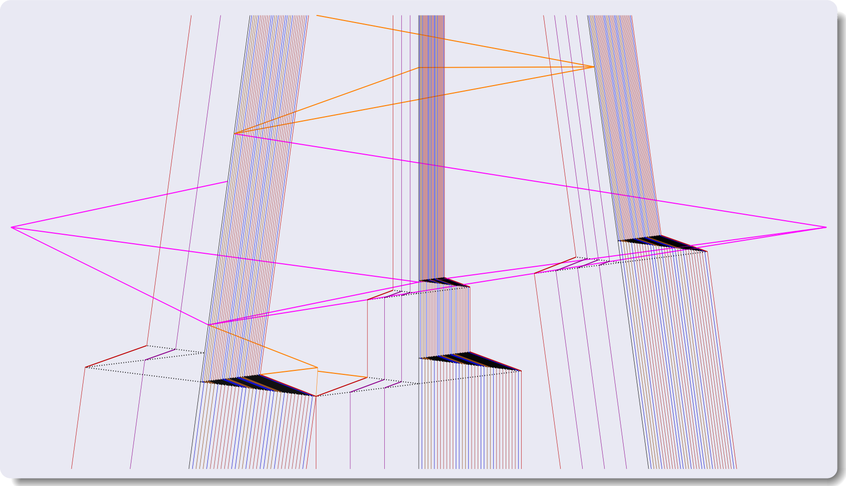
Checking the right positioning of Other main$^{\emptyset}$ Signals



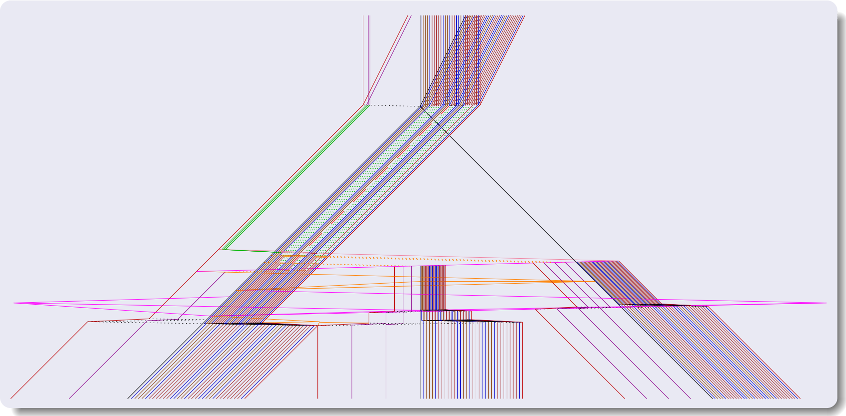
Detecting Potential Overlaps



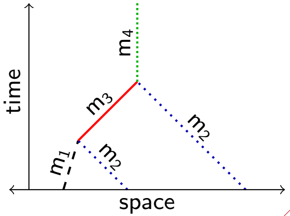
Whole Preparation

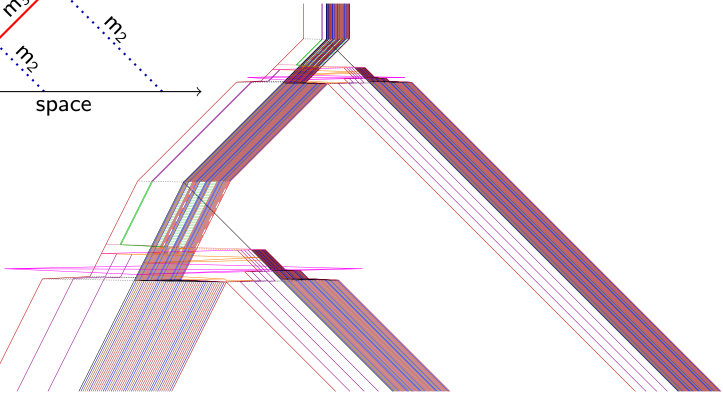


Exact 3-signal collision

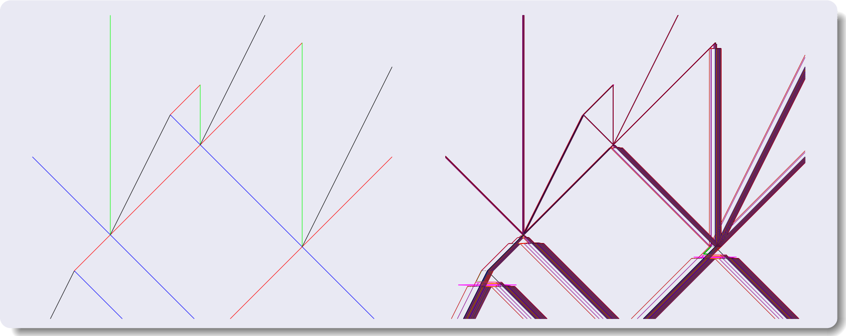


Some examples





Some examples



Some examples

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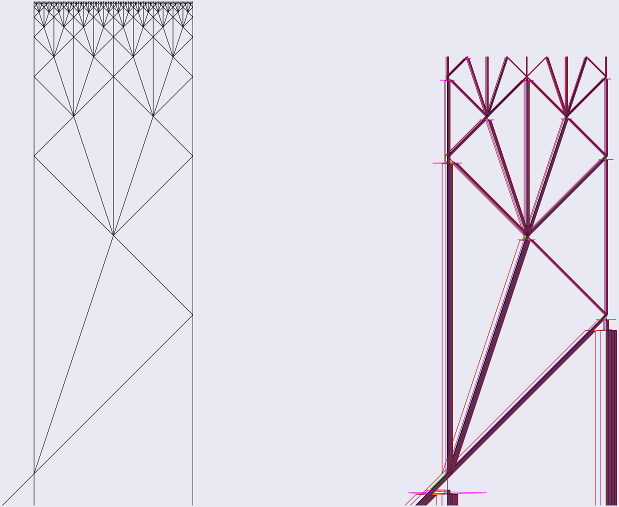


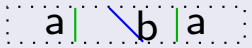
Non-determinism in rule output

Meta-signals

a	0
b	-1
c	1

Collision rules

$\{ a, b \} \rightarrow \{ a, c \}$
$\{ a, b \} \rightarrow \{ b \}$
$\{ c, a \} \rightarrow \{ b \}$
$\{ c, a \} \rightarrow \{ a \}$



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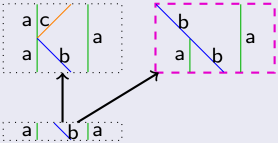

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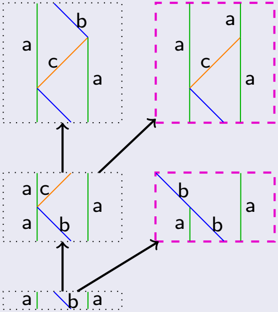

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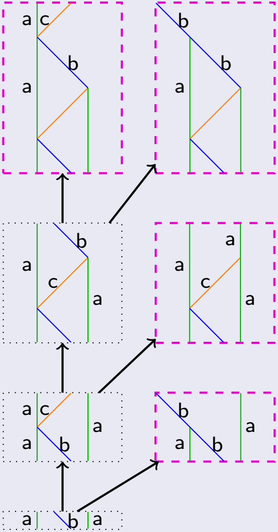

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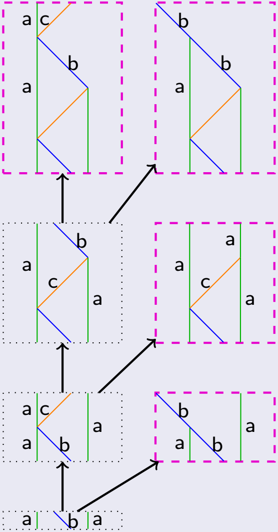
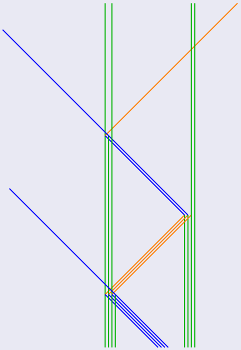


Shifted superposition



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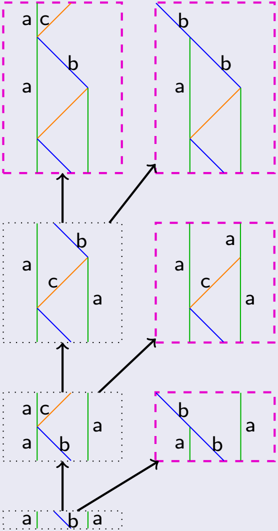

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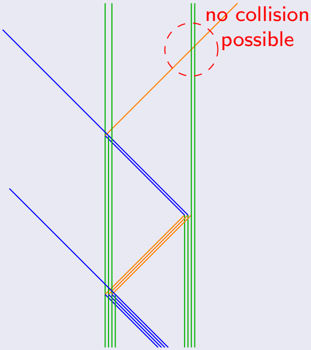
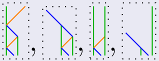


All possible universes

$\mathcal{U} = \{$  $\}$

Unbounded signals

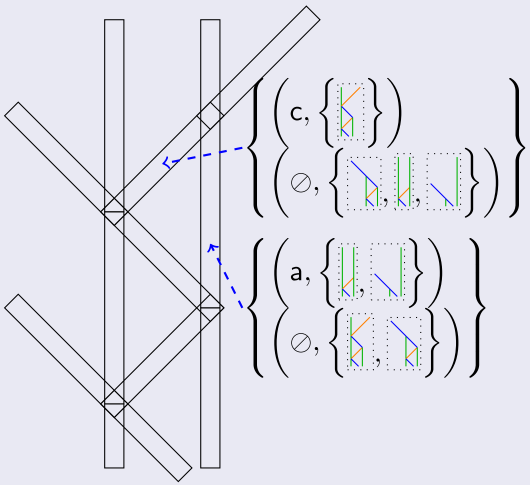
Information held:

$$\{(v_\alpha, u_\alpha)\}_\alpha$$

such that:

$$v_\alpha \in \{\oslash, \mathrm{a}, \mathrm{b}, \mathrm{c}\}$$

$$\biguplus_\alpha u_\alpha = \mathcal{U}$$



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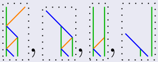
$\mathcal{U} = \{$  $\}$

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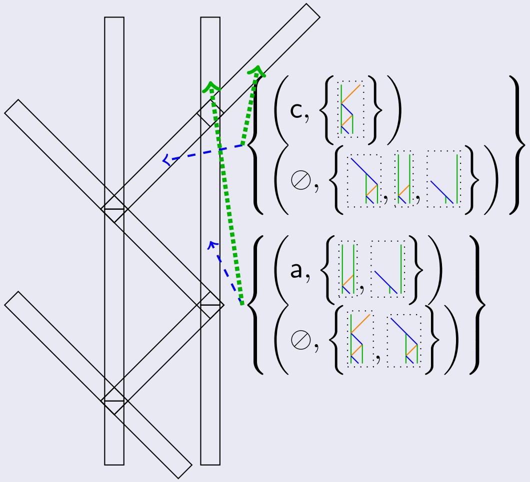

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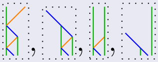
$\mathcal{U} = \{$  $\}$

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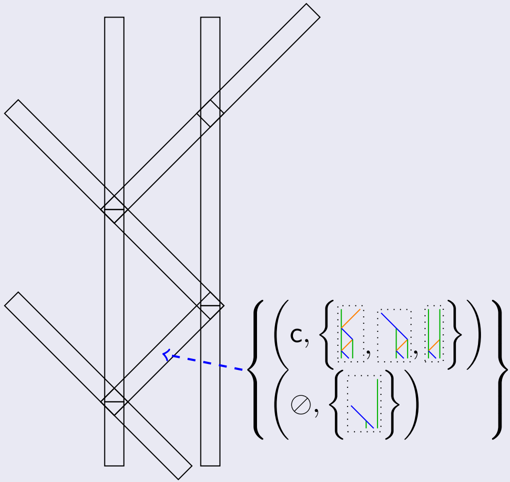
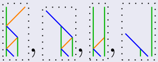
Future unknown



All possible universes

$\mathcal{U} = \{$  $\}$

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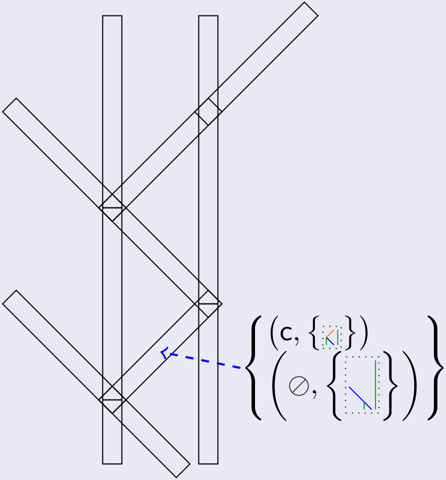

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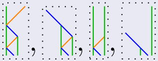
$\mathcal{U} = \{$  $\}$

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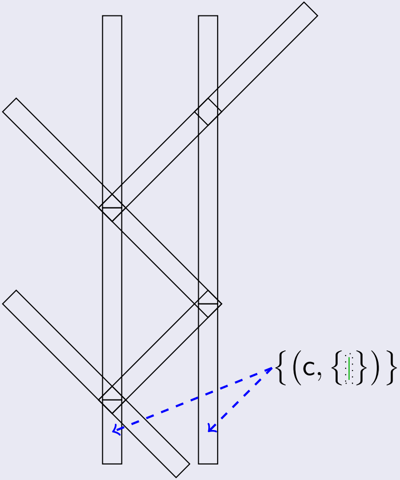
Distinguish



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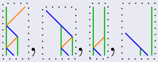
$\mathcal{U} = \{$  $\}$

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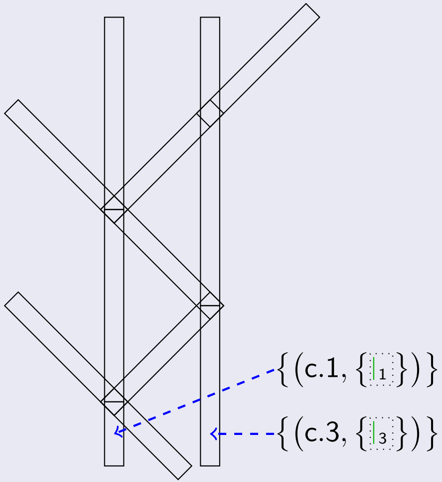

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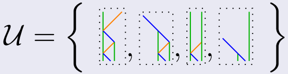

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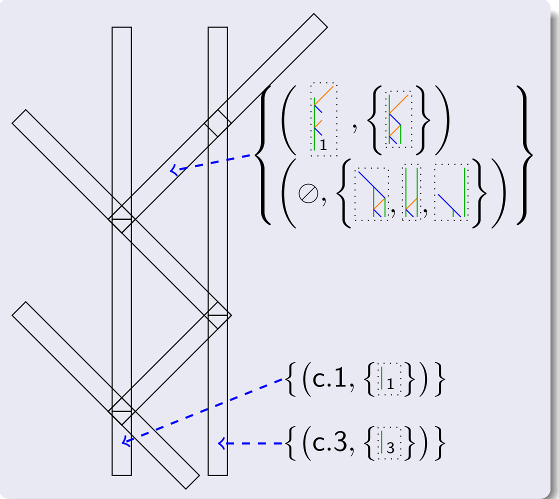


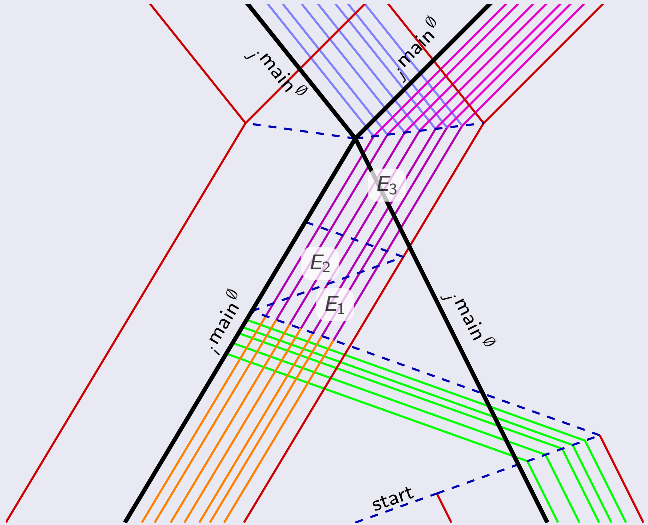
Macro-collision

$$\{\,(v_\alpha, u_\alpha)\,\}_\alpha \text{ meets } \left\{\,\left(v'_\beta, u'_\beta\right)\,\right\}_\beta$$

1. $E_1 = \left\{\,\left((v_\alpha, v'_\beta), u_\alpha \wedge u'_\beta\right)\,\right\}_{\alpha,\beta}$ $\qquad \mathcal{U} = \biguplus_{\alpha,\beta} u_\alpha \wedge u'_\beta$
2. $E_2 = \{\,((v, v'), u \wedge u') \in E_1 \mid u \wedge u' \neq \emptyset\,\}$
3. $E_3 = \{\,(\emptyset, w) \mid ((\oslash, \oslash), w) \in E_2\,\}$ $\cup \{\,(\{\mu\}, w) \mid ((\mu, \oslash), w) \in E_2 \vee ((\oslash, \mu), w) \in E_2\,\}$ $\cup \{\,(\rho^+.\mu, \rho(\mu, \nu) \wedge w) \mid ((\mu, \nu), w) \in E_2 \wedge \rho^- = \{\mu, \nu\}\,\}$
4. $Out\_Speed = \{\,Speed(\mu) \mid \exists \mu, (F, w) \in E_3, \mu \in F\}$
5. $\forall s \in out,$ $Out_s = \{\,(\mu, w) \mid \exists F, (F, w) \in E_3 \wedge \mu \in F, Speed(\mu) = s\}\,\}$ $\cup \{\,(\oslash, w) \mid \exists F, (F, w) \in E_3 \wedge \forall \mu \in F, Speed(\mu) \neq s\}\,\}$

- Compatible string encodings

Displaying the operations



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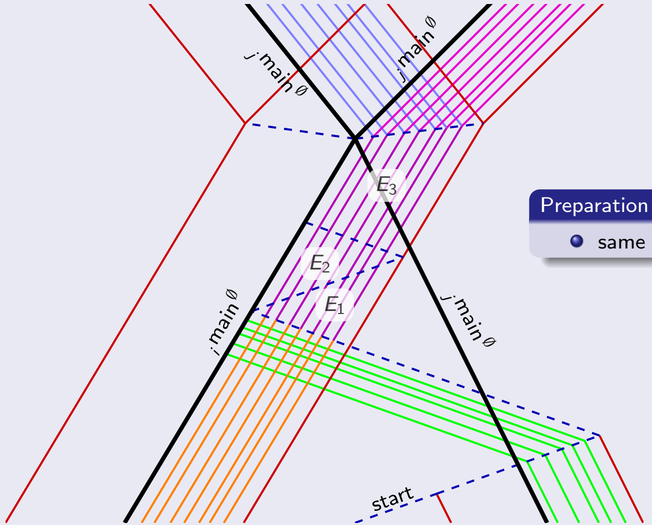

Preparation

- same as before

- Very rich setting

- Intrinsically universal family of signal machines

- Non-deterministic signal machine are not “more powerful”

- Very rich setting

- Intrinsically universal family of signal machines
- What is the complexity?

- Non-deterministic signal machine are not “more powerful”
- How to extract “result”?
- What is the complexity?

- Very rich setting

- Intrinsically universal family of signal machines
- What is the complexity?

- Non-deterministic signal machine are not "more powerful"
- How to extract "result"?
- What is the complexity?

- Augmented signal machines

That's all folks!

Thank you for your attention

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