Representing reversible cellular automata with reversible block cellular automata

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Context

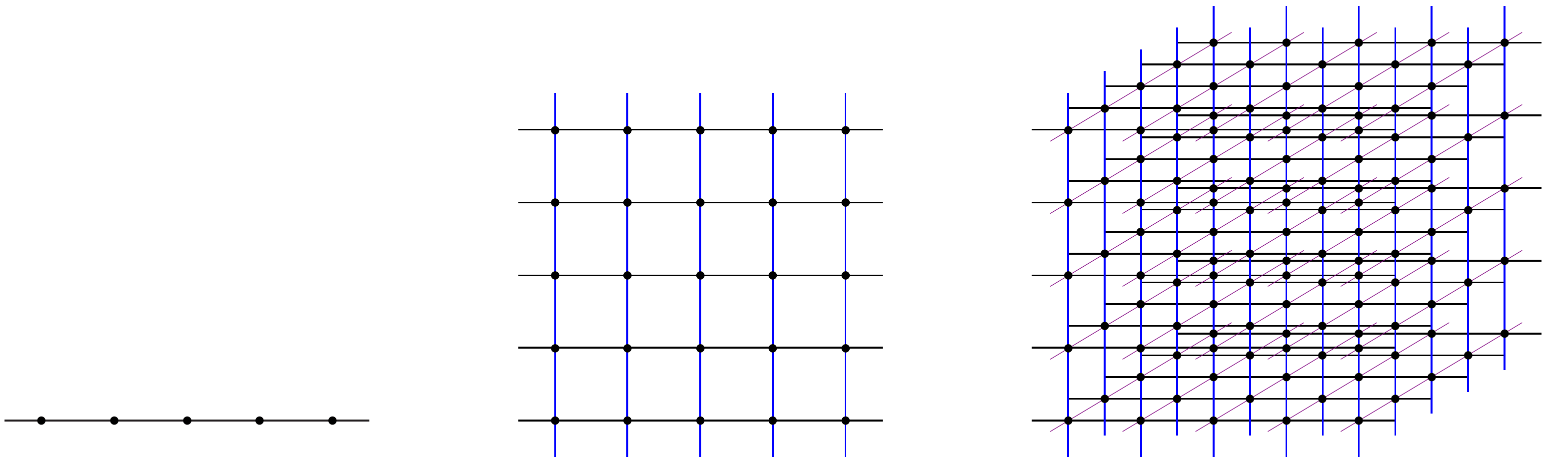
History and definitions

Cellular automata as block cellular automata

Reversible CA as reversible BCA

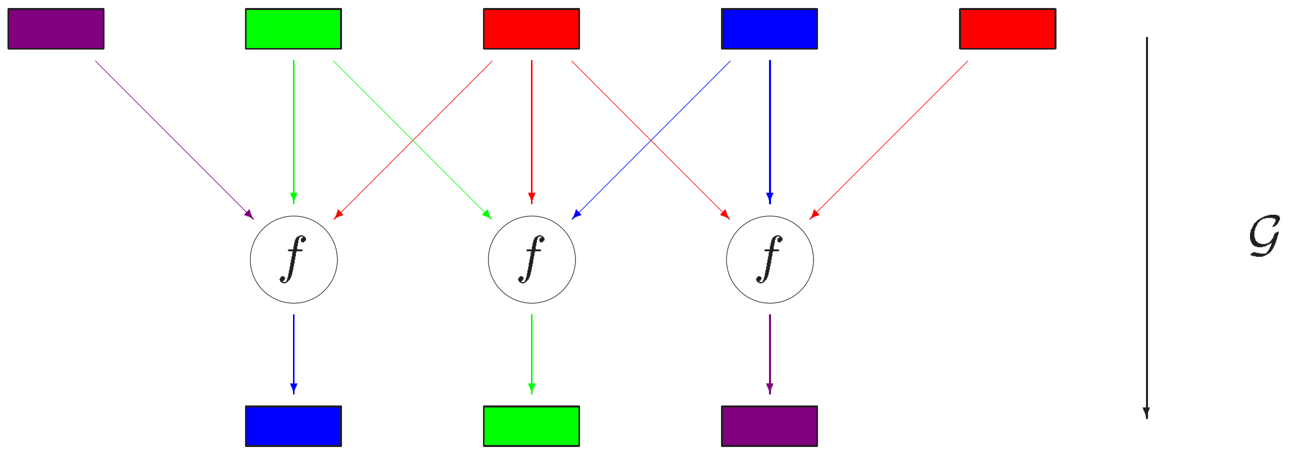
Context — Cellular automata (1)

Model → Computation (von Neumann), parallelism
→ Physical phenomena



Infinite d-dimensional underlying lattice

Context — Cellular automata (2)



Local and synchronous updating

Context — Reversibility

Physical phenomena are reversible at macroscopic level

Irreversibility $\longleftrightarrow$ heating

$$\left(\mathcal{S}^{\mathbb{Z}^d}, \mathcal{G}\right) \text{ reversible } \iff \mathcal{G}^{-1} \text{ exists}$$

Questions $\longrightarrow$ Properties of such systems

Questions $\longrightarrow$ Computing abilities

History — Decidability

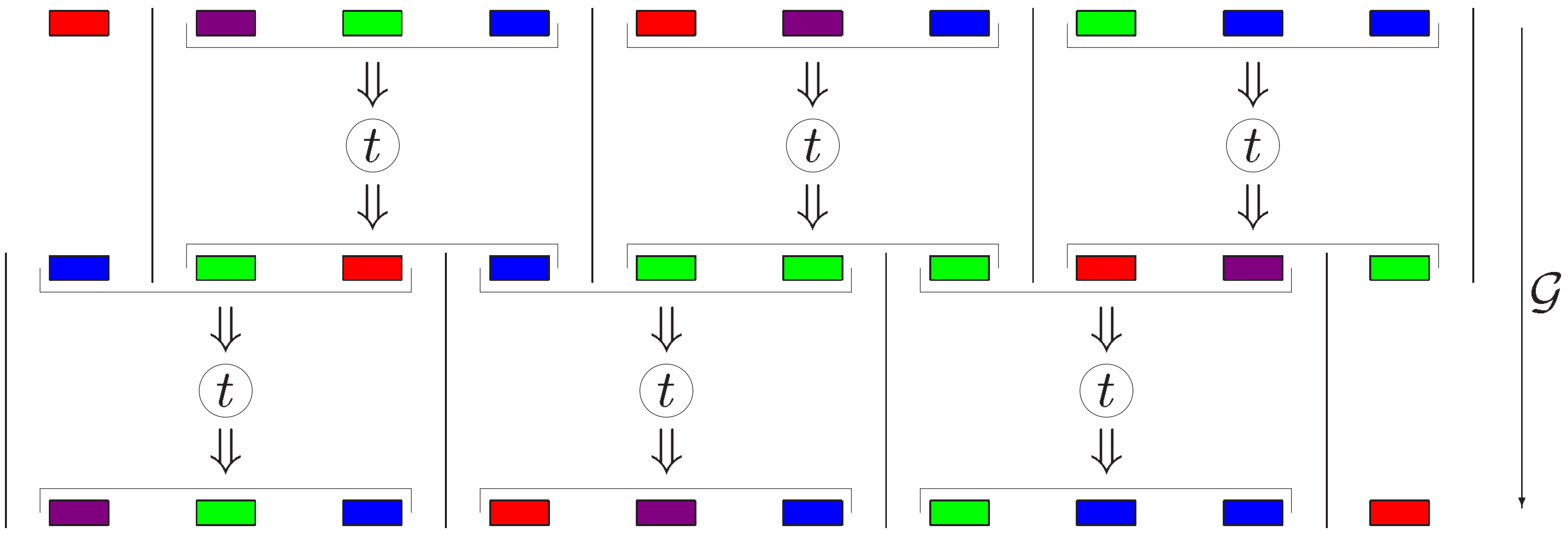
Amoroso and Patt, 1972
Reversibility is decidable in dimension 1
(linked to connected components of a finite graph)
“ ... should be translatable to higher dimension ... ”

Kari 1989
Reversibility is undecidable from dimension 2
(linked to tiling in two dimension)

History — Block cellular automata

Margolus 1983

Billiard ball model $\xrightarrow{\text{Generalization}}$ Block cellular automata



Reversibility easily checkable

History — Computing power

Margolus 1983

Billiard ball model is Turing universal

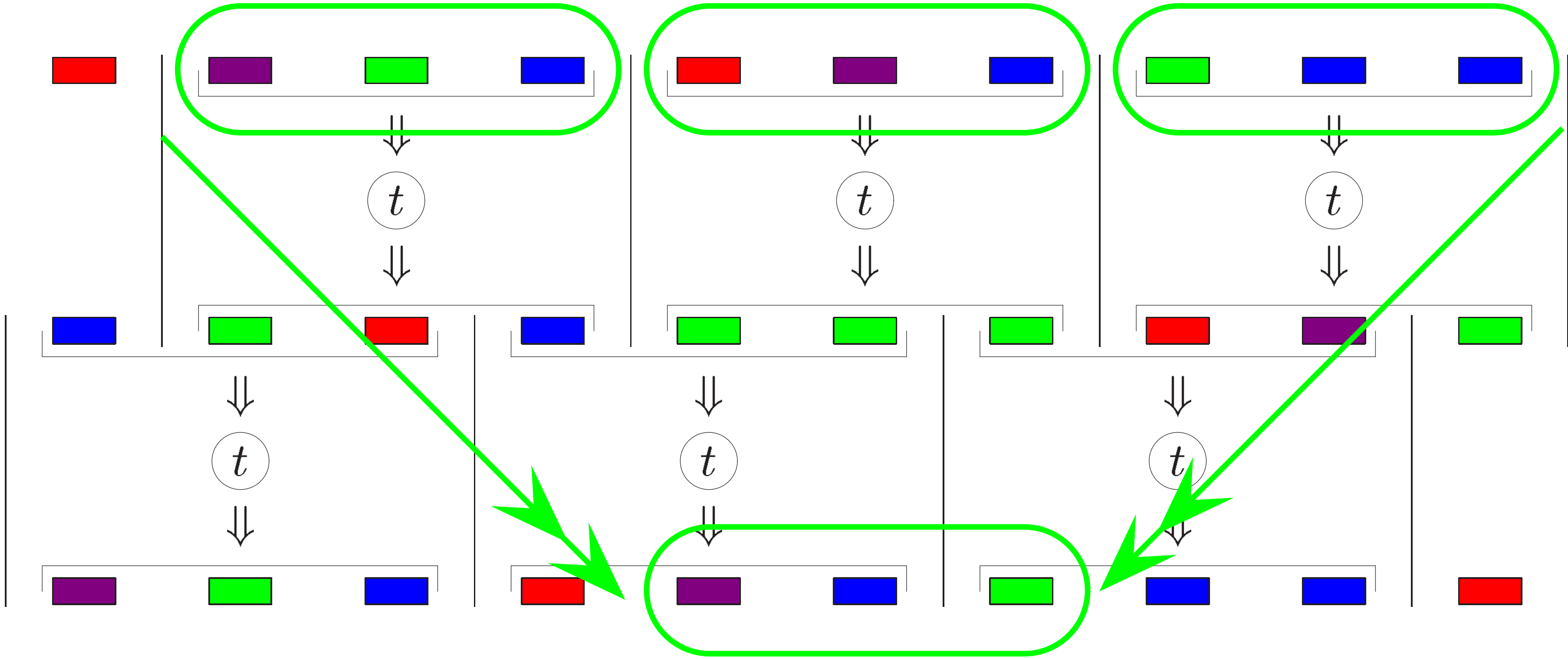
$$\text{BCA} \xrightarrow{\text{Change of scale}} \text{CA}$$

$\rightsquigarrow$ existence of 2-dimensional CA both reversible and universal

Morita 1989

Existence of 1-dimensional CA both reversible and universal

Identifying BCA as CA

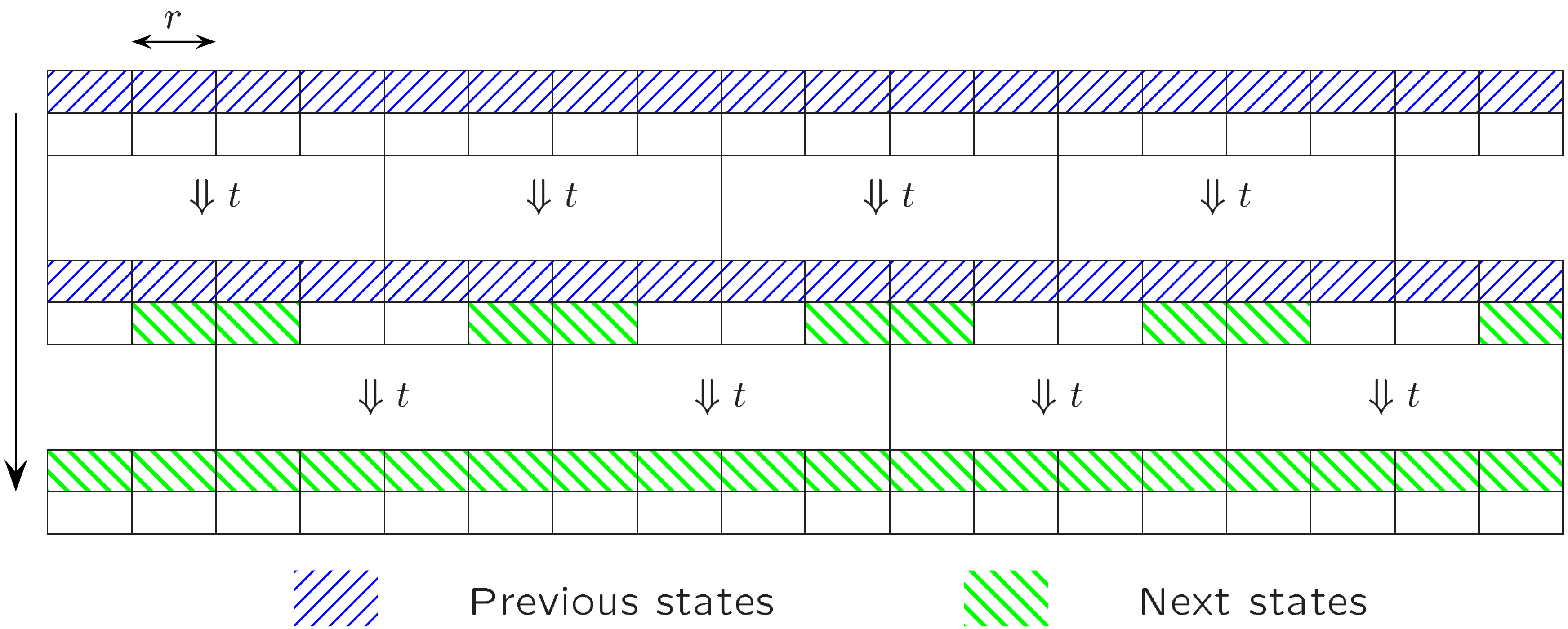


BCA are CA

Reversibility is preserved

CA as BCA

Yes, with a larger set of states: $\mathcal{S} \rightsquigarrow \mathcal{S}^2$



Reversbile CA by reversible BCA ?

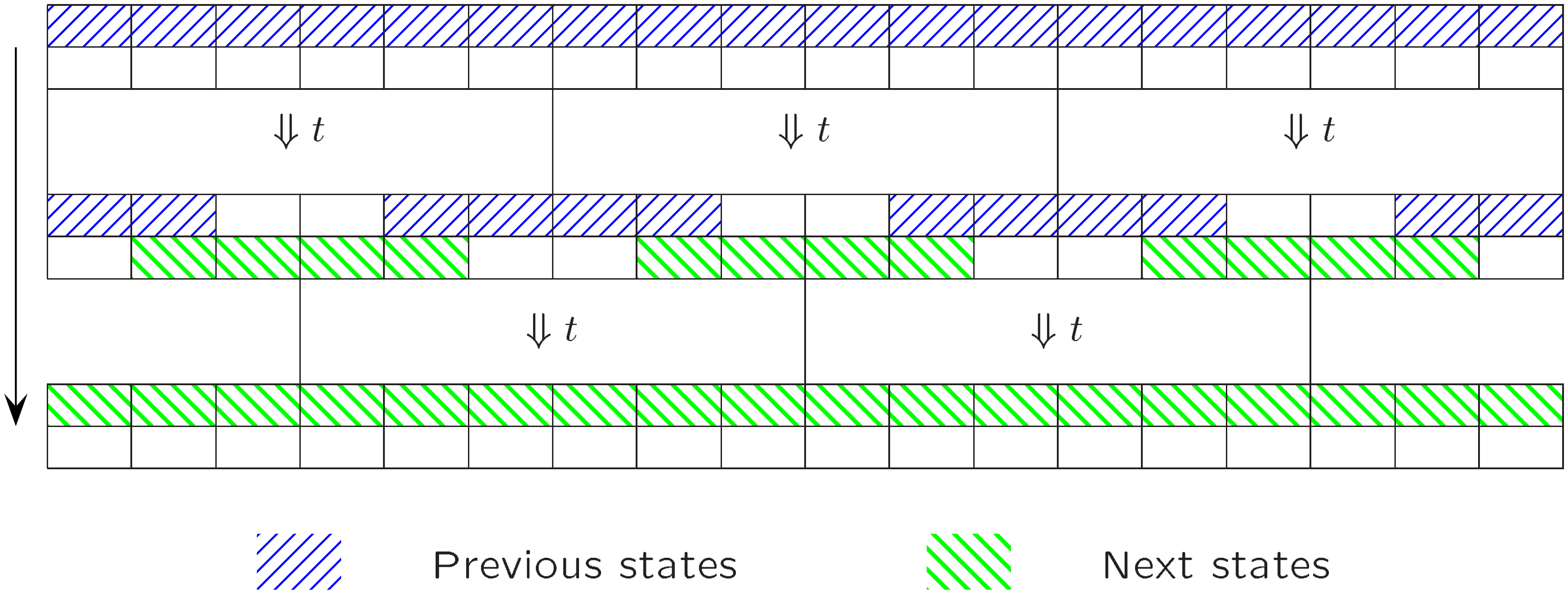
Previous construction leads to non reversible BCA

Toffoli and Margolus 1990
Conjectures that it is possible

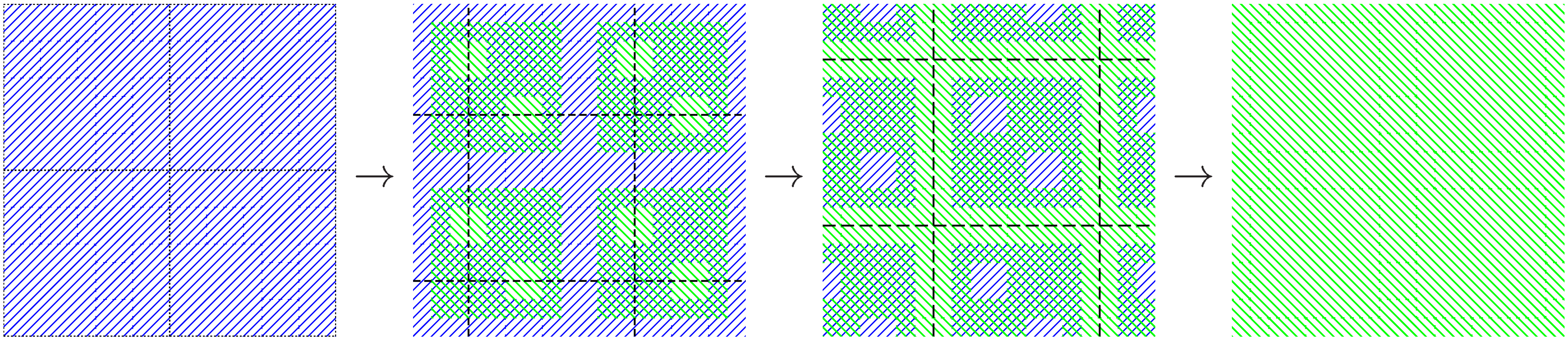
Yes,
1. find the inverse (*)
2. set radius large enough

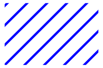
(*) This can be done since the cellular automaton IS reversible (complexity can not be bound by any computable function)

Reversible CA by reversible BCA



Reversible CA by reversible BCA 2D





Previous states

Next states

Reversible CA by reversible BCA

Possible in any dimension

Number of partitions:	$d + 1$	$2^{d+1} - 1$
Size of blocks:	$(6rd)^d$	$(4r)^d$

Open problem

Is it possible *without increasing the number of states*?

— no extra storage —

Kari 96
Yes in dimension 1 and 2
Complex transformation (algebraic tools), uneasy to generalized

Still open for dimension 3 and higher