Abstract geometrical computation for Black hole computation

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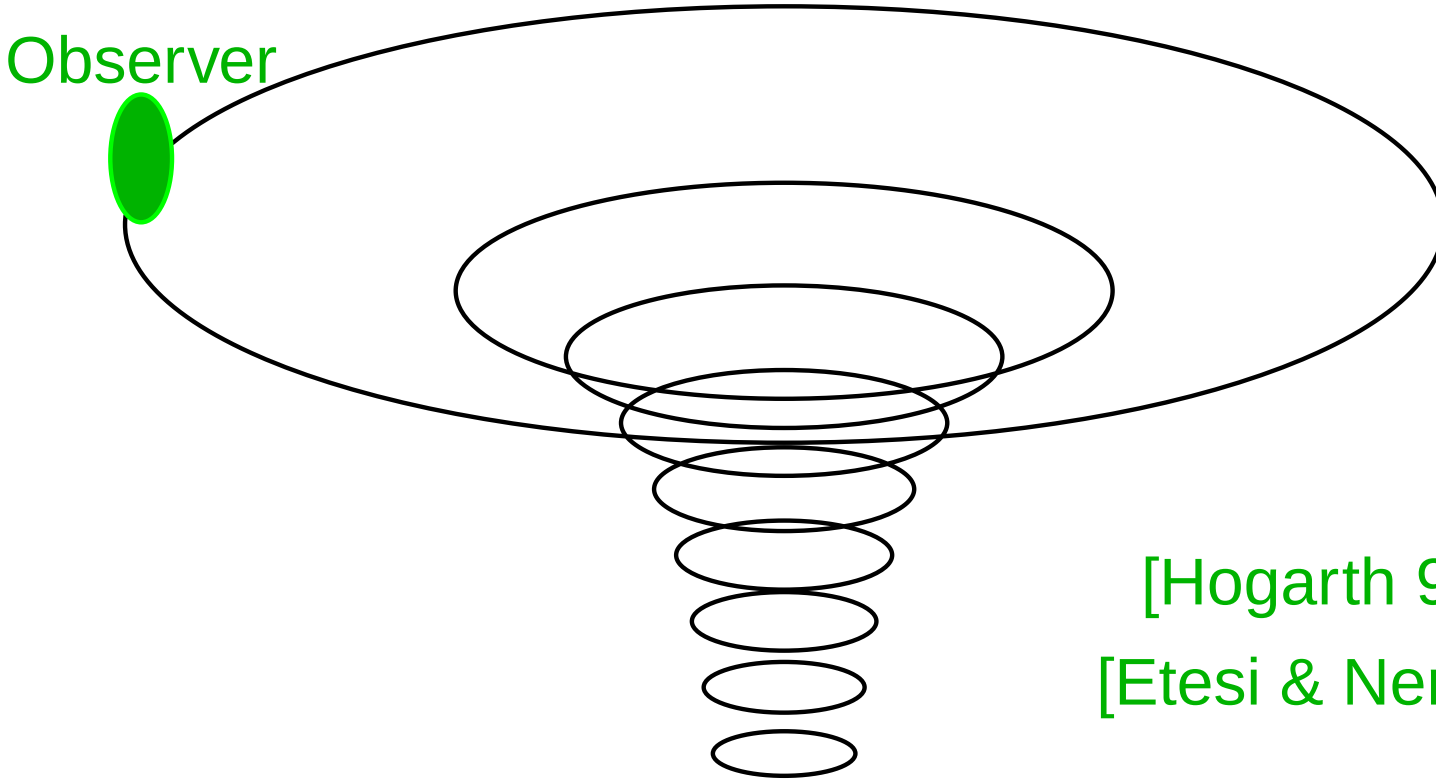
Outline

1. Black hole computation
2. Cellular automata to Abstract geometrical computation
3. Signal machine and restriction
4. Turing-computing power
5. Black hole effect
6. Conclusion and extension

Black hole computation

Black hole model

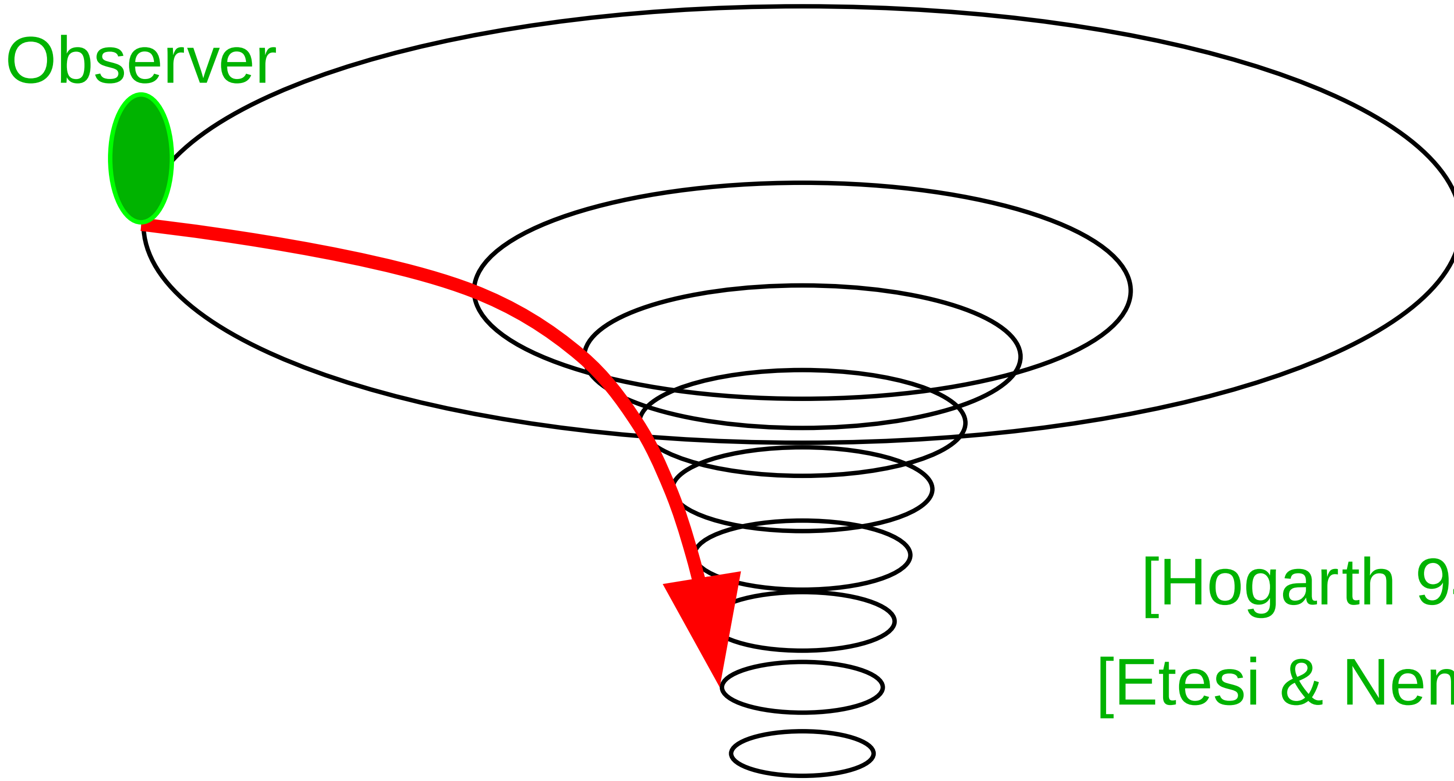


[Hogarth 94 & 00]

[Etesi & Nemeti 02]

1. Observer at the “edge”

Black hole model

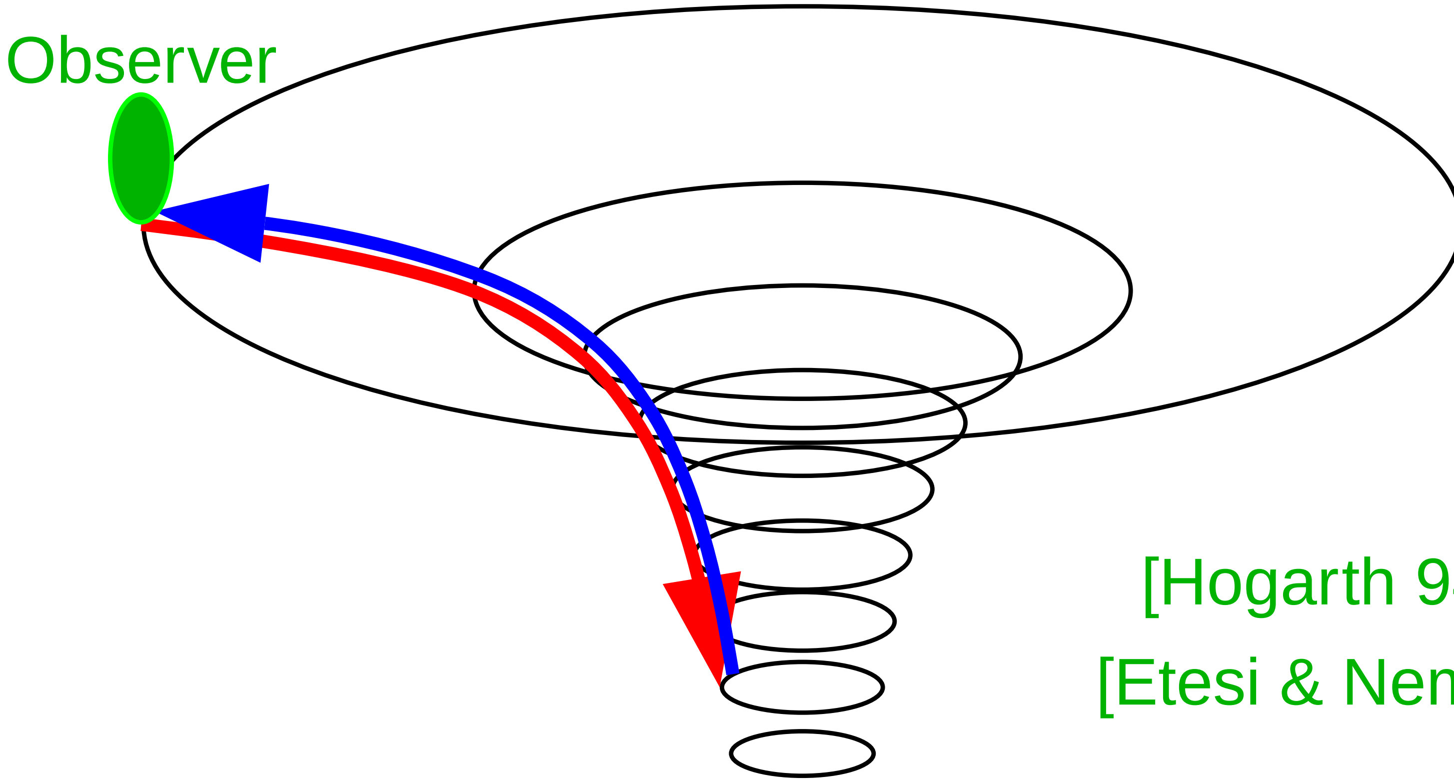


[Hogarth 94 & 00]

[Etesi & Nemeti 02]

1. Observer at the “edge”
2. Machine sent into the black hole *infinitely accelerated*

Black hole model

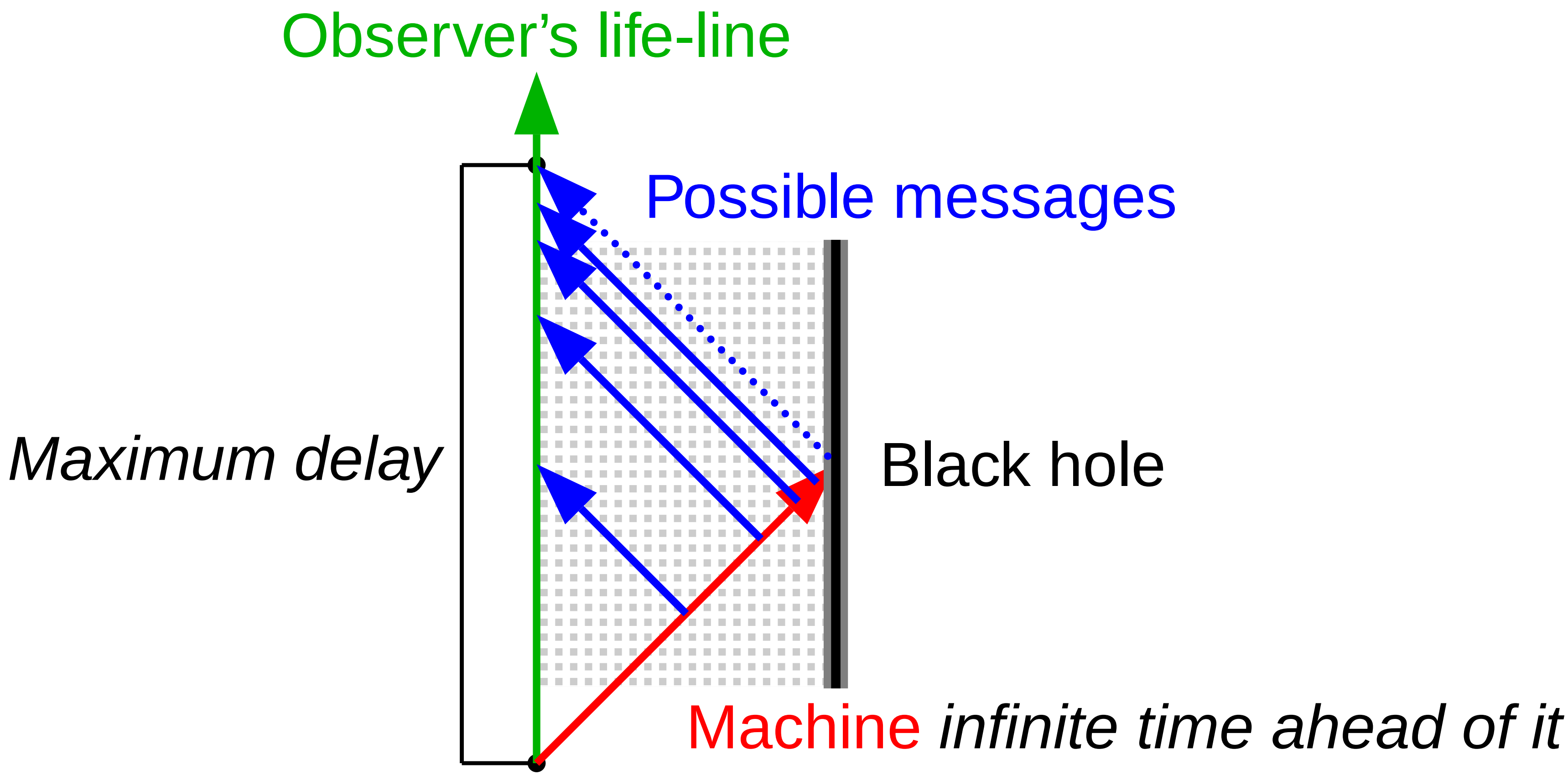


[Hogarth 94 & 00]

[Etesi & Nemeti 02]

1. Observer at the “edge”
2. Machine sent into the black hole *infinitely accelerated*
3. Message sent by the machine received by the observer *within a bounded delay*

Malament-Hogarth space-time



Message indicates the result of the computation

After the delay, the observer knows whether the computation stops

Any recursively enumerable problem can be decided!

Related models

Main idea: infinitely many “iterations” on a sub-time-scale

Can be achieved with a transfinite ordinal scale as in:

Infinite time Turing machines

[Hamkins 02]

Or with a “Zeno” sub-scale as in:

Piecewise constant derivative systems

[Asarin & Maler 95, Bournez 99]

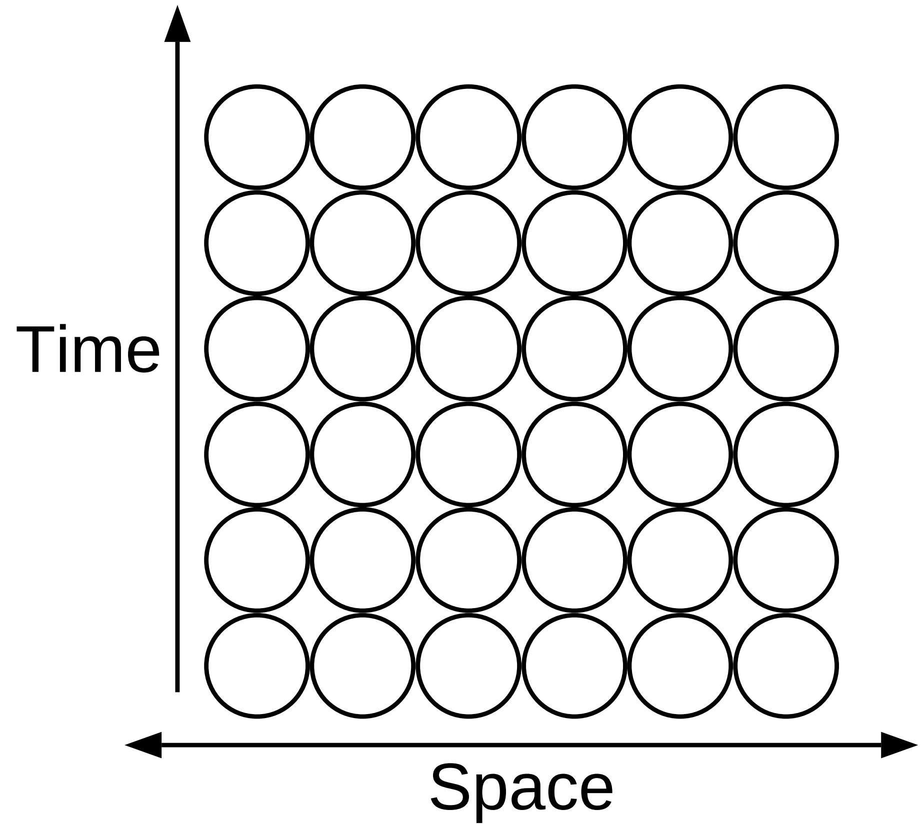
We use the last approach

Cellular automata to
Abstract geometrical computation

Basis

Well-known Model for parallelism, biology, physics. . .

Discrete time and space

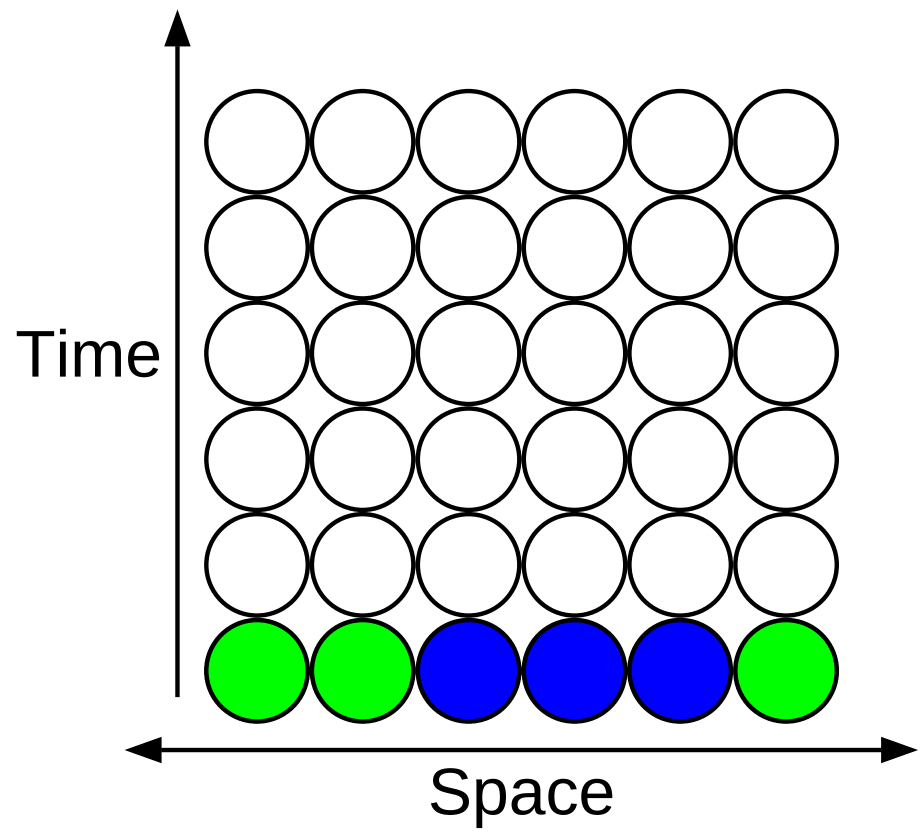


Basis

Well-known Model for parallelism, biology, physics. . .

Discrete time and space

Locally finitely many states



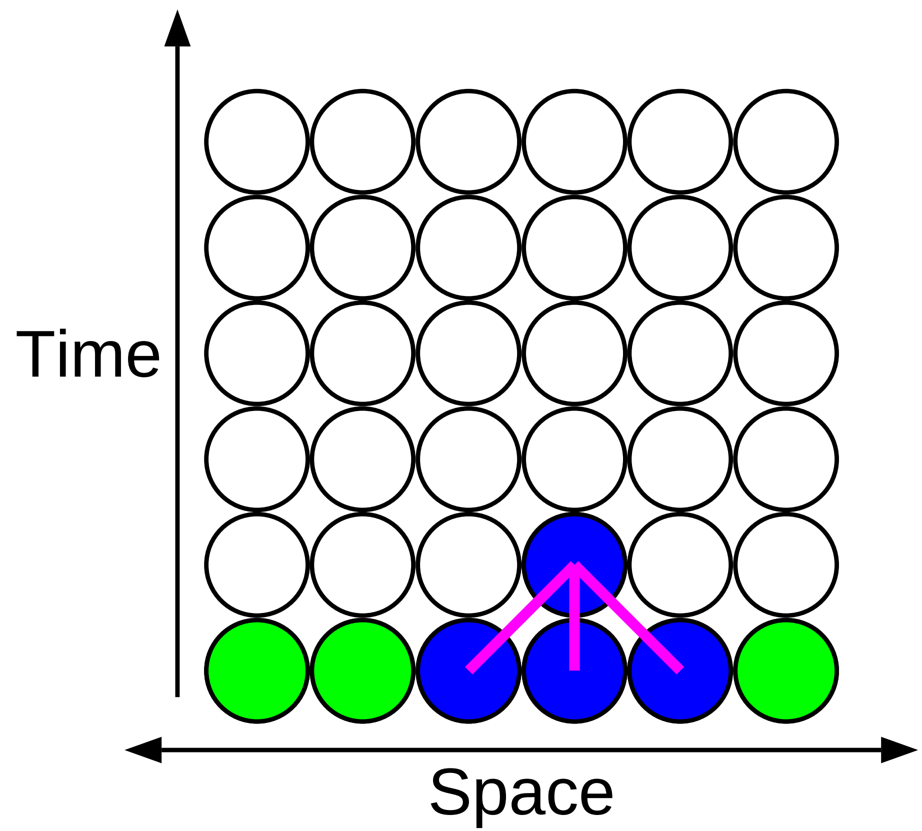
Basis

Well-known Model for parallelism, biology, physics. . .

Discrete time and space

Locally finitely many states

Local interaction



Basis

Well-known Model for parallelism, biology, physics. . .

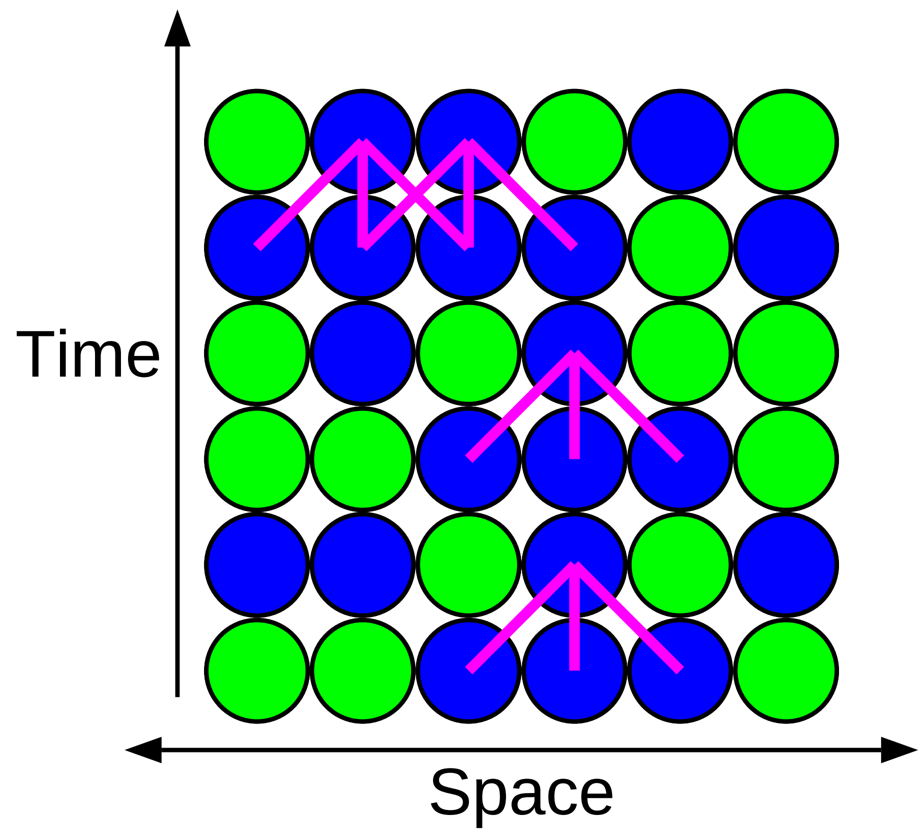
Discrete time and space

Locally finitely many states

Local interaction

Uniform in space and time

Turing-universal model



Space-time diagrams understanding

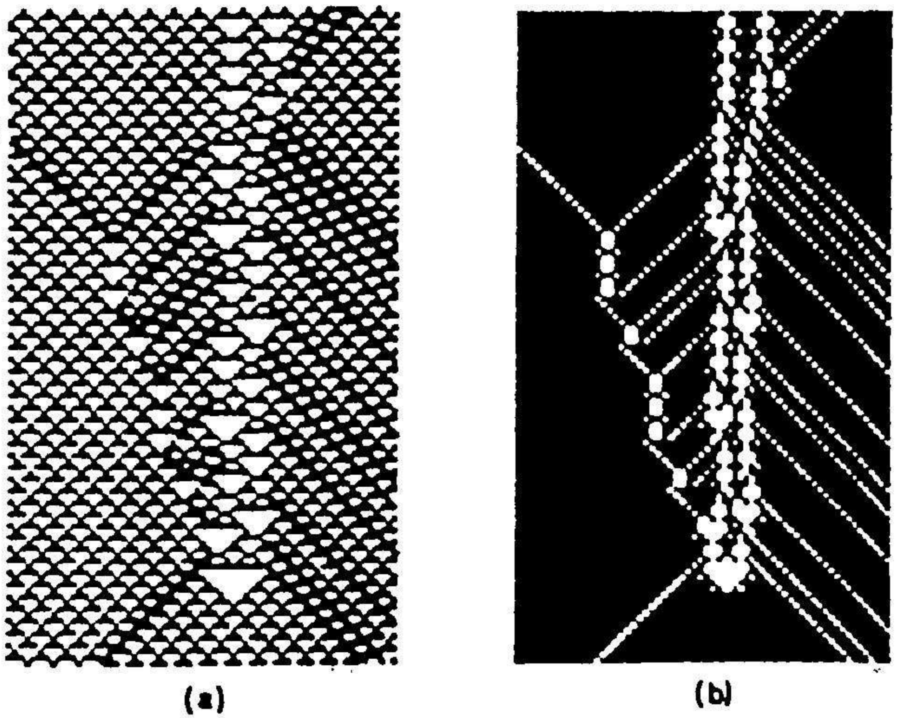


FIG. 7. Rule 54. (a) Annihilation of the radiating parti-
le. (b) The same as (a) with the mapping defined in Fig. 6.

[Boccara, Nasser & Roger 91]

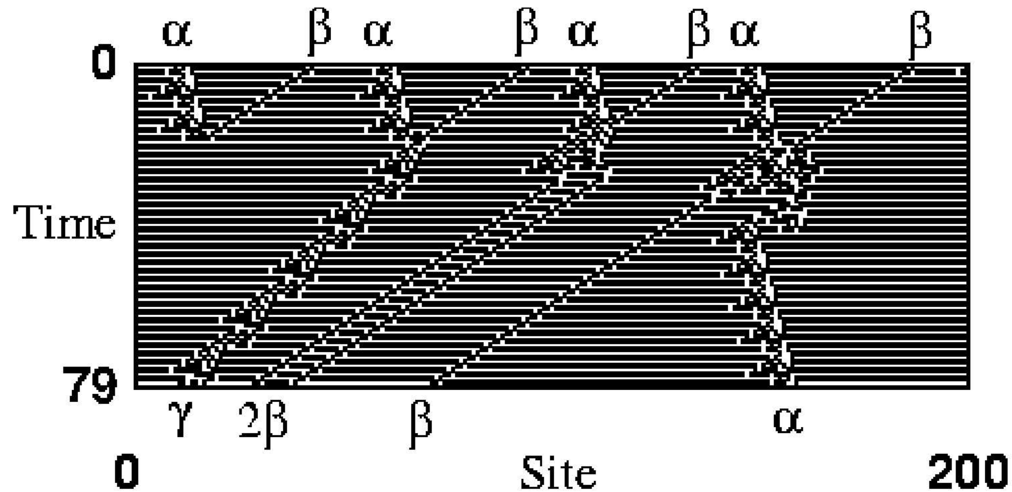


FIG. 7. The four different (out of 14 possible) interaction products for the $\alpha + \beta$ interaction

[hordijk-shalizi-crutchfi eld01

Observation of discrete lines ⇝ keys to dynamic

Space-time diagrams designing

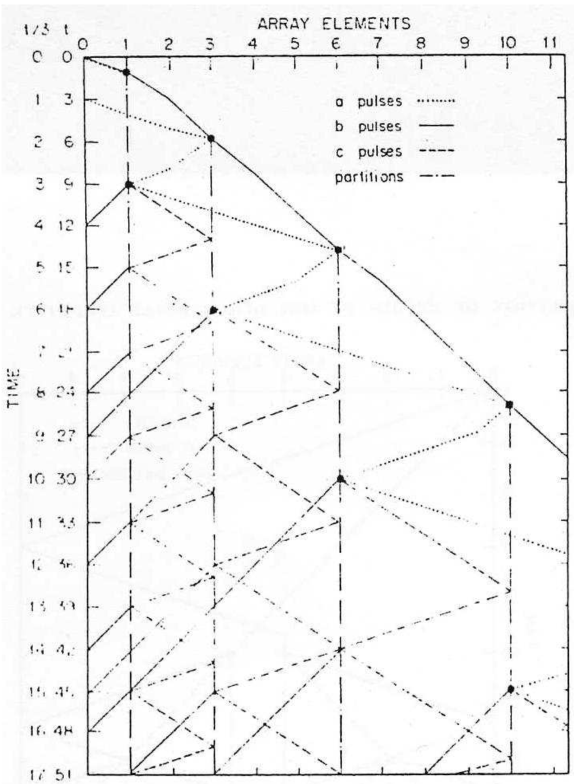
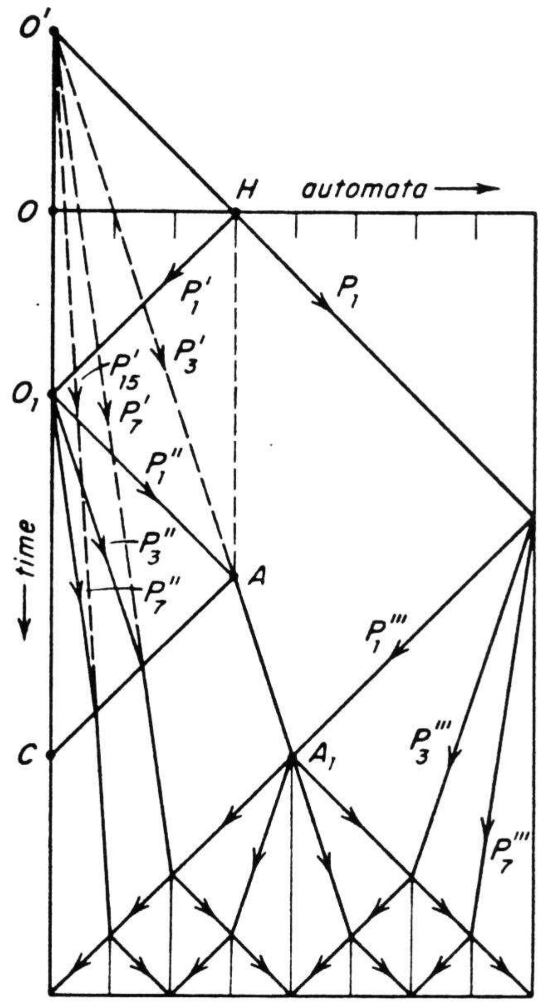
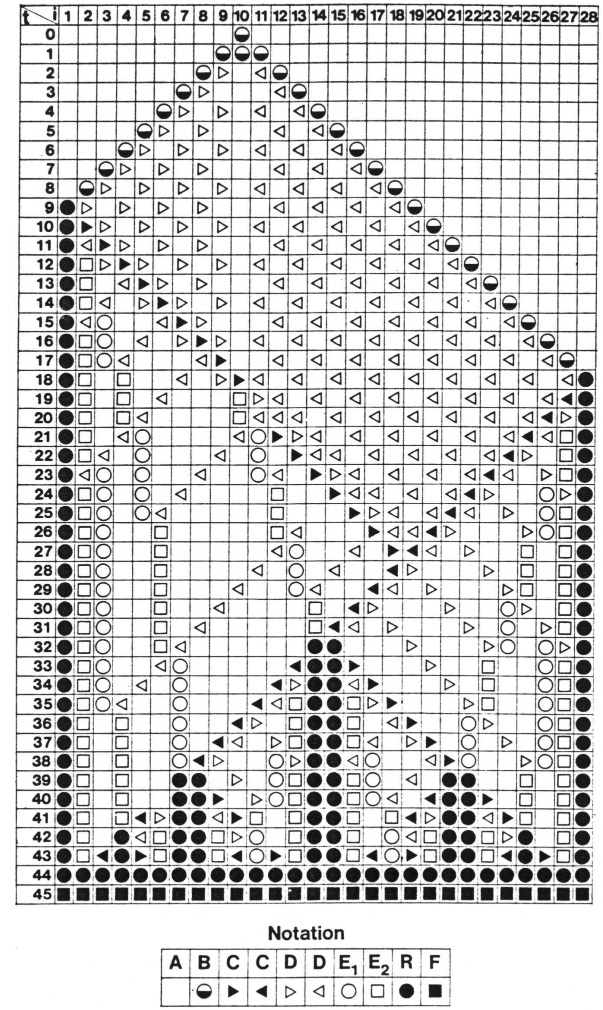


FIG. 2. Solution to the prime problem

[Fischer 65]



[Varshavsky *et al.* 70]



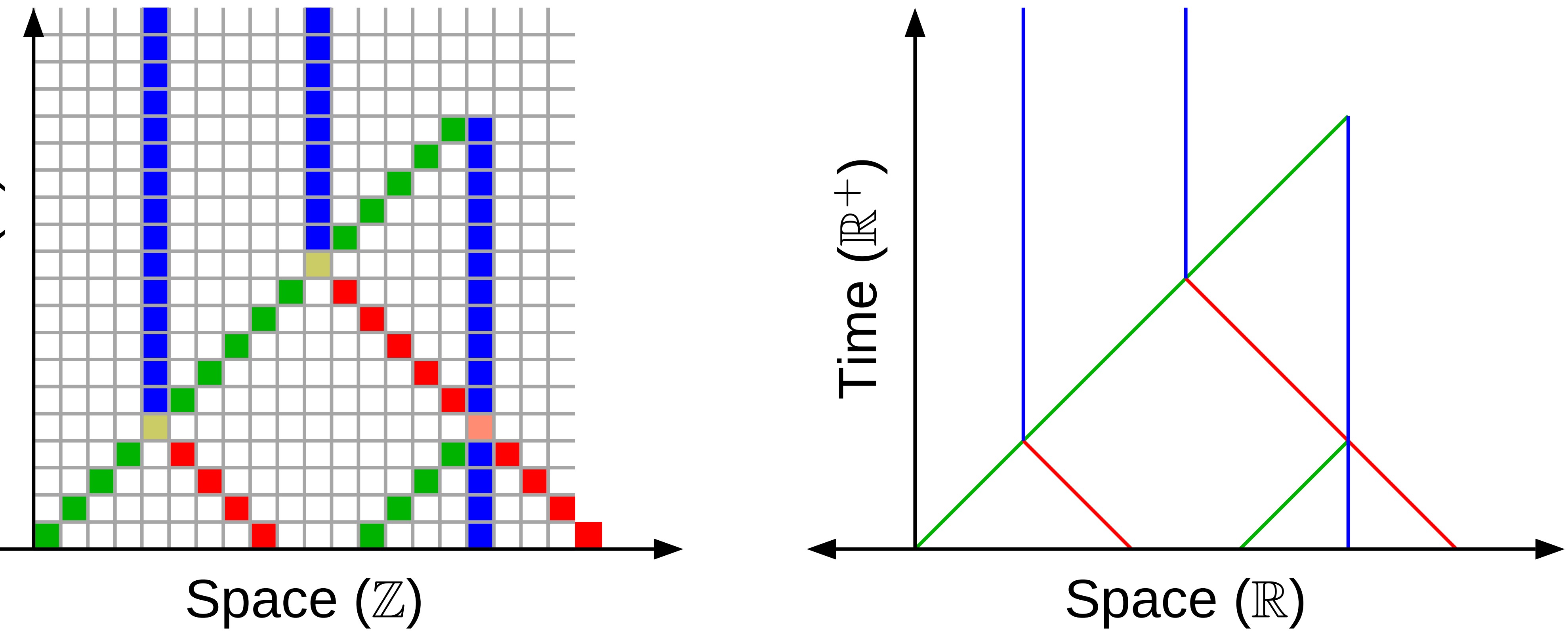
[Varshavsky *et al.* 70]

Easily generated discrete lines ⇝ special purposes CA design

Continuous abstraction

Signal: important notion, often used in literature

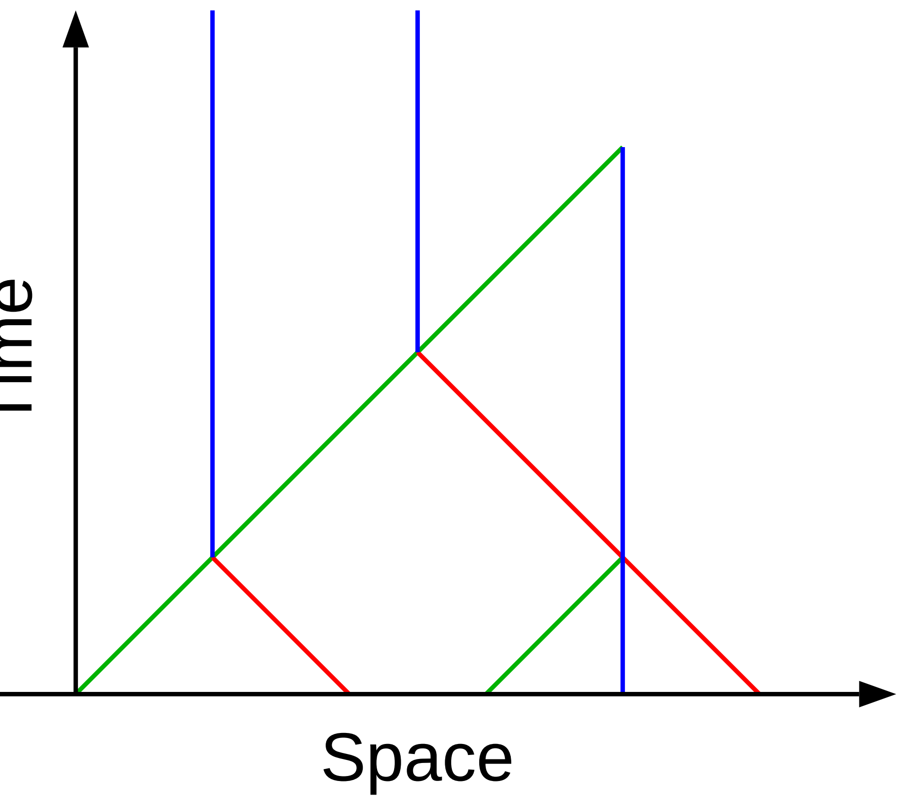
- to describe
- to design



Signal machine and restriction

Model definition

$\cancel{\mathbb{Z} \times \mathbb{N}}$ $\qquad \mathbb{R} \times \mathbb{R}^+$



Model definition

$\cancel{\mathbb{Z} \times \mathbb{N}}$ $\qquad \mathbb{R} \times \mathbb{R}^+$

Signal

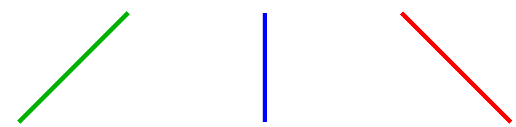
(Meta-signal, position)

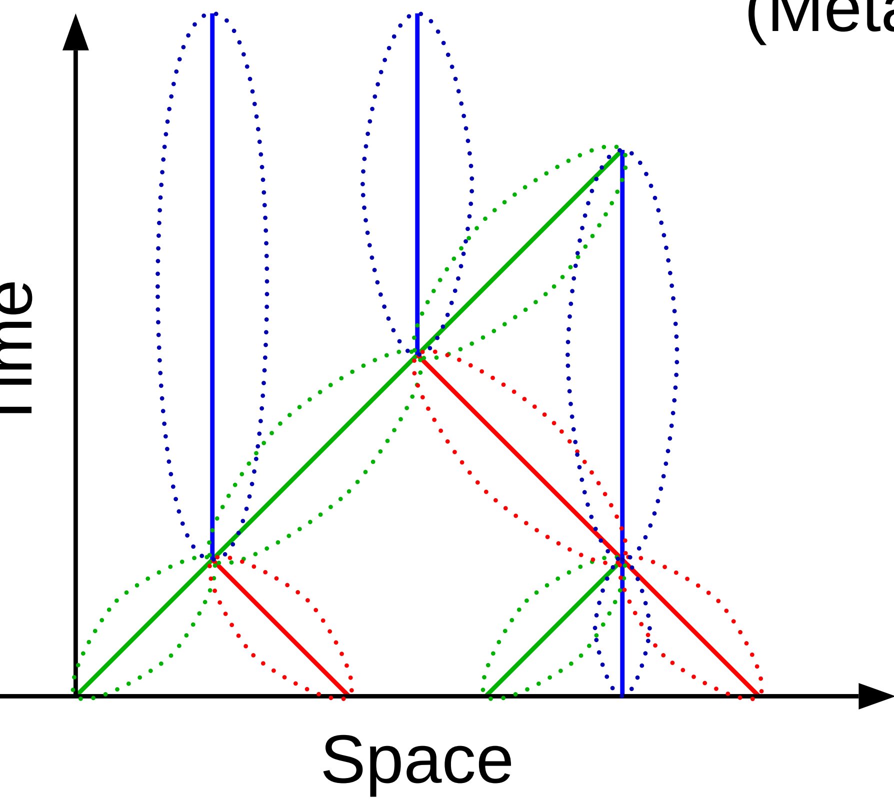
Position

(x, t)

Meta-signal

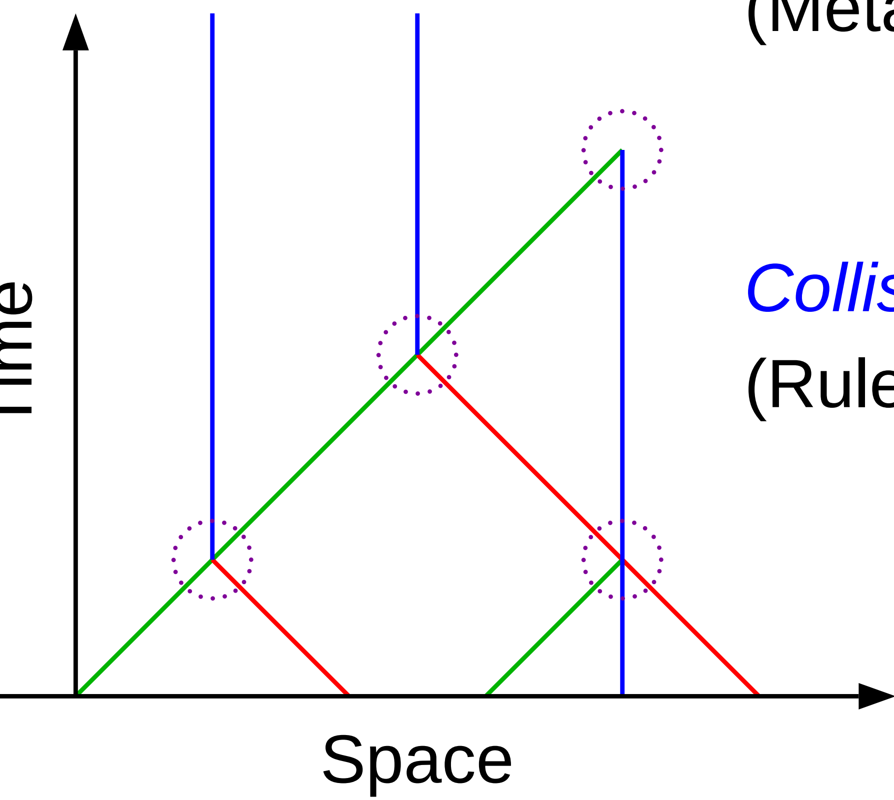
$\mu = (\iota, \nu)$





Model definition

$\cancel{\mathbb{Z}\times\mathbb{N}}$ $\qquad \mathbb{R}\times\mathbb{R}^+$



Signal

(Meta-signal, position)

Position

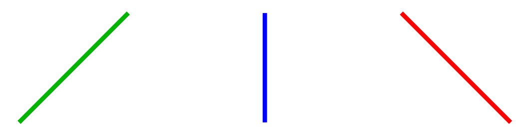
(x,t)

Collision

(Rule, position)

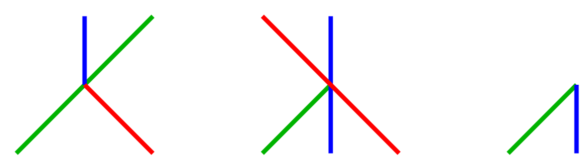
Meta-signal

$\mu = (\iota, \nu)$



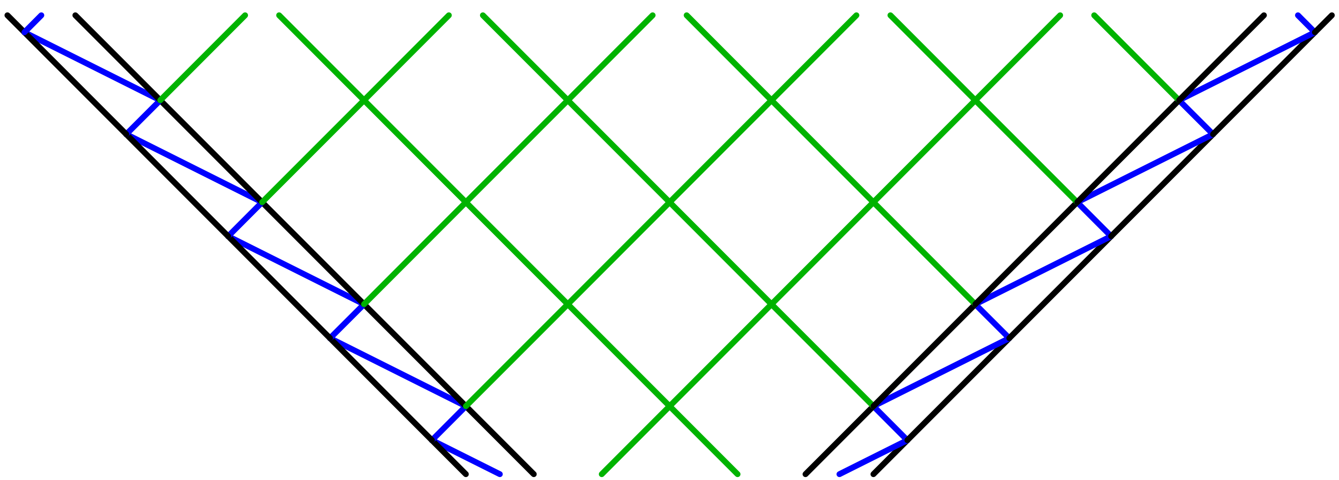
Rule

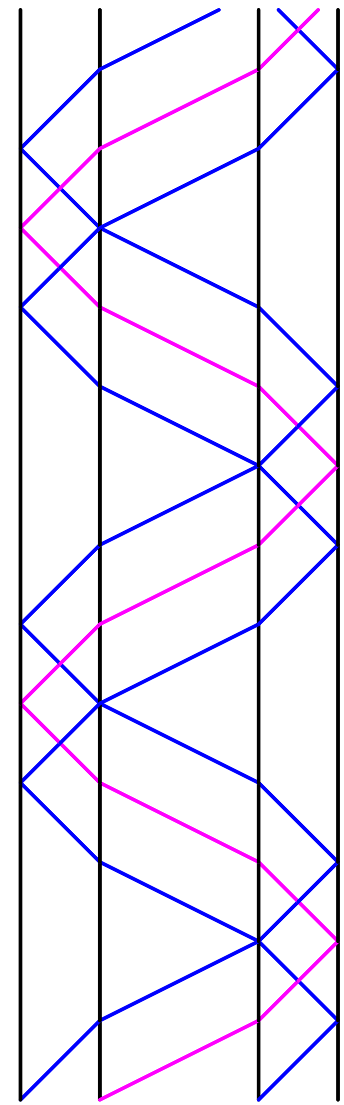
$\{\mu_i^-\}_i \rightarrow \{\mu_j^-\}_j$



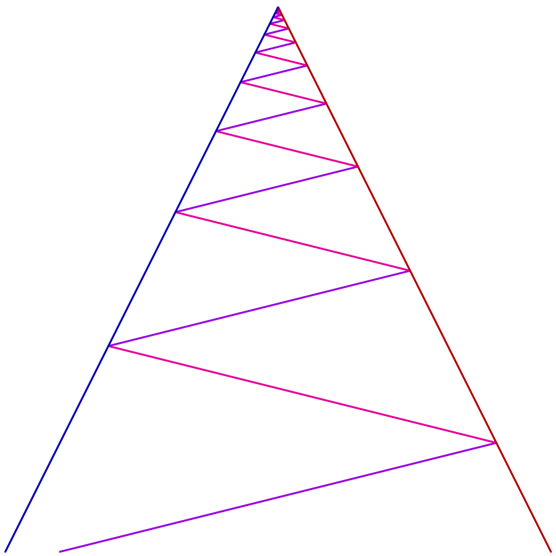
Properties and examples

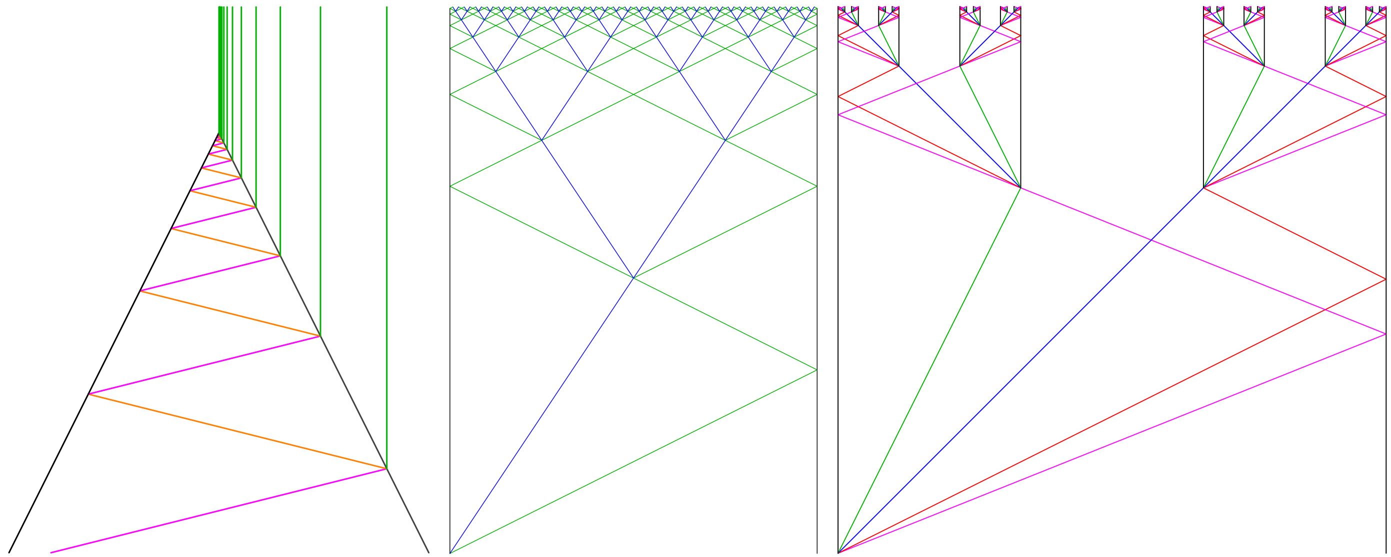
- Finite number of values & rules
- Light cone
- Local interaction
- Uniform in space and time
- Continuous space and time





Strange space-time diagrams





Zeno artifact

Unwanted cases

Unwanted because

- The number of signals is bursting to infinity
  (free creation of mater/energy)
- Difficulty (if not impossibility) to define continuation there

Restriction

- Energy : $\mu \longrightarrow E(\mu) \in \mathbb{N}^*$
- $\forall \rho = \{\mu_i^-\}_i \rightarrow \{\mu_j^+\}_j, \quad \sum E(\mu_i^-) \geq \sum E(\mu_i^+)$
- $E($ configuration $)$ = $\sum E($ existing signals $)$
- Total energy quantified and bounded
- The total number of signals is bounded

All energies equal
$\rightsquigarrow$ the number of signals is preserved by a collision

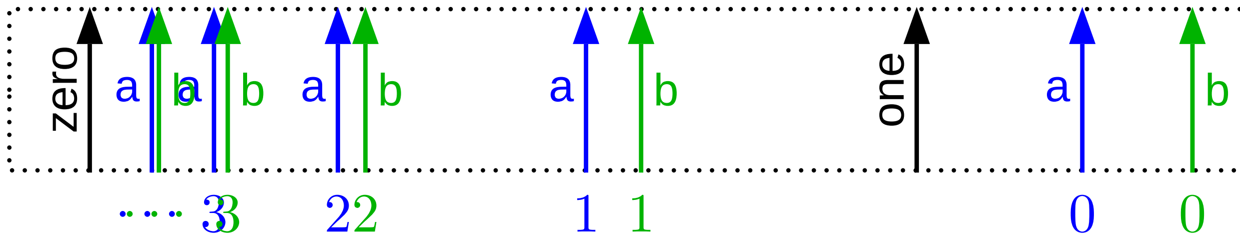
Turing-computing power

Simulating 2-counter automata

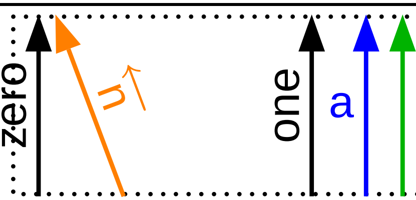
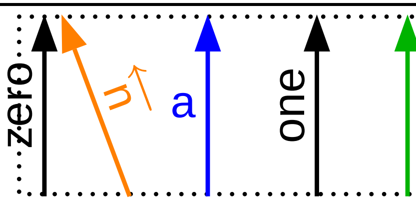
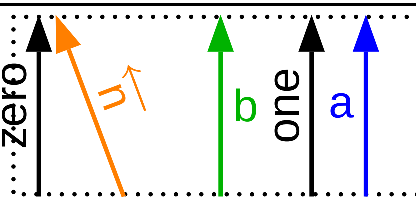
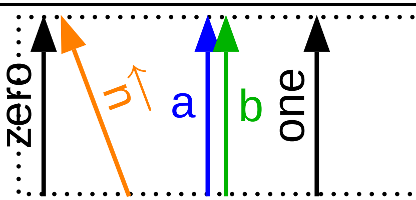
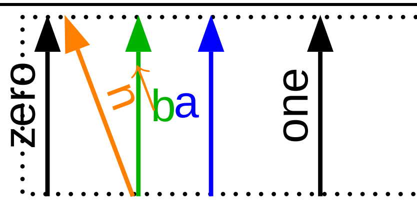
```
  be: B++
      A--
      IF A!=0 be1
      IF B!=0 im
 be1: A--
      IF A!=0 be
  pa: B--
      A++
      IF B!=0 pa
      IF A!=0 be
  im: B--
      A++
      A++
      IF B!=0 im1
      IF A!=0 be
 im1: B--
      A++
      A++
      A++
      IF B!=0 im1
      IF A!=0 be
```

2 non-negative counters $\times$ 3 operations

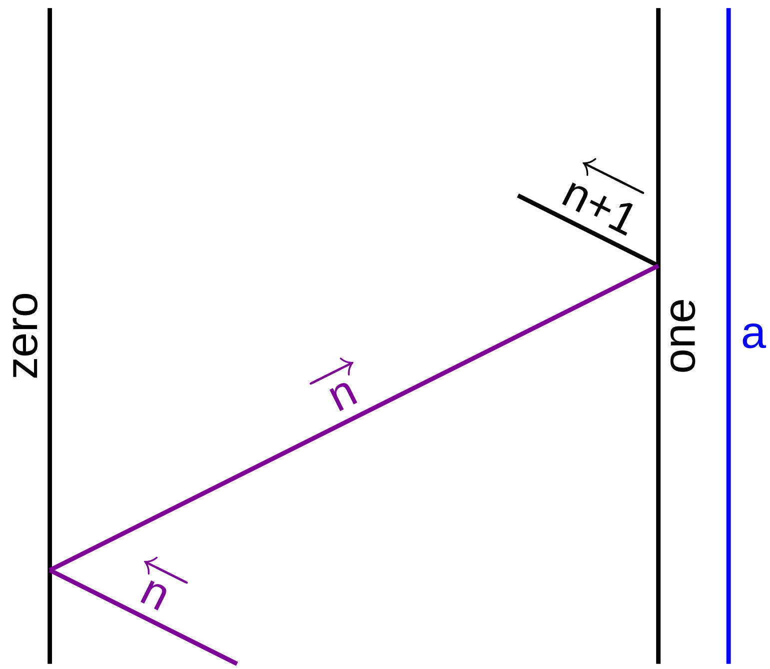
Encoding positions of counters



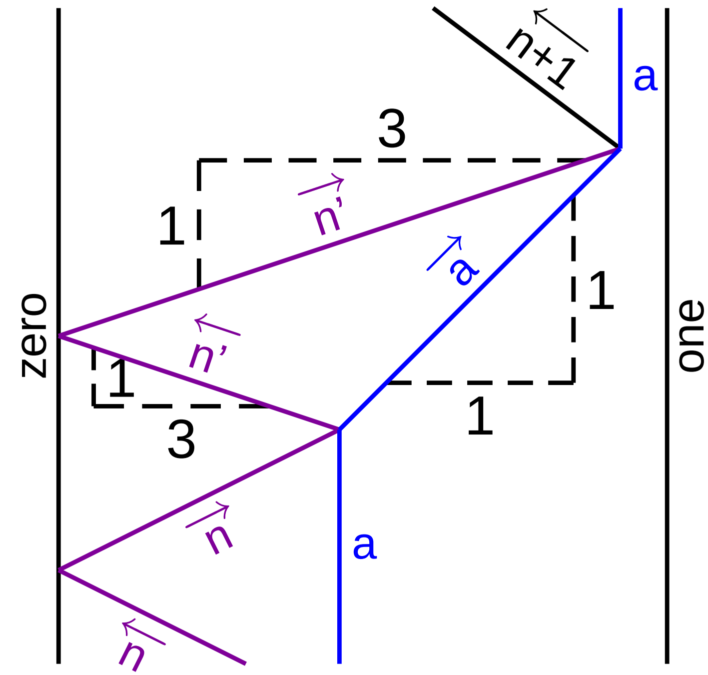
Encoding of configurations

	$a = 0$	$0 < a$
$b=0$		
$0<b$		 

Implementation of A- -



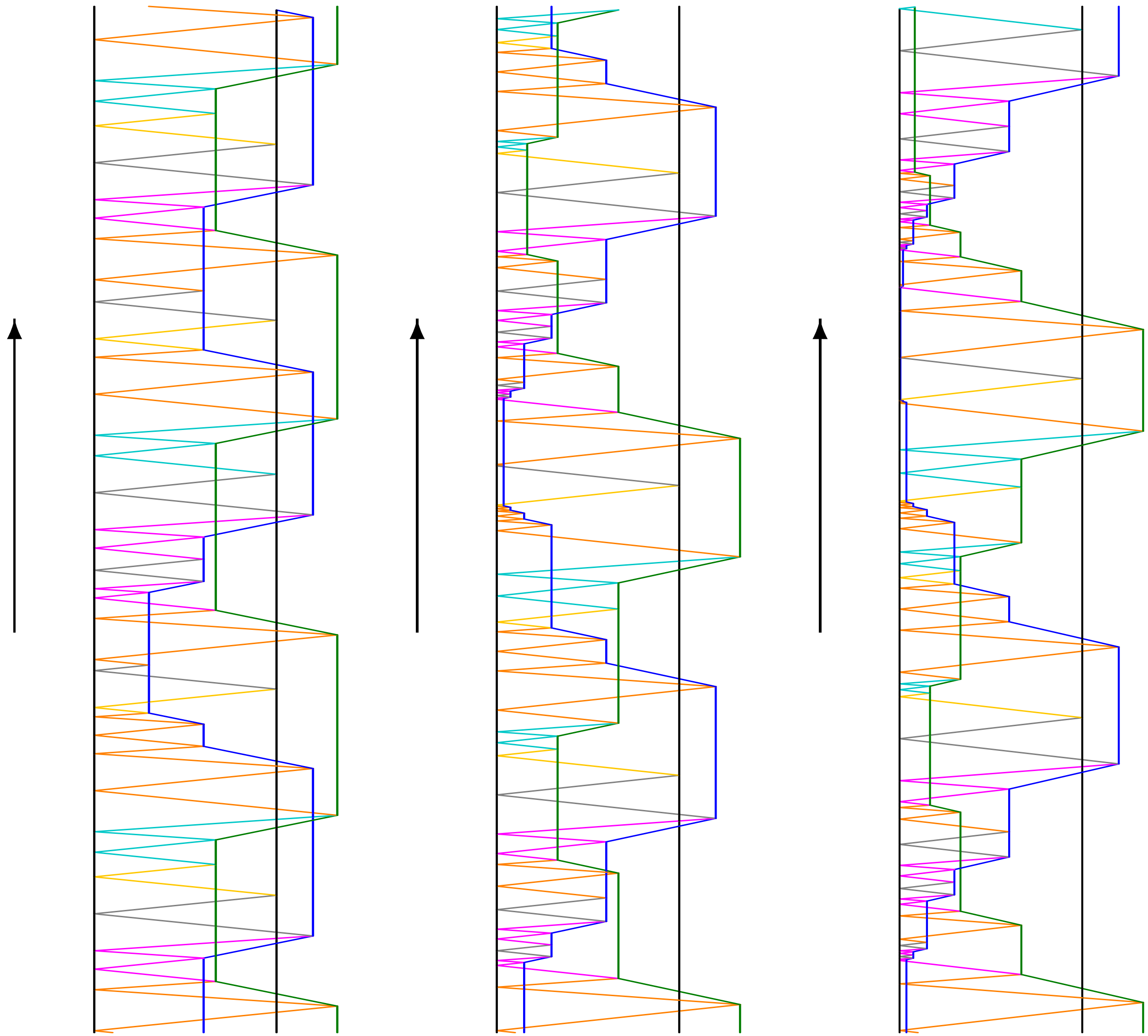
$a = 0$



$0 < a$

Some Examples

```
 be: B++
     A--
     IF A!=0 be1
     IF B!=0 im
be1: A--
     IF A!=0 be
 pa: B--
     A++
     IF B!=0 pa
     IF A!=0 be
 im: B--
     A++
     A++
     IF B!=0 im1
     IF A!=0 be
im1: B--
     A++
     A++
     A++
     IF B!=0 im1
     IF A!=0 be
```



$a{=}1 \quad b{=}0$ $\qquad$ $a{=}3 \quad b{=}0$ $\qquad$ $a{=}5 \quad b{=}0$

Handling the halt

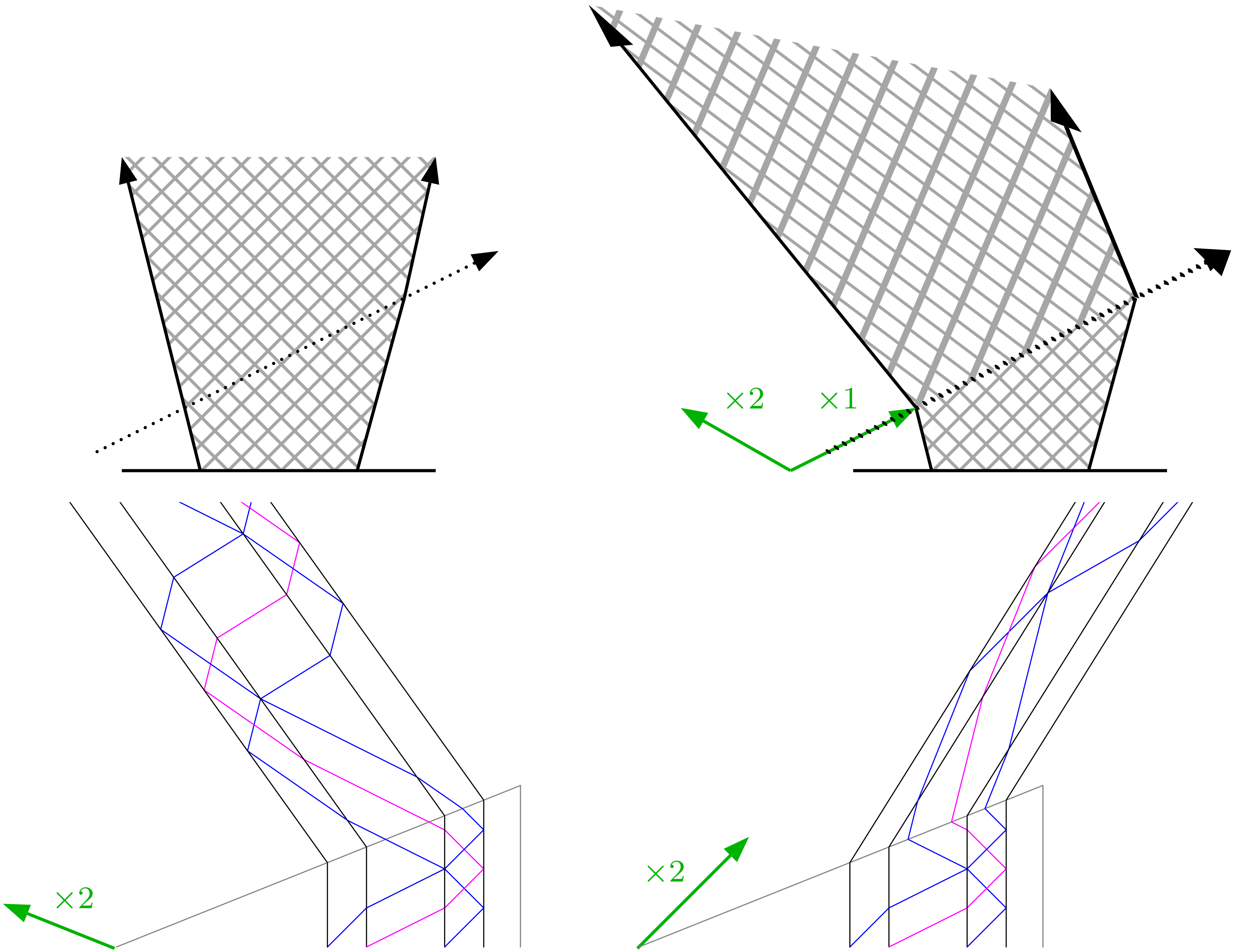
Restriction is always satisfied but. . .

what about halting?

The instruction turns into a `yes/no` signal leaving on the left

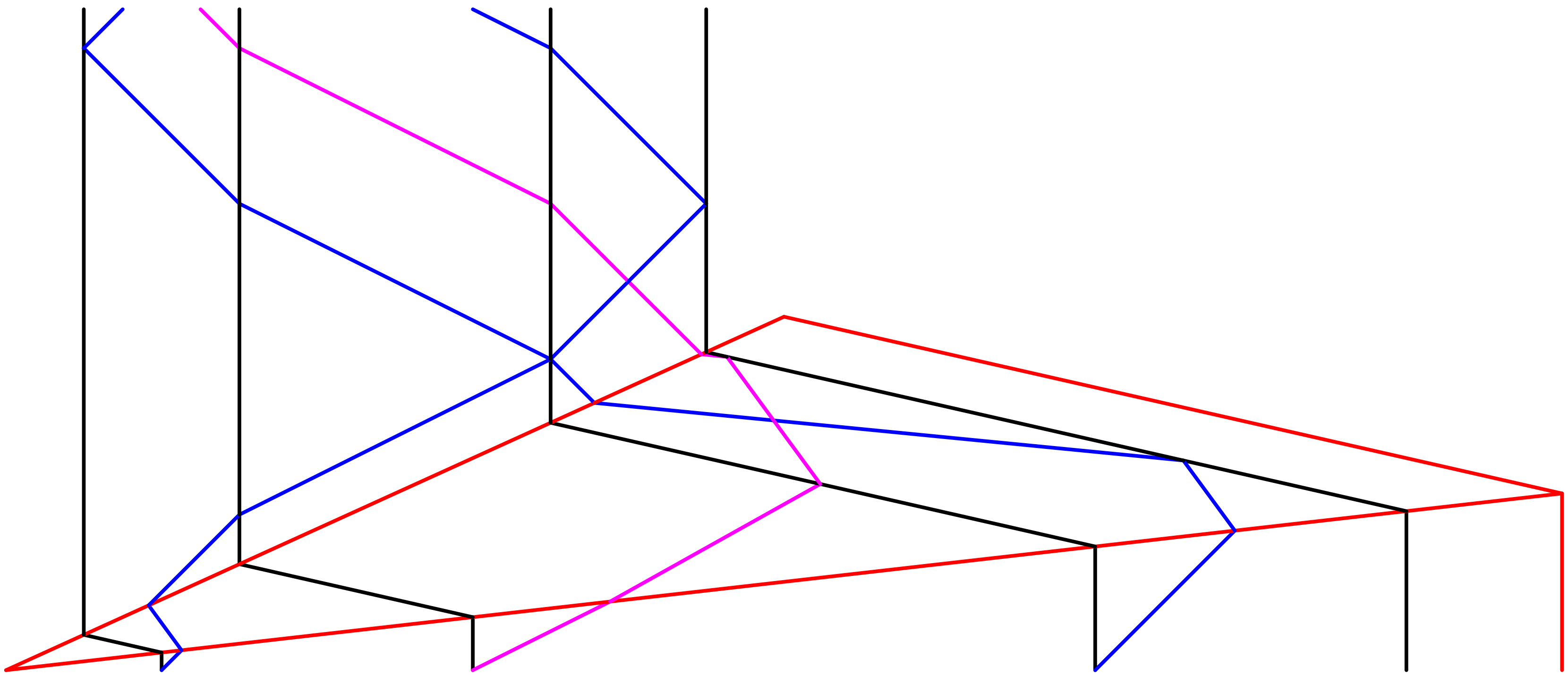
Black hole effect

Providing a strain



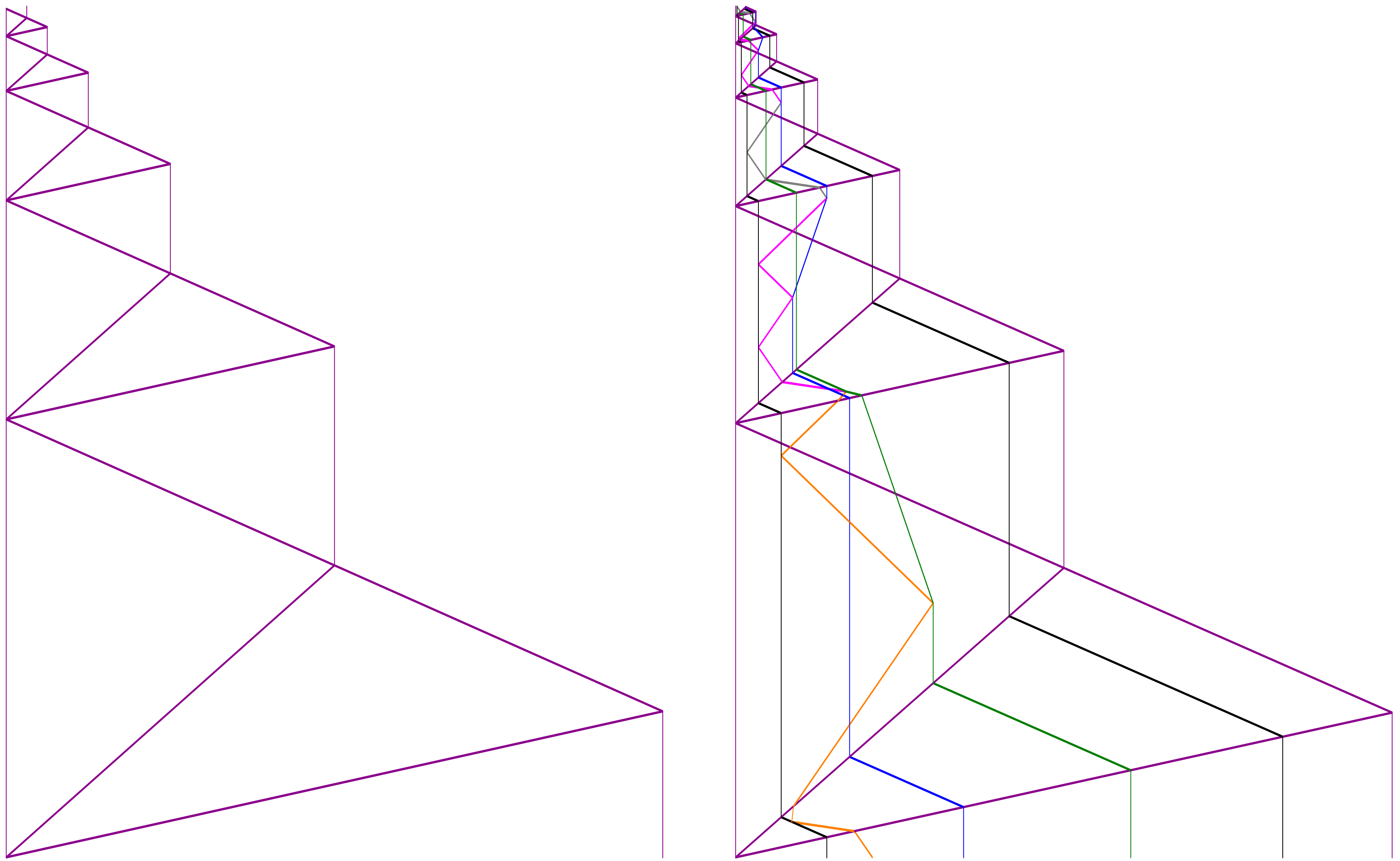
Providing a shrinking

Two consecutive strains with the same directions
coefficient $1/2$ on one direction then the other



Iterating possible if spatially bounded

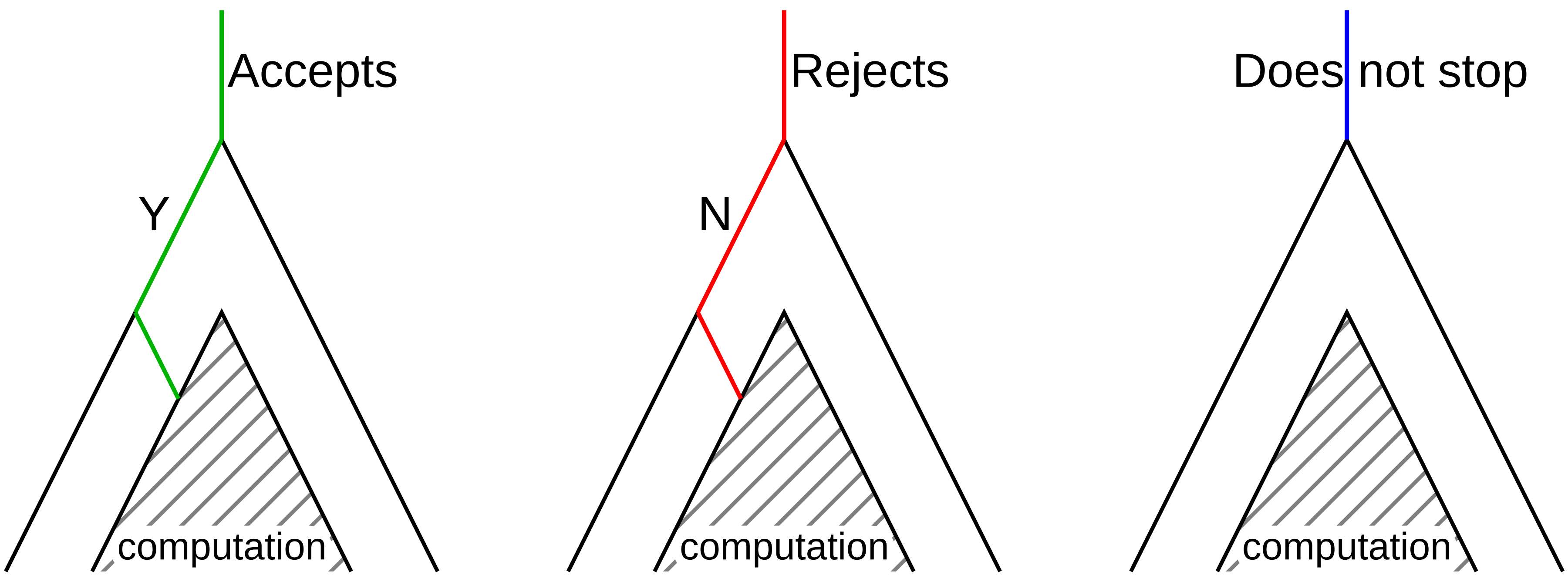
Iterating shrinking



(For a spatially bounded computation)

Bounding delay

Simulation & iterated shrinking construction satisfy the restriction



Bounding signals indicate when it is too late to get any answer

Conclusion and extension

Conclusion

- Turing computation power in a continuous space and time model
- Geometric model where geometric constructions allow Zeno effects
- Similarity with the Black hole model
- Rational numbers are enough to get all this (*i.e.* distinction lies in continuity and not in cardinality)

Extension

- Second (and higher) accumulation could be generated by lifting the restriction (hierarchy climbing)
- Real values
  - could be used as oracles
  - analog model