Reversible conservative rational abstract geometrical computation is Turing-universal

Jérôme Durand-Lose

Laboratoire d'Informatique Fondamentale d'Orléans,
Université d'Orléans, Orléans, FRANCE

CiE 2006 – Swansea, Wales UK – 30 June - 5 July, 2006

Problematics of reversibility

Interest

- Backtracking a phenomenon to its origin
- Physics is reversible at some level (*e.g.* quantum level)
- Irreversible means
  - heat to dissipate
  - energy to provide

Challenge

- Build reversible computing devices
- Compute with them

History of reversibility / inversibility

Known universal reversible. . .

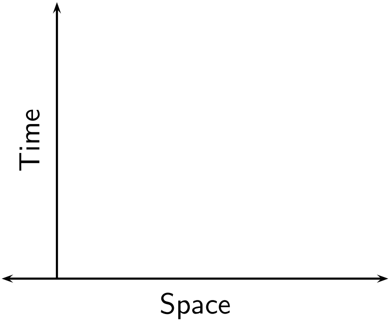
- [Lecerf, 1963, Bennett, 1973] Turing machines
- [Fredkin and Toffoli, 1982] logical gates
- [Morita, 1996] two-counter machines
- [Jacopini and Sontacchi, 1990] model on continuous space

On cellular automata

- [Toffoli, 1977] universal reversible CA for dimension 2+
- [Morita and Harao, 1989] universal dim 1 reversible CA
- [Kari, 1990] undecidability of reversibility
- [Toffoli and Margolus, 1990] survey on reversible CA

Continuous space-time

~~$\mathbb{Z} \times \mathbb{N}$~~ $\mathbb{R} \times \mathbb{R}^+$ (or $\mathbb{Q} \times \mathbb{Q}^+$)

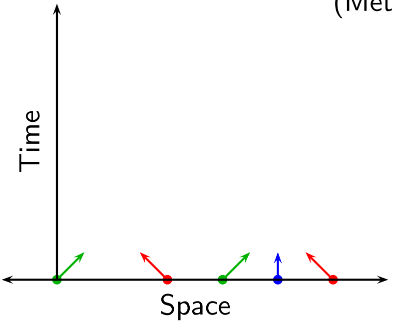


Continuous space-time, signals

~~$\mathbb{Z} \times \mathbb{N}$~~ $\mathbb{R} \times \mathbb{R}^{+}$ (or $\mathbb{Q} \times \mathbb{Q}^{+}$)

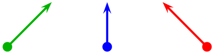
Signal

(Meta-signal, position)



Meta-signal

$\mu = (\iota, \nu)$

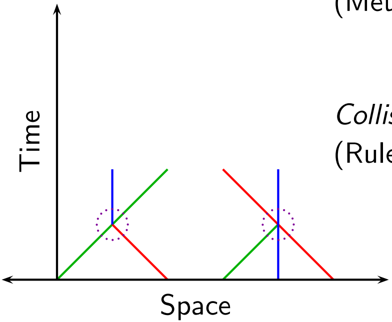


Continuous space-time, signals and collisions

~~$\mathbb{Z} \times \mathbb{N}$~~ $\mathbb{R} \times \mathbb{R}^+$ (or $\mathbb{Q} \times \mathbb{Q}^+$)

Signal

(Meta-signal, position)

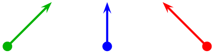


Collision

(Rule, position)

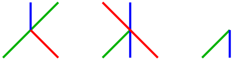
Meta-signal

$\mu = (\iota, \nu)$



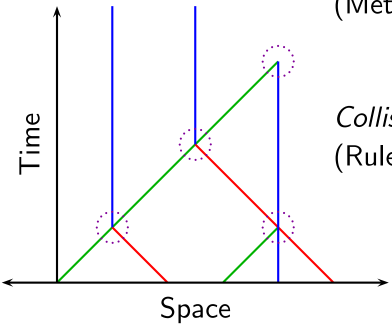
Rule

$\rho = \{\mu_i^-\}_i \rightarrow \{\mu_j^+\}_j$



Continuous space-time, signals

~~$\mathbb{Z} \times \mathbb{N}$~~ $\mathbb{R} \times \mathbb{R}^+$ (or $\mathbb{Q} \times \mathbb{Q}^+$)



Signal

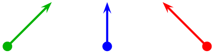
(Meta-signal, position)

Collision

(Rule, position)

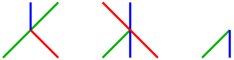
Meta-signal

$\mu = (\iota, \nu)$



Rule

$\rho = \{\mu_i^-\}_i \rightarrow \{\mu_j^+\}_j$



Reversible signal machines

Going backward possible and unique

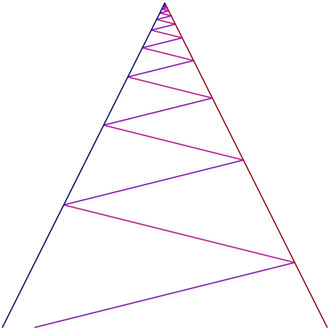
- away from any collision... automatic
- at a collision...
  - impossible to guess if no or just one signal has left (reverse of *at least 2 signals for a collision*)

Necessary and sufficient condition

Rules form a bijection over sets of 2 or more signals

- easy to check
- easy to complete any 1-to-1 set of rules

Link with the black hole model of computation

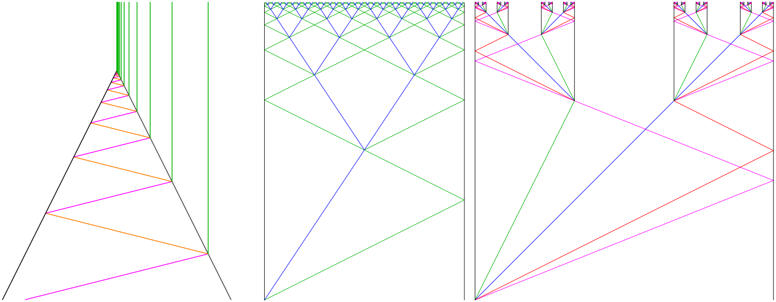


Zeno's paradox

- Infinitely many steps in finite time
- Infinite computation in finite time?
- Halting problem solvable in finite time [Durand-Lose, 2005]

(infinite-time Turing machine,
computation on ordinals. . .)

Unwanted diagrams



Unwanted because

- The number of signals is bursting to infinity
  (free creation of mater/energy)
- Difficulty (if not impossibility) to define continuation there

Conservativeness condition on rules (not presented here)

What is known

- Can compute any recursive function
  with conservativeness constrain
  [Durand-Lose, 2005, MCU]

  together with reversibility constrain. . . *this talk !*

- Forecasting accumulation is Σ_2^0-complete (arithmetical hier.)
  (even with conservativeness constrain)
  [Durand-Lose, 2006, TAMC]

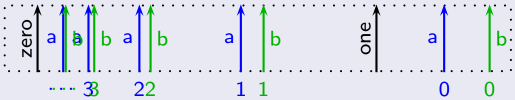
Simulating 2-counter automata

2 non-negative counters
× 3 operations

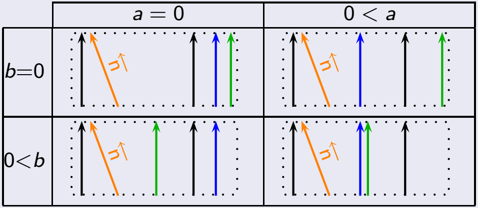
Code

```
        A != 0  notZ
        A++
glob    B != 0  loop
        A != 0  fin
loop    B--
        A++
        A != 0  glob
notZ    A--
        B++
fin     stop
```

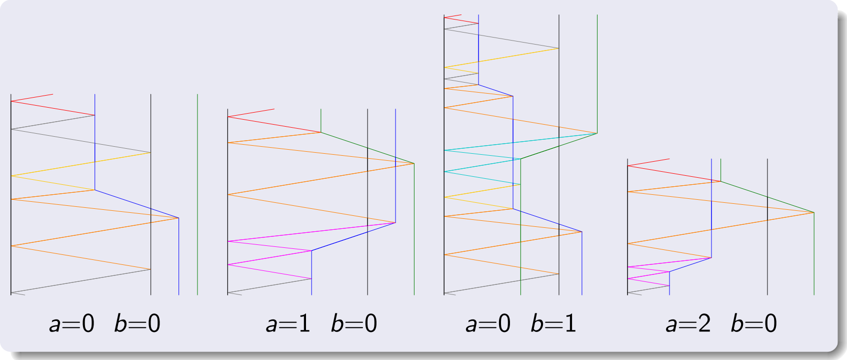
Encoding positions of counters



Encoding of configurations



Two-counter automata simulations



Two-counter automata + stack for reversibility

Two-counter automata intrinsically irreversible

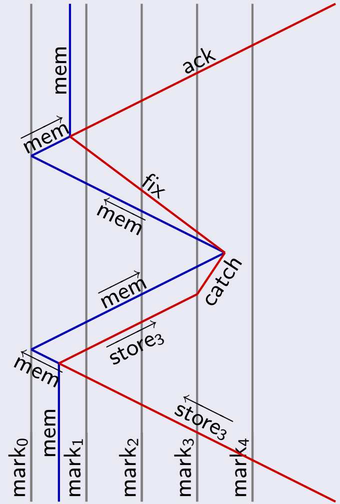
- **Previous counter value** $0{-}{-} = 0 = 1{-}{-}$
- **Previous line number** conditional jumps

Memory trick

- Record the previous values
  - of a counter reaching 0
  - of the line number
- Use a **stack**
  - push onward
  - pop backward
- (Invertible as long as the stack is not empty)

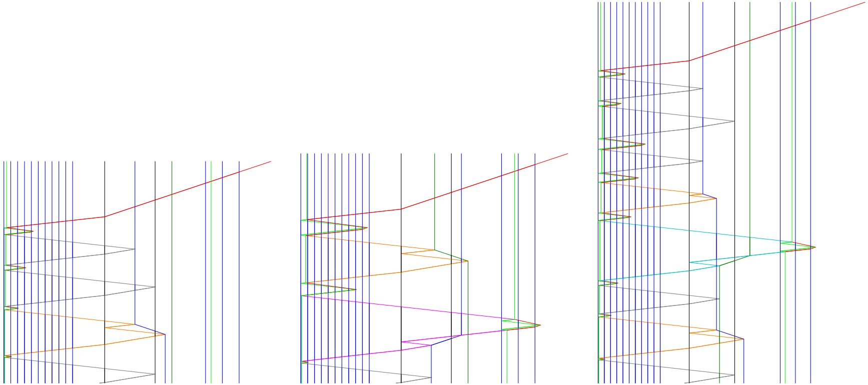
Reversible stack implementation

Fixed alphabet/number of possible values



Rules
$\{\text{mem}, \overleftarrow{\text{store}_v}\} \rightarrow \{\overleftarrow{\text{mem}}, \overrightarrow{\text{store}_v}\}$
$\{\text{mark}_0, \overleftarrow{\text{mem}}\} \rightarrow \{\text{mark}_0, \overrightarrow{\text{mem}}\}$
$\{\overrightarrow{\text{store}_v}, \text{mark}_v\} \rightarrow \{\text{mark}_v, \text{catch}\}$
$\{\overrightarrow{\text{mem}}, \text{catch}\} \rightarrow \{\overleftarrow{\text{mem}}, \text{fix}\}$
$\{\overrightarrow{\text{mem}}, \text{fix}\} \rightarrow \{\text{mem}, \text{ack}\}$
$\{\text{mark}_i, \overleftarrow{\text{store}_v}\} \rightarrow \{\overleftarrow{\text{store}_v}, \text{mark}_i\}$
$v < i,\ \{\overrightarrow{\text{store}_v}, \text{mark}_i\} \rightarrow \{\text{mark}_i, \overrightarrow{\text{store}_v}\}$
$\{\overrightarrow{\text{mem}}, \text{mark}_i\} \rightarrow \{\text{mark}_i, \overrightarrow{\text{mem}}\}$
$\{\text{mark}_i, \overleftarrow{\text{mem}}\} \rightarrow \{\overleftarrow{\text{mem}}, \text{mark}_i\}$
$\{\text{mark}_i, \text{fix}\} \rightarrow \{\text{fix}, \text{mark}_i\}$
$\{\text{ack}, \text{mark}_i\} \rightarrow \{\text{mark}_i, \text{ack}\}$

All together



a=0 b=0 a=1 b=0 a=0 b=1

Results

Theorem

Rational reversible conservative signal machine can compute any recursive function

Theorem

As long as there are finitely many signals and no accumulation they can be simulated by Turing machines

Black hole effect is still available