Forecasting black holes in Abstract geometrical computation is highly unpredictable

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TAMC, Beijing, 2006

Discrete lines in cellular automata

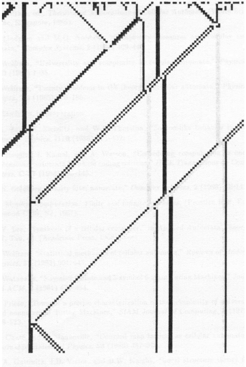


Figure 4: The $k = 4$, $r = 2$ universal cellular automaton of table 4 simulated starting from a random initial state. The symbols 0, 1, ␣, and + are represented by

[Lindgren and Nordahl, 1990, Fig. 3]

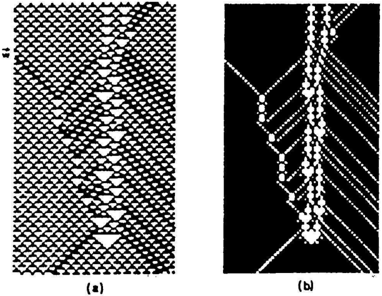


FIG. 7. Rule 54. (a) Annihilation of the radiating particle. (b) The same as (a) with the mapping defined in Fig. 6.

[Boccara et al., 1991, Fig. 7]

Discrete lines in cellular automata

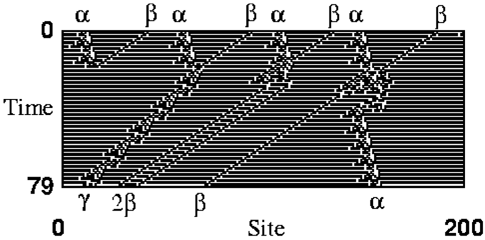


FIG. 7. The four different (out of 14 possible) interaction products for the $\alpha + \beta$ interaction.

[Hordijk et al., 2001, Fig. 7]

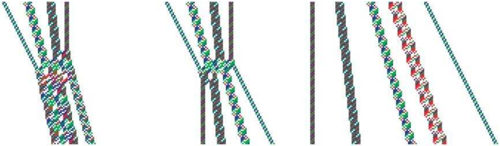
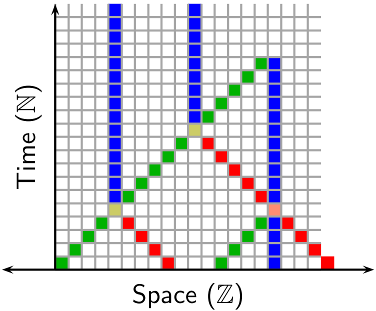
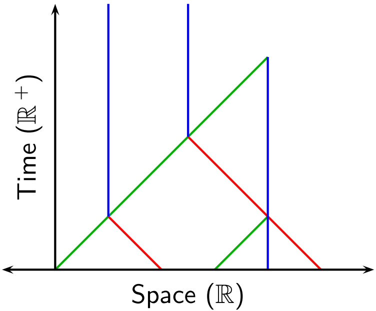


Figure 5. Two collisions of filtrons, and five free filtrons supported by the FPS model; ST diagram applies $q = 1$.

[Siwak, 2001, Fig. 5]

Abstracting signal machines

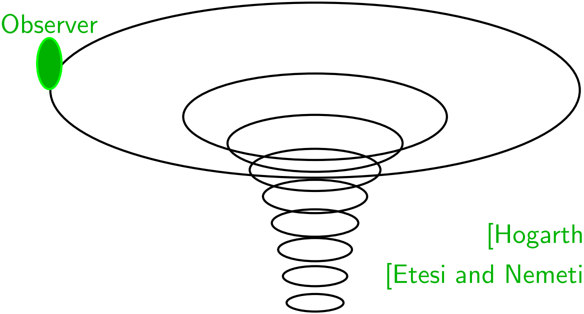




Accumulating is now possible

- Infinitely many steps in finite time
- Infinite computation in finite time?

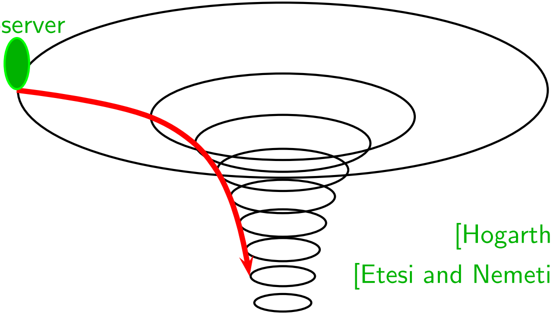
Black hole model



[Hogarth, 2000]
[Etesi and Nemeti, 2002]

1. Observer at the “edge”

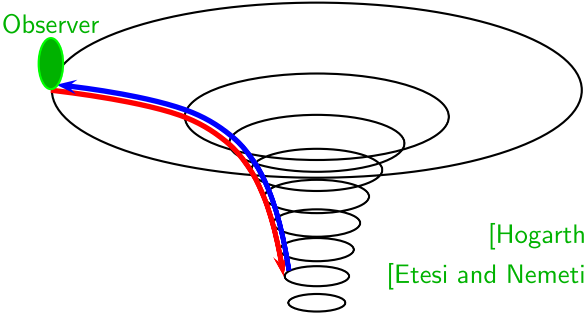
Black hole model



[Hogarth, 2000]
[Etesi and Nemeti, 2002]

1. Observer at the “edge”
2. Machine sent into the black hole *infinitely accelerated*

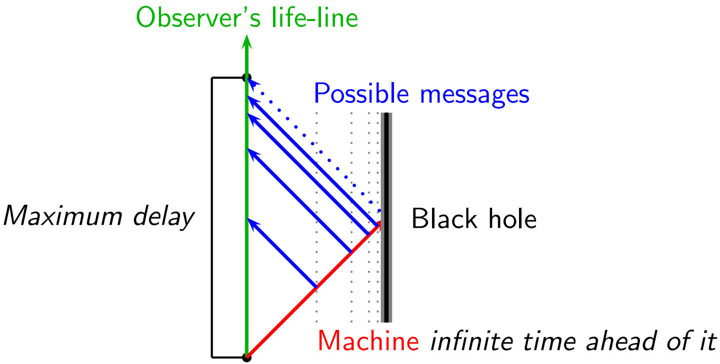
Black hole model



[Hogarth, 2000]
[Etesi and Nemeti, 2002]

1. Observer at the "edge"
2. Machine sent into the black hole *infinitely accelerated*
3. Message sent by the machine received by the observer *within a bounded delay*

Malament-Hogarth space-time



Message indicates the result of the computation

After the delay, the observer knows whether the computation stops

Any recursively enumerable problem can be decided!

In the abstract geometrical computation context. . .

- Can accumulations be used as black hole?

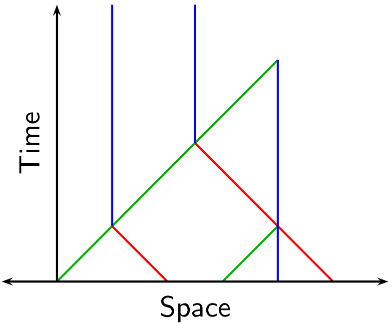
  YES! [Durand-Lose, 2005]

- Are accumulations predictable?

  this communication. . .

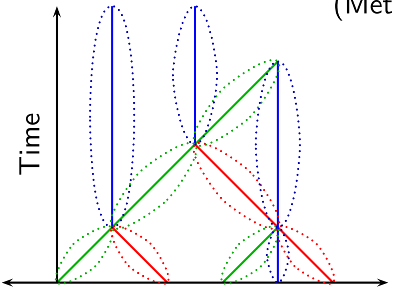
Continuous space-time

~~$\mathbb{Z} \times \mathbb{N}$~~ $\mathbb{R} \times \mathbb{R}^+$ (or $\mathbb{Q} \times \mathbb{Q}^+$)



Continuous space-time, signals

~~$\mathbb{Z} \times \mathbb{N}$~~ $\mathbb{R} \times \mathbb{R}^+$ (or $\mathbb{Q} \times \mathbb{Q}^+$)



Signal

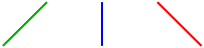
(Meta-signal, position)

Position

(x, t)

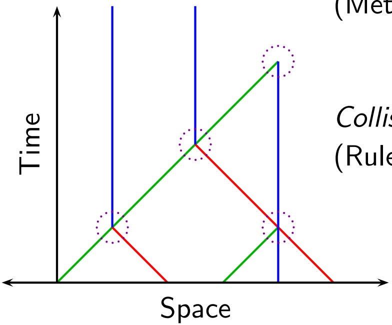
Meta-signal

$\mu = (\iota, \nu)$



Continuous space-time, signals and collisions

~~$\mathbb{Z} \times \mathbb{N}$~~ $\mathbb{R} \times \mathbb{R}^+$ (or $\mathbb{Q} \times \mathbb{Q}^+$)



Signal

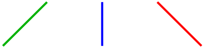
(Meta-signal, position)

Position

(x, t)

Meta-signal

$\mu = (\iota, \nu)$

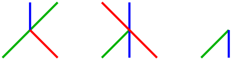


Collision

(Rule, position)

Rule

$\rho = \{\mu_i^-\}_i \rightarrow \{\mu_j^+\}_j$



Problem definition

Rational signal machine

All speeds and initial positions are rational numbers
$\rightsquigarrow$ computations remain in $\mathbb{Q}$
$\rightsquigarrow$ exact encoding and manipulation in classical theory

AGC-accumulation-Forecasting

Instance

$\mathcal{M}$: rational signal machine, and
c: (rational) configuration for $\mathcal{M}$.

Question

Is there any accumulation in the space-time generated by $\mathcal{M}$ from c?

Arithmetical hierarchy

Hierarchy of *undecidable* problems

Logical definition

by an alternation of $\forall$ and $\exists$ on $\mathbb{N}$
quantifying a recursive total predicate

Examples

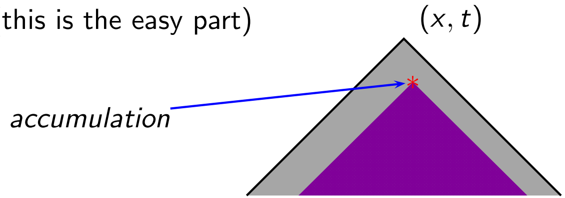
- Σ_0^0 corresponds to recursive sets
- Σ_1^0 corresponds to recursively enumerable sets
- Σ_2^0 corresponds to sets definable by

$$\{\ x \mid \exists n_1, \forall n_2,\ \ \phi(x, n_1, n_2)\ \}$$

  where ϕ is total and recursive

Σ_2^0-membership

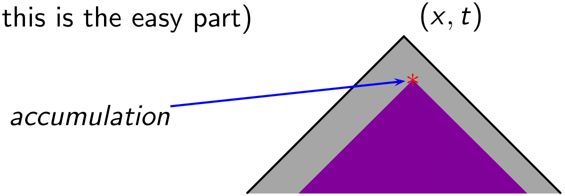
(this is the easy part)



$\exists(x,t) \in \mathbb{Z} \times \mathbb{N},\ \forall n \in \mathbb{N},$ There is at least n collisions in the light cone ending in (x,t)

Σ_2^0-membership

(this is the easy part)



recursive total predicate

$\exists (x,t) \in \mathbb{Z} \times \mathbb{N},\ \forall n \in \mathbb{N},$ There is at least n collisions in the light cone ending in (x,t)

$\rightsquigarrow$ in Σ_2^0

Problems to be reduced

Σ_1^0-complete

The Halting problem

(M, x) *s.t.* $\exists n$, Turing machine M stops on x in n iterations

Σ_2^0-complete

Non total recursive function

(M) *s.t.* $\exists x, \forall n$, TM M does not stop on x in n iterations

Turing equivalent model

Turing machine can be replaced by *2-counter automaton*

2-counter automata

```
 beg: B++
      A--
      IF A != 0 beg1
      IF B != 0 imp
beg1: A--
      IF A != 0 beg
pair: B--
      A++
      IF B != 0 pair
      IF A != 0 beg
 imp: B--
      A++
      A++
      IF B != 0 imp1
      IF A != 0 beg
imp1: B--
      A++
      A++
      A++
      IF B != 0 imp1
      IF A != 0 beg
```

Turing-universal

A, B counters (values in $\mathbb{N}$)

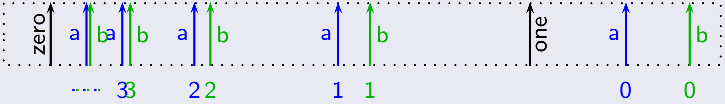
Operations

A++	B++
A--	B--
A != 0 m	B != 0 m

a configuration $\rightsquigarrow$ (n, a, b)

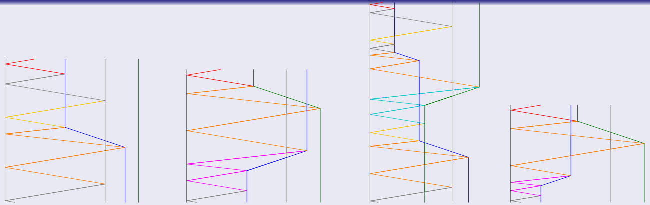
Encoding and simulation

Encoding with positions



Example

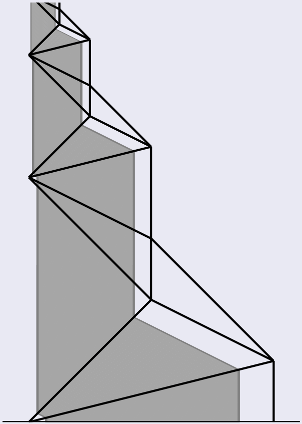
```
      A != 0  notZ
      A++
glob  B != 0  loop
      A != 0  fin
loop  B--
      A++
      A != 0  glob
notZ  A--
      B++
fin   stop
```



$a{=}0 \quad b{=}0$ $a{=}1 \quad b{=}0$ $a{=}0 \quad b{=}1$ $a{=}2 \quad b{=}0$

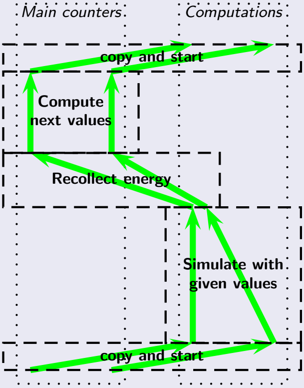
co-Σ_1^0-Hardness

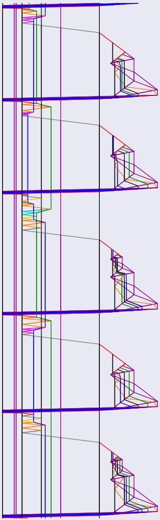
Embedding inside an accumulating structure



- The structure iteratively shrinks the computation
- An accumulation is produced
- Rules are modified so that: computation stops
  $\Rightarrow$ structure is stopped
- This reduction shows the co-Σ_1^0-Hardness

Σ_2^0-Hardness: Try all entries for not total





Results

Theorem

Forecasting an accumulation is Σ_2^0*-complete*