Abstract geometrical computation: small Turing universal signal machines

Jérôme Durand-Lose

Laboratoire d'Informatique Fondamentale d'Orléans,
Université d'Orléans, Orléans, FRANCE

IW Complexity of Simple Programs
December 6-7, 2008 – Cork, Irland

Context

Collision based computing

Idealization

- continuous space
- continuous time
- dimensionless particles/signals

Abstract geometrical computation

Signal machines

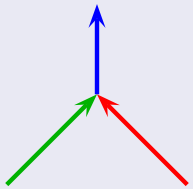
- meta-signals (finitely many)
- their speed/velocity
- collision rules

Signals, e.g.

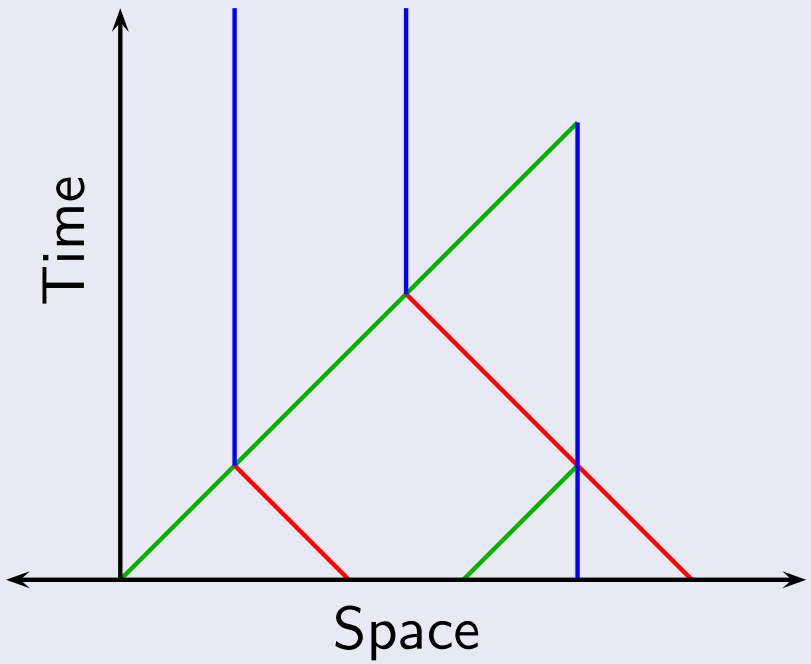
- `red` (with speed 1) at position xx
- `blue` (with speed -1) at position yy

Collision, e.g.

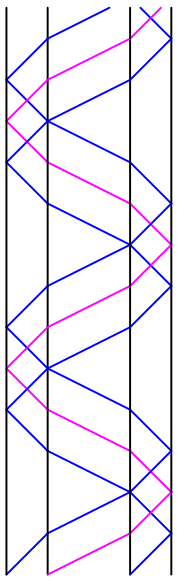
- rule $\{\texttt{green}, \texttt{red}\} \longrightarrow \{\texttt{blue}\}$
- application

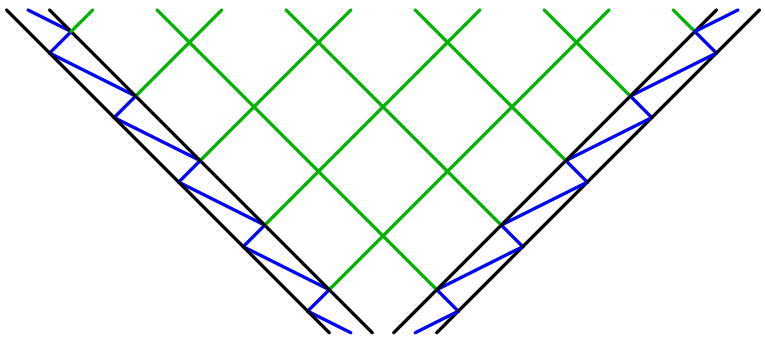


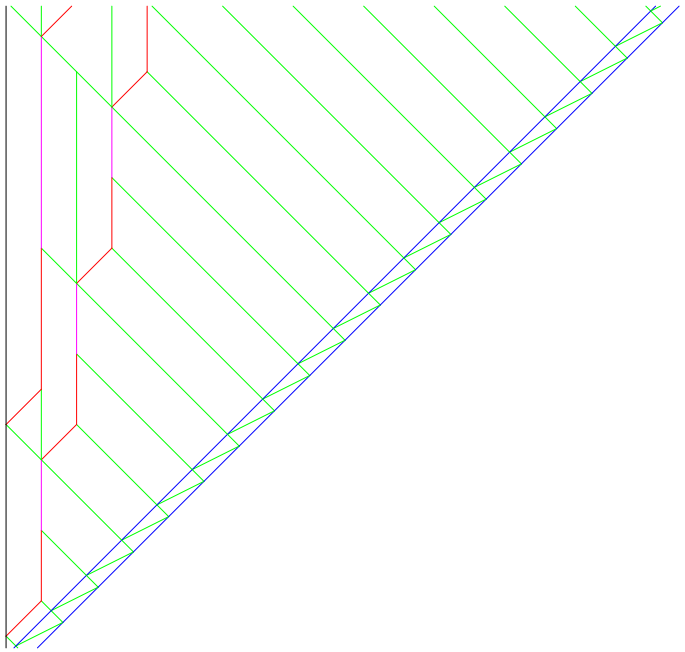
An example



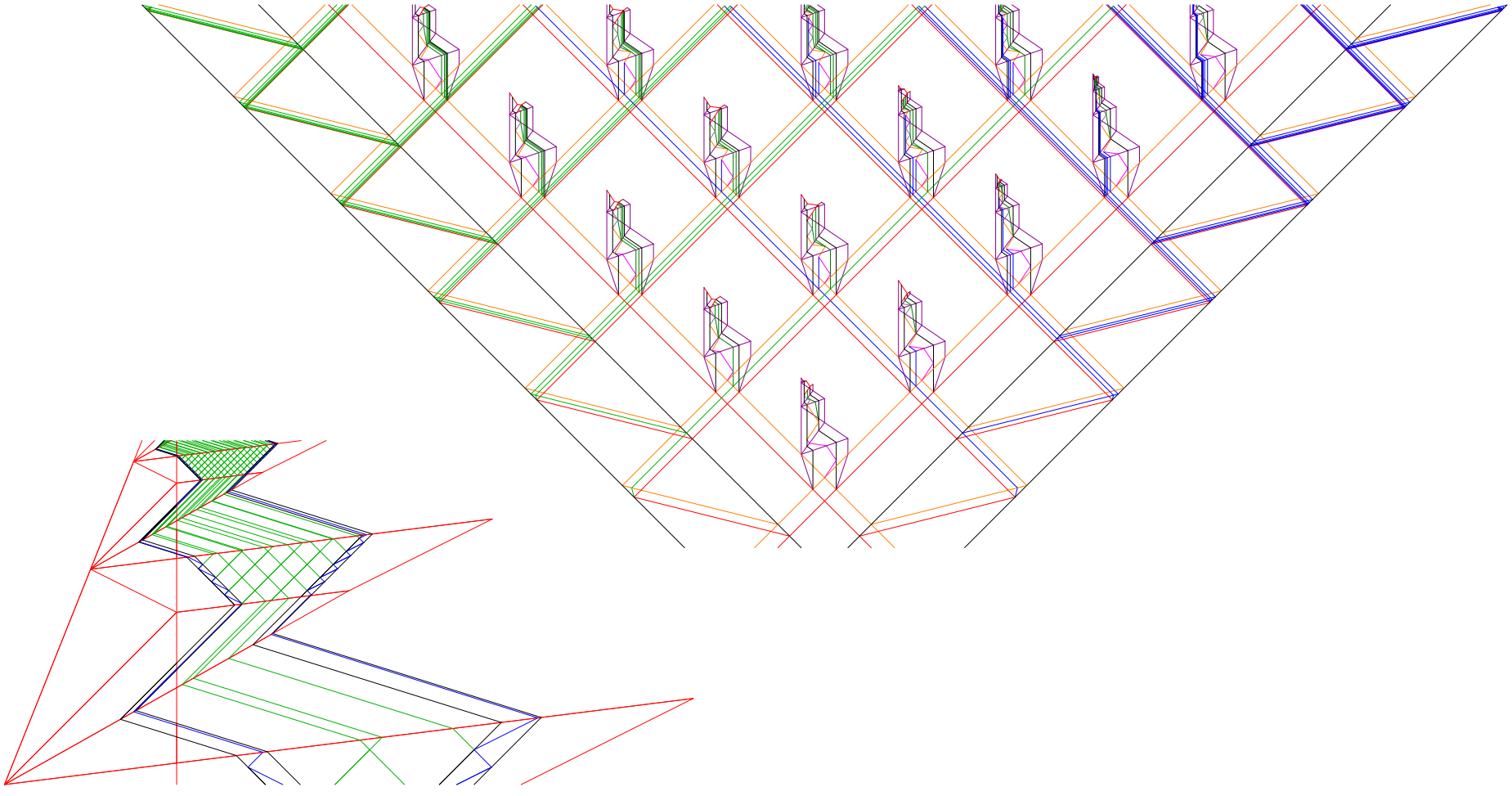
More examples



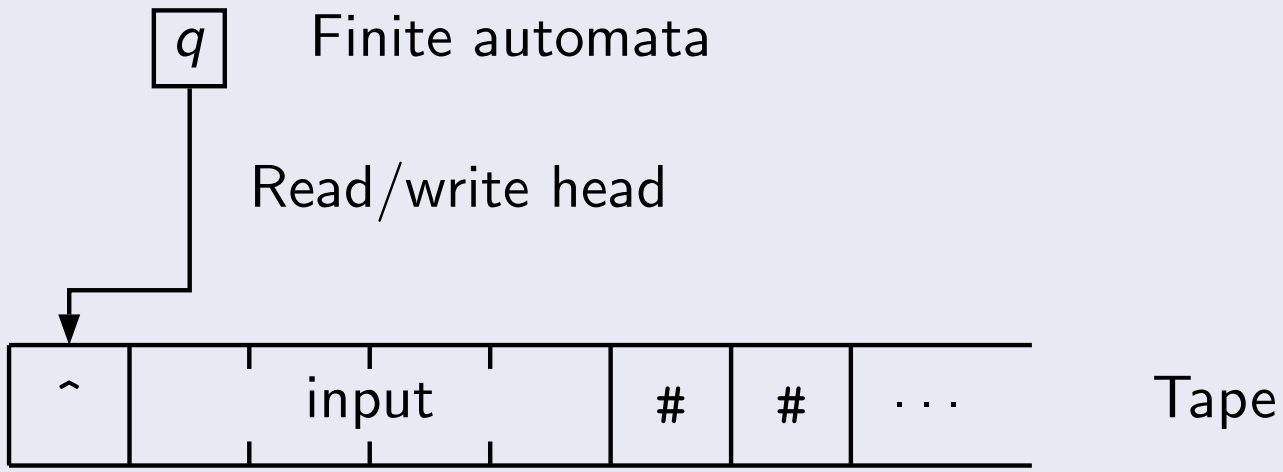




More complex examples

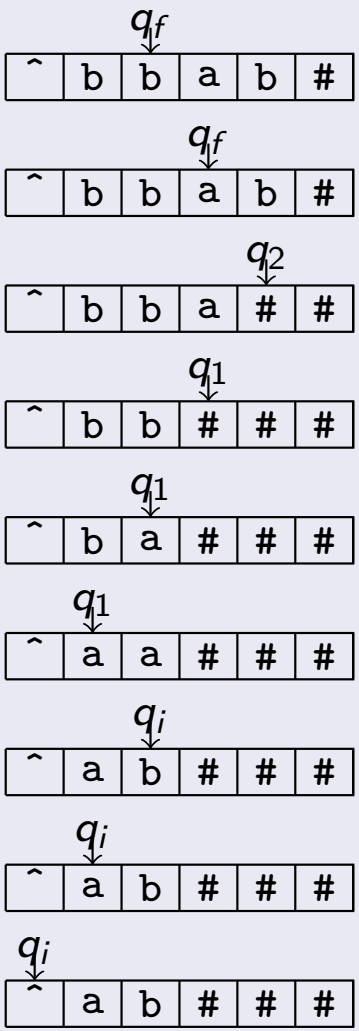


Turing machines?

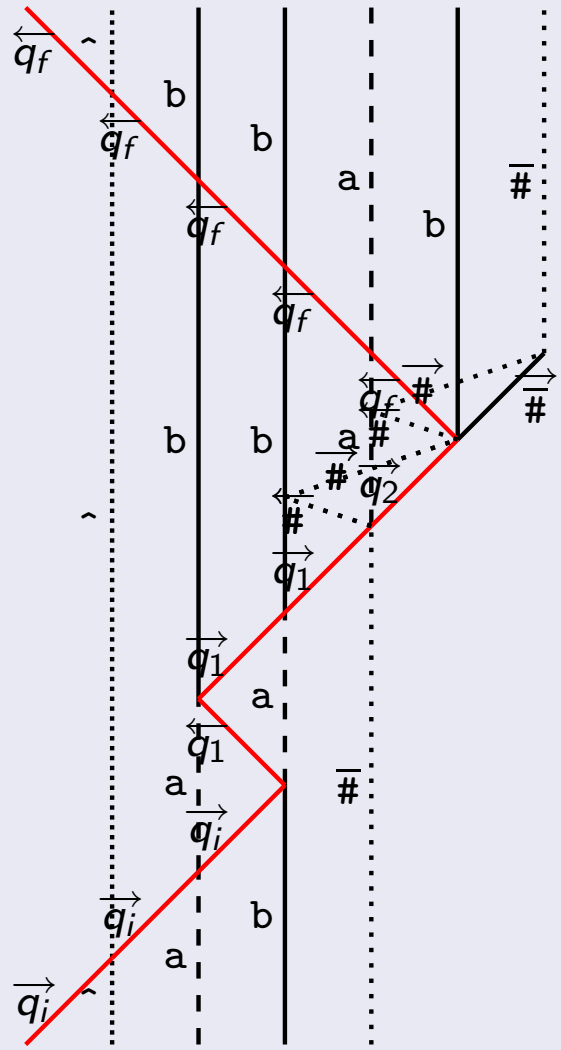


Simulation

Iterations of a Turing machine



Corresponding signal machine



How many meta-signals?

		Meta-signal
1 symbol	$\rightsquigarrow$	1
1 state	$\rightsquigarrow$	2
for finitness		
$\overline{\#}$, $\overleftarrow{\#}$, $\overrightarrow{\#}$, $\overrightarrow{\overline{\#}}$	$\rightsquigarrow$	4

$$|\Gamma| + 2|Q| + 4$$

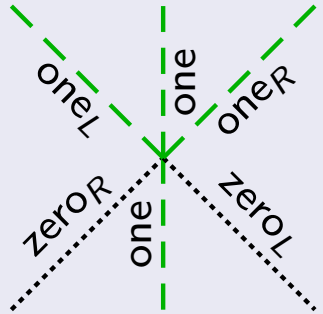
Results

universal	**semi-universal**
18 (Woods and Neary, 2007)	**7** (Smith, 2007)

Cellular automata

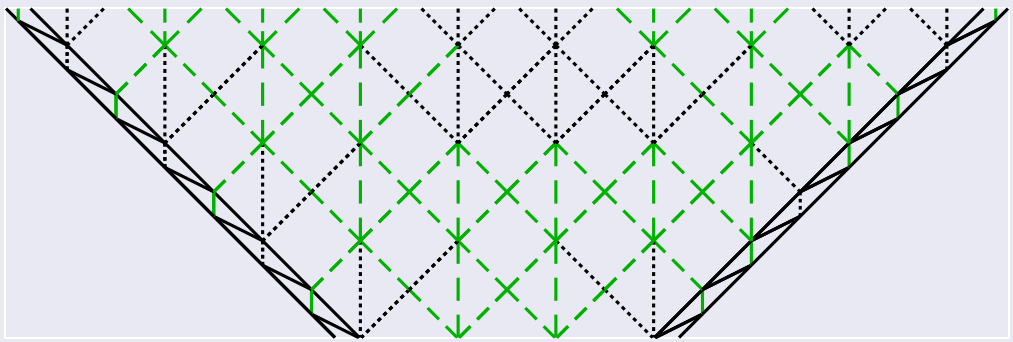
Rule 110 and one transition implementation

Output	0	1	1	0	1	**1**	1	0
Input	111	110	101	100	011	**010**	001	000



Evolution and simulation on 11 framed by $^{\omega}(10)$ and $(011)^{\omega}$

01110011**0**0
1011000**11**
101111**10**
10*11***01**



How many meta-signals?

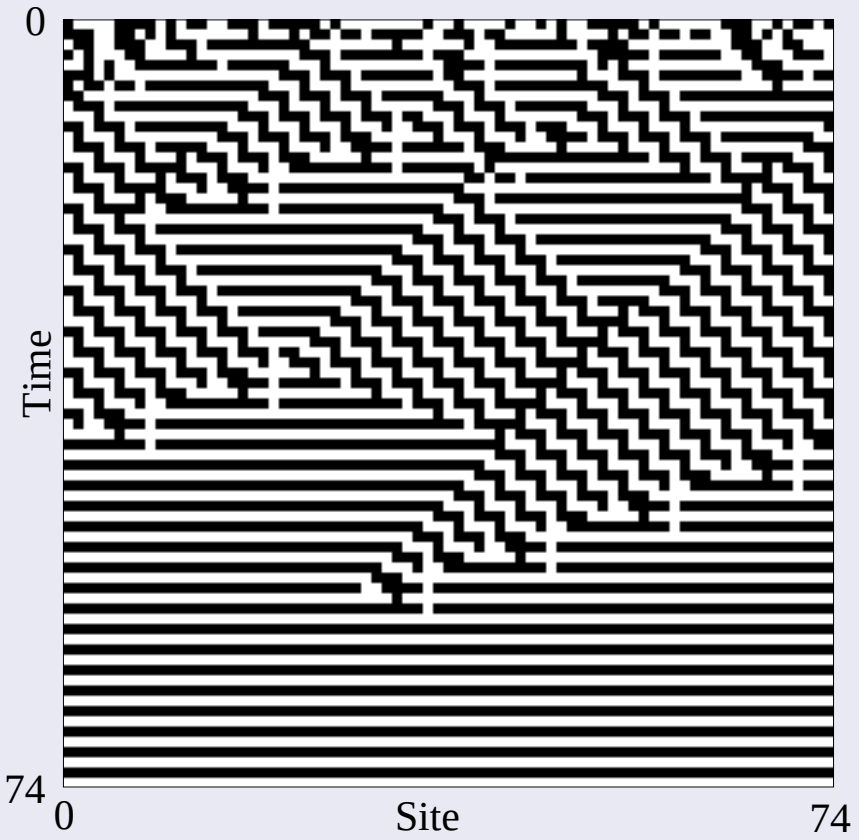
		Meta-signal
1 state	$\rightsquigarrow$	3
for finitness		
Regular pattern on both side	$\rightsquigarrow$	expensive

$$3|Q|+???$$

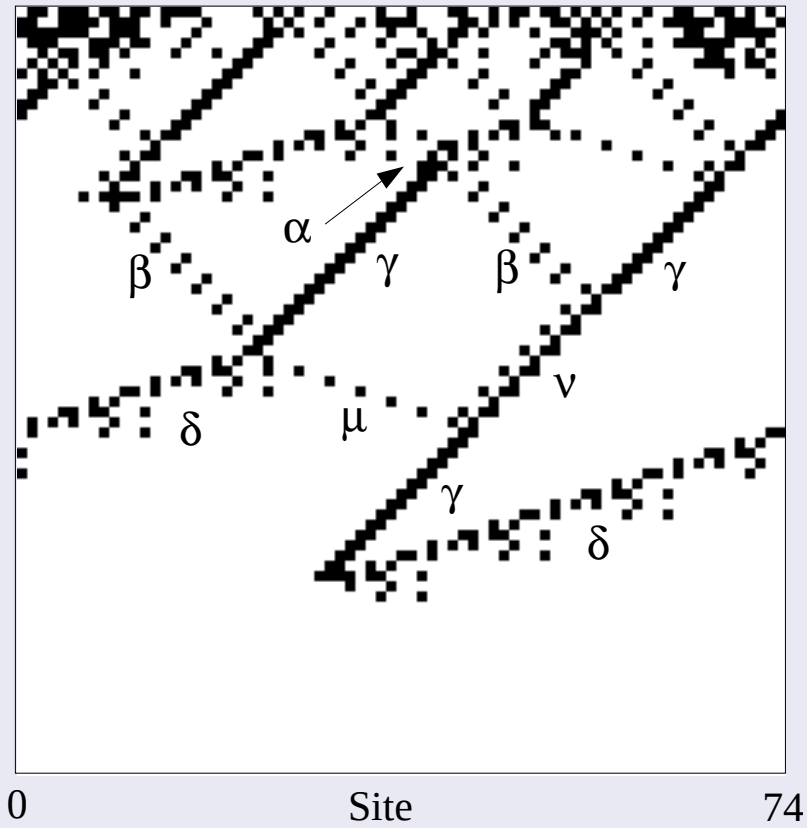
Results

universal	**semi-universal**
not interesting	**6** (Cook, 2004)

Link CA-ACG



(a) Space-time diagram.



(b) Filtered space-time diagram.

Cyclic tag system?

Definition

- a binary word
- a circular list

Dynamics

101
011 :: h :: 0110 :: 01011

Halt

- empty word
- halt appendant (here h)
- cycle (too expensive to test)

Cyclic tag system?

Definition

- a binary word
- a circular list

Dynamics

~~1~~01**011**
~~011~~ :: h :: 0110 :: 01011 :: 011

Halt

- empty word
- halt appendant (here h)
- cycle (too expensive to test)

Cyclic tag system?

Definition

- a binary word
- a circular list

Dynamics

~~10~~1011
~~011~~ :: ~~h~~ :: 0110 :: 01011 :: 011 :: h

Halt

- empty word
- halt appendant (here h)
- cycle (too expensive to test)

Cyclic tag system?

Definition

- a binary word
- a circular list

Dynamics

~~101~~011**0110**

~~011~~ :: ~~h~~ :: ~~0110~~ :: 01011 :: 011 :: h :: 0110

Halt

- empty word
- halt appendant (here h)
- cycle (too expensive to test)

Cyclic tag system?

Definition

- a binary word
- a circular list

Dynamics

~~1010~~110110
~~011~~ :: ~~h~~ :: ~~0110~~ :: ~~01011~~ :: 011 :: h :: 0110 :: 01011

Halt

- empty word
- halt appendant (here h)
- cycle (too expensive to test)

Cyclic tag system?

Definition

- a binary word
- a circular list

Dynamics

~~1010~~10110**011**

~~011~~ :: ~~h~~ :: ~~0110~~ :: ~~01011~~ :: ~~011~~ :: h :: 0110 :: 01011:: 011

Halt

- empty word
- halt appendant (here h)
- cycle (too expensive to test)

Cyclic tag system?

Definition

- a binary word
- a circular list

Dynamics

~~101011~~**0110011**

~~011~~ :: ~~h~~ :: ~~0110~~ :: ~~01011~~ :: ~~011~~ :: ~~h~~ :: 0110 :: 01011:: 011 :: h

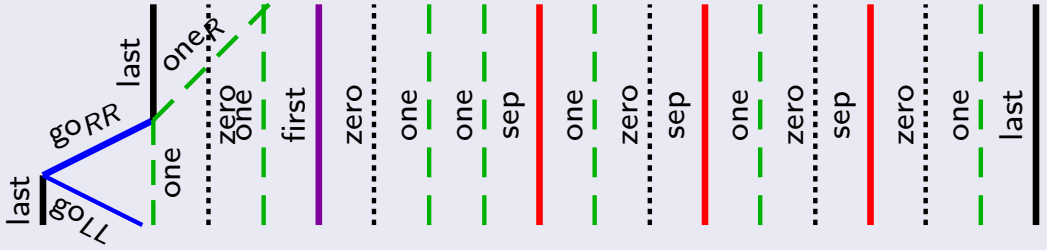
Halt

- empty word
- halt appendant (here h)
- cycle (too expensive to test)

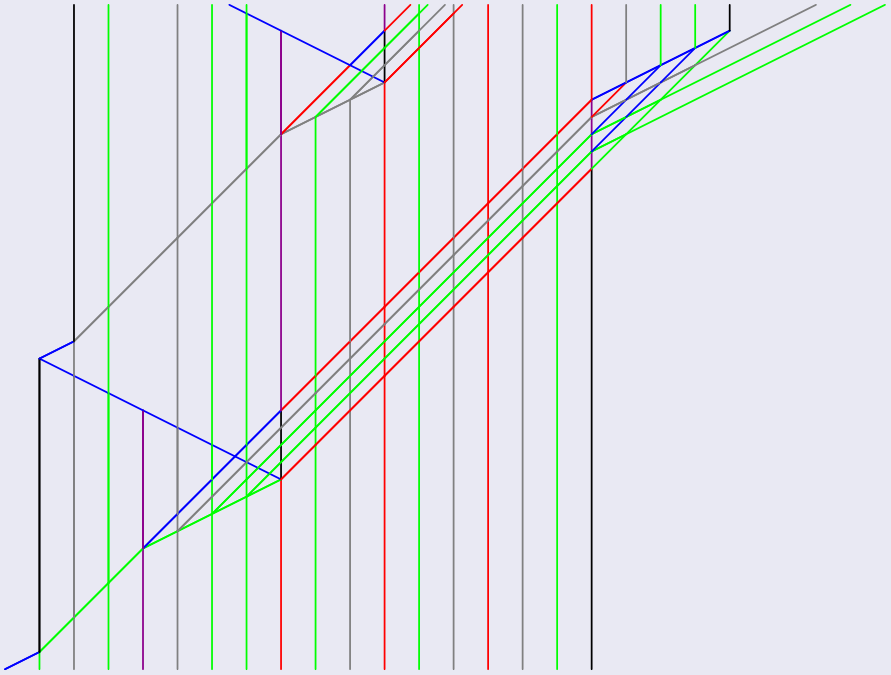
Simulation

101 and 011 :: 10 :: 10 :: 01

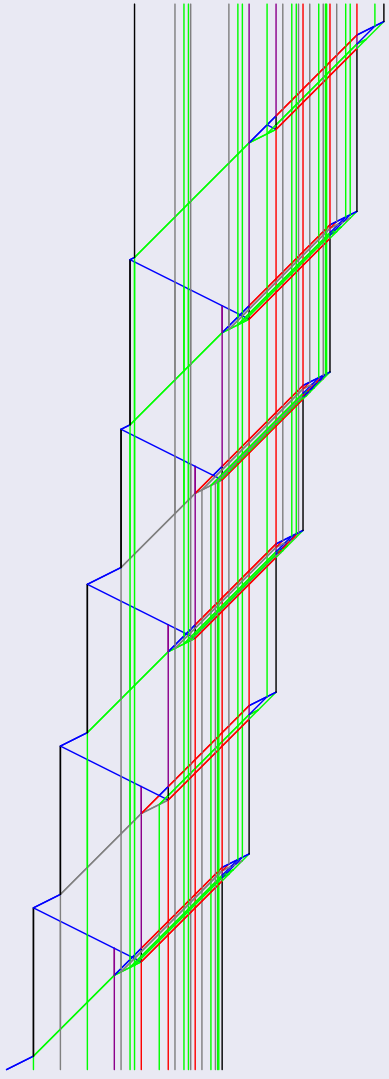
- initial configuration



- one iteration



101 & 011 :: h :: 0110 :: 01011



How many meta-signals?

Universality

- 13 meta-signals
- 21 non-blank rules

Conclusion

Bounds

universal	semi-universal
13	**6**

Future work

- lower this bound
- bounds for
  - accumulation (3 or 4)
  - black hole implementation (13 ? 14 ? more)
  - analog computation (BSS or computable analysis)