Abstract Geometrical Computation and Computable Analysis

Jérôme DURAND-LOSE

LABORATOIRE D'INFORMATIQUE FONDAMENTALE D'ORLEANS

Laboratoire d'Informatique Fondamentale d'Orléans,
Université d'Orléans, ORLÉANS, FRANCE

L'UNIVERSITÉ N'EST PAS UNE ENTREPRISE
LE SAVOIR N'EST PAS UNE MARCHANDISE

Unconventional Computation '09
Sep. 9th, 2009 — Ponta Delgada, Azores, Portugal

Context: Analog computation

Computing over $\mathbb{R}$

- no Church-Turing Thesis
- how does the various models relate?

Computable analysis [Weihrauch, 2000]

- based on converging sequences of approximations
- approximations are discrete values handled in the classical context

Abstract Geometrical Computation…

- continuous counterpart of cellular automata
- idealized collision based computing

…is a new model that needs more introduction

Signal machines

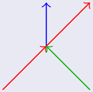
- meta-signals (finitely many)
- their speed/velocity
- collision rules

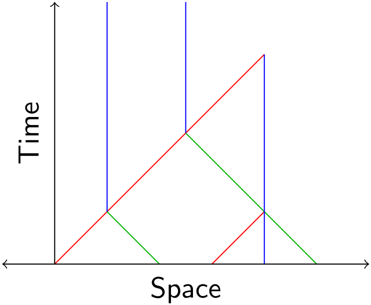
Signals, e.g.

- Red (speed 1) at position 0
- Green (speed -1) at position 2

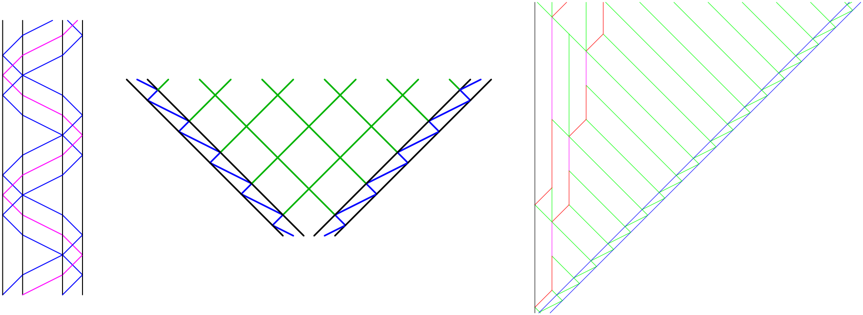
Collision, e.g.

- rule $\{\text{Red}, \text{Green}\} \longrightarrow \{\text{Blue}, \text{Red}\}$
- application

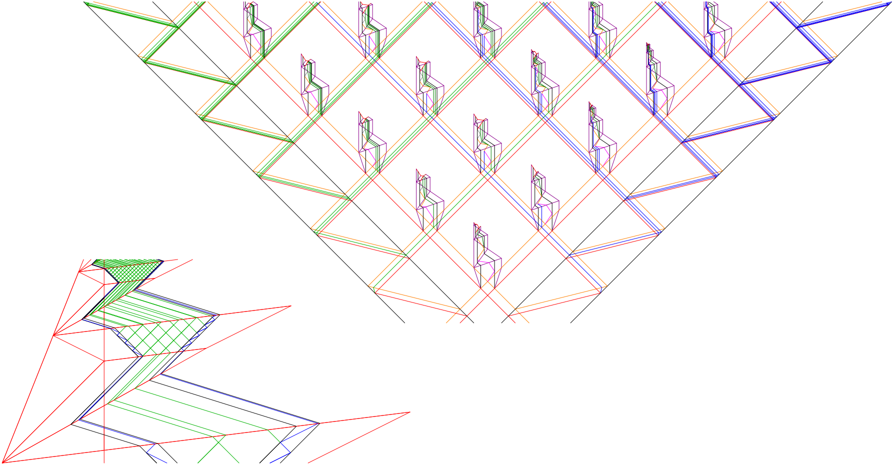




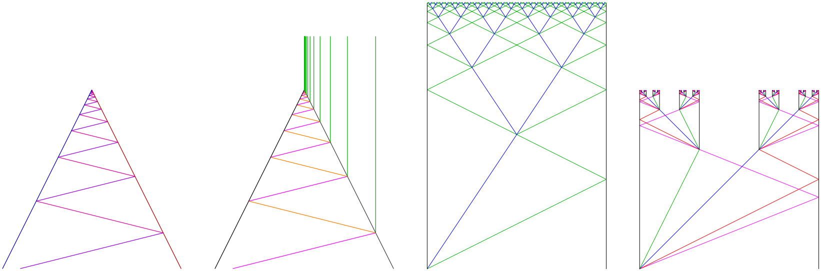
Space-time diagram examples



Space-time diagram examples

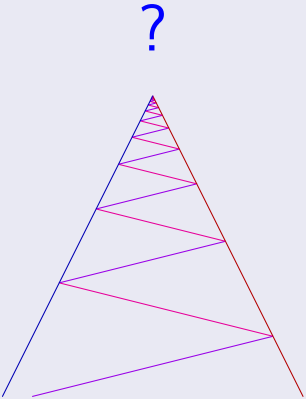


Problematic examples

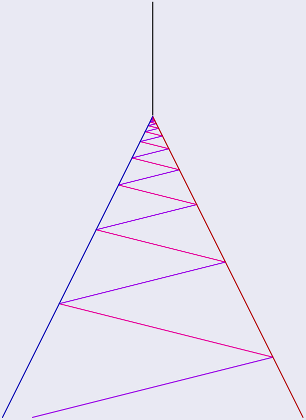


⇝ *extended signal machines*

Simple, “isolated” accumulations



Simple, “isolated” accumulations

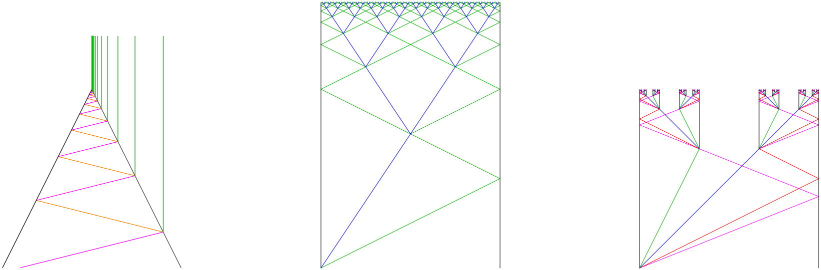


Special value at the accumulation point: ❄

⇝ replaced by a signal

Always the same emerging meta-signal

Other accumulations



✺ on the accumulation points that are not simple
They are called *singularities*

Nothing comes out from ✺

Representation in computable analysis

Principle

- converging infinite sequence of approximations
- encoded as an infinite word
  larger prefix ⇒ better approximation

Signed binary representation [Weihrauch, 2000, Def. 7.2.4 p. 206]

$$\begin{array}{rcl} \{\overline{1},0,1\}^{*}\bullet\{\overline{1},0,1\}^{\omega} & \longrightarrow & \mathbb{R} \\ n_k n_{k-1}\ldots n_0\bullet d_1 d_2 d_3 \ldots d_n \ldots & \longmapsto & \displaystyle\sum_{0\leq i\leq k} n_i.2^i + \sum_{1\leq i} \frac{d_i}{2^i} \end{array}$$

Back to AGC

Problem: infinitely many signals

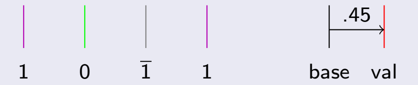
- scattered on the entire line *or*
- there is already an accumulation point (singularity)

Finitely many signals

- integer: $n_k n_{k-1} \dots n_0$ represented by $k+1$ signals
- rest: ε in $[-1, 1]_{\mathbb{R}}$ encoded by two parallel signals

  their distance “is” $\displaystyle\sum_{1 \leq i} \frac{d_i}{2^i}$

Example $7.45 = 8 - 2 + 1 + .45$



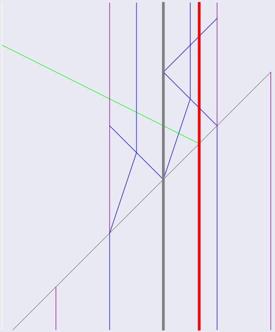
Treatment of ε is the crux

Extracting signed bit on demand

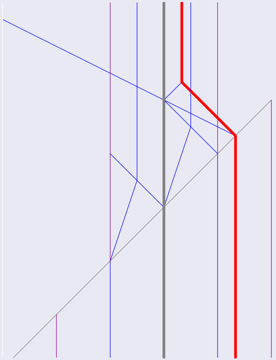
Scheme: extract, shift and shrink the approximating structure

$-1 < \varepsilon \leq -\frac{1}{2}$

$-\frac{1}{2} < \varepsilon < \frac{1}{2}$

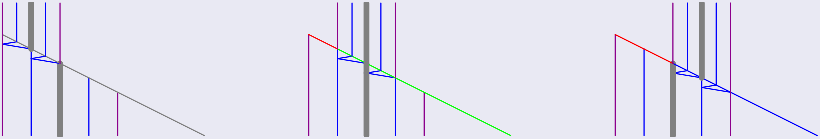


$\frac{1}{2} \leq \varepsilon < 1$

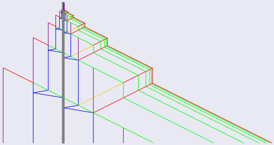


Storing the signed bits, improving the approximation

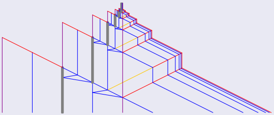
Scheme: reverse roles with the same structure



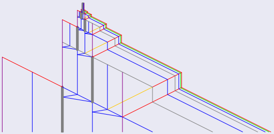
0 0 0 0 0 0



1 1 1 1 1 1



1 $\bar{1}$ 1 $\bar{1}$ 1 0



Computing in the classical understanding

Computable analysis principle

- the infinite entry word is read
  while
  the infinite output word is generated
- output is write-once
  i.e. once a symbol is written it is never changed
- carried out by any classical model of computation

Input / output

- decode / encode real as previously shown
- sending decoding / encoding signals is trivial

Turing machine simulation in bounded space

First transitions

$\boxed{\text{b}}\boxed{\text{b}}\boxed{\text{c}}\boxed{\text{a}}\boxed{\text{a}}\overset{q_1}{\boxed{\#}}\dots$

$\boxed{\text{b}}\boxed{\text{b}}\boxed{\text{c}}\boxed{\text{a}}\overset{q_1}{\boxed{\#}}\dots$

$\boxed{\text{b}}\boxed{\text{b}}\boxed{\text{c}}\overset{q_1}{\boxed{\#}}\dots$

$\boxed{\text{b}}\boxed{\text{b}}\overset{q_1}{\boxed{\text{b}}}\dots$

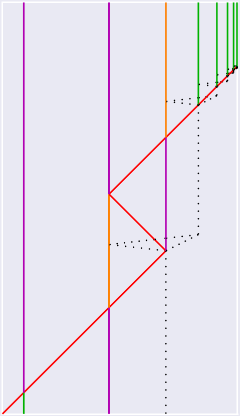
$\boxed{\text{b}}\overset{q_1}{\boxed{\text{c}}}\boxed{\text{b}}\dots$

$\boxed{\text{b}}\boxed{\text{c}}\overset{q_0}{\boxed{\#}}\dots$

$\boxed{\text{b}}\overset{q_0}{\boxed{\text{b}}}\dots$

$\overset{q_0}{\boxed{\text{a}}}\boxed{\text{b}}\dots$

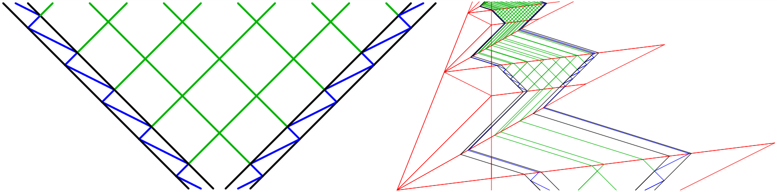
Simulation



No accumulation there since it has to make I/O infinitely often

Generating the output in finite time

There exist operators to shrink space and time while preserving the ratio inside (linked to the Black hole model simulation [Durand-Lose, 2006, Durand-Lose, 2009])

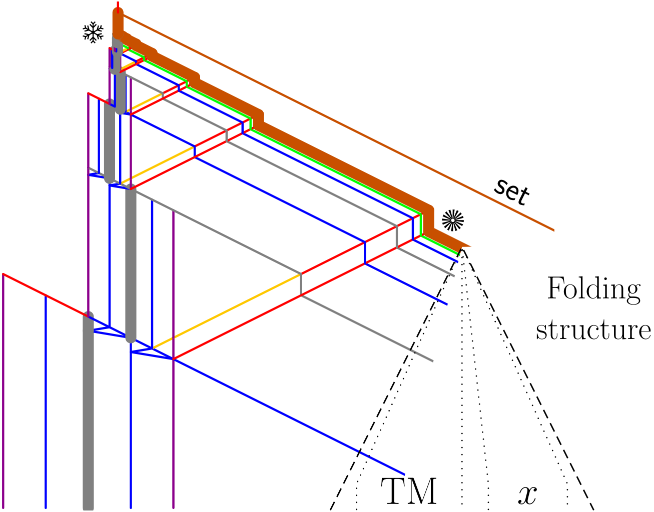


- Input: inside the contracting structure
- Output: has to be outside, infinitely many signals ⇝ *convoy*

Convoy frontier: non isolated accumulations

Always there, prevent any signal above ⇝ impassable barrier

Global picture



Result

- Computable analysis is implemented
- moreover, the representation of real numbers is compatible with the one used to implements the Blum, Shub and Smale model [Durand-Lose, 2007, Durand-Lose, 2008]

- Two incomparable models
- Characterize the computing power