Jérôme Durand-Lose. Drawing numerable linear orderings. Poster presented at the 16th Int. Conf. Unconventional Computation and Natural Computation, UCNC 2023, Jacksonville, FL, USA, 2023.

Drawing numerable linear orderings

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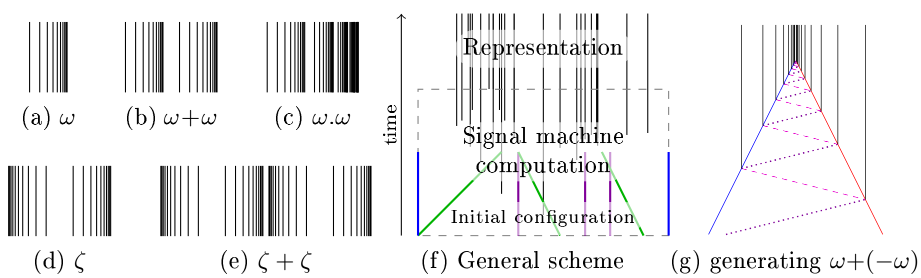
A *linear ordering* on a set is a reflexive, transitive and anti-symmetric binary relation so that each two elements are comparable. For examples ω denotes the ordering of the natural numbers $(0, 1, 2, \ldots)$ and ζ denotes the one of the integers $(\ldots, -2, -1, 0, 1, 2, \ldots)$. The sum of two orderings is the union of the sets with all elements of the first set before the elements of the second set; $\omega + \omega$ is $0, 1, 2, \ldots 0', 1', 2', \ldots$ ($0'$ is a distinct copy of 0). The product of two orderings is the Cartesian product of the sets with the lexicographical order; $\omega.\omega$ is $(0, 0), (0, 1), (0, 2), \ldots (1, 0), (1, 1), (1, 2), \ldots$.

We propose a way to geometrically generate graphical representations of numerable linear orderings by a ray of (identical) parallel lines (of zero width) such that the sequence is generated for left to right as in Fig. 1. The representation is isomorphic to the ordering. If there are finitely many elements between two elements/lines, then there should be exactly the same number of lines between (same if infinitely many). The continuity of space and time allow to put infinitely many lines in finite space.



(a) ω (b) $\omega+\omega$ (c) $\omega.\omega$ (d) ζ (e) $\zeta + \zeta$ (f) General scheme (g) generating $\omega+(-\omega)$

Fig. 1: Example of basic representations.

As depicted in Fig. 1f, some space-time is used to geometrically construct the ray and only the ray emerges of it. The scheme is to repeatedly divide space by finding an element in-between line/element and adding a line.

The device used to draw is a *signal machine*. It operates by extending coloured line segments until they intersect and then replace them by other segment(s) or nothing. Continuity and exact precision are used to ensure that all computations end before a given date.

If all positions and speeds are rational, then it is possible to draw any decidable (numerable) ordering. If some irrationality is allowed then any (numerable) ordering can be generated through its encoding in some *oracle* real number.

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