

Apériodicité, Substitutions, Automaticité, Pavages

accueil en délégation CNRS

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INSTITUT
DE RECHERCHE
EN INFORMATIQUE
FONDAMENTALE

Where do I come from?

1999–2000 DEA Algo, ENS de Cachan

2000–2002 PhD Thesis

LIP, ENS Lyon

MC2 Team



2003–2011 Maître de Conférences

LIF, Univ. Provence (Aix-Marseille I)

Escape Team



2007–2008 Délégation and HDR

IS5, Univ. Nice Sophia Antipolis

MC3 Team



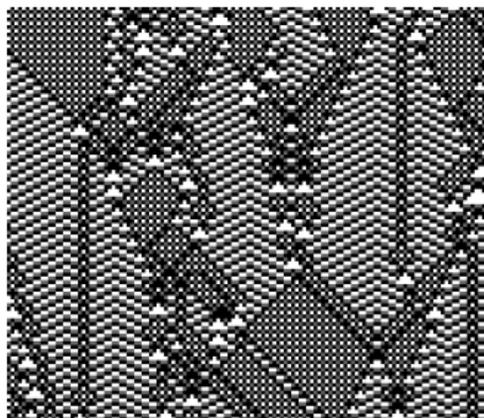
Since 2011 Prof. at LIFO, Univ. Orléans

Équipe **G**raphes,
Algorithmes
et **M**odèles
de **C**alcul

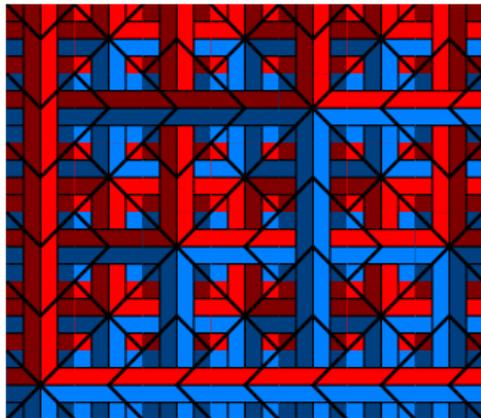


What objects do I work with?

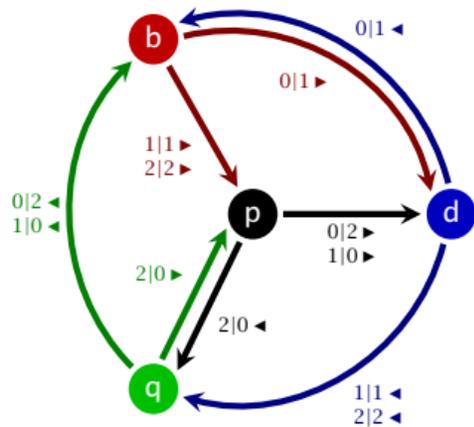
My work focus on **discrete dynamical models** where **local computations** might generate **complex global dynamics** leading to the **undecidability** of simple dynamical properties.



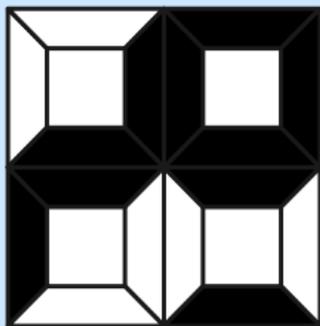
Cellular Automata



Tilings



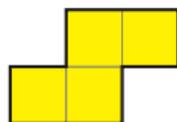
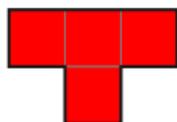
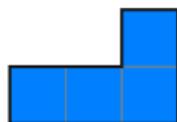
Turing Machines



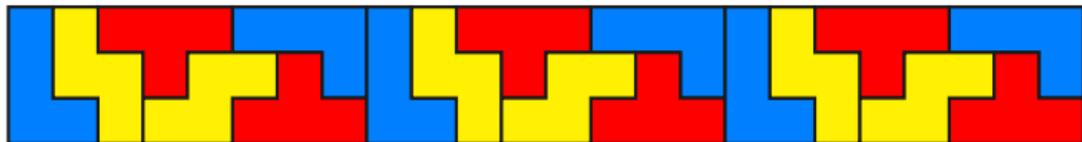
1. Tiling

Tiles and tilesets

Definition A **tileset** is a finite collection of simply connected tiles.



Definition A **tiling** of a region by a tileset is a partition of the region into images of the tiles by isometries.

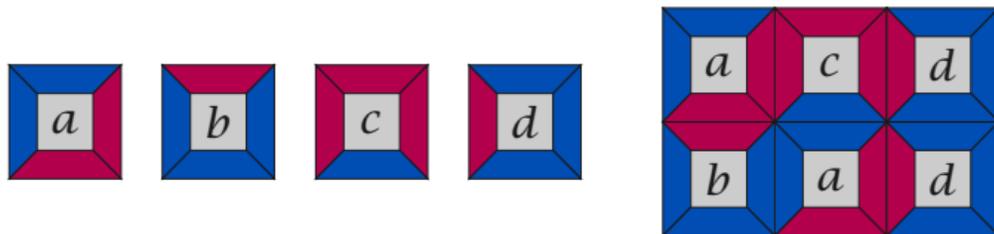


Remark Without loss of generality, one can consider **decorated tiles** with decorations that must match edge-to-edge. One might also want to restrict isometries to **translations only**.

The Domino Problem

“Assume we are given a finite set of square plates of the same size with edges colored, each in a different manner. Suppose further there are infinitely many copies of each plate (plate type). We are not permitted to rotate or reflect a plate. The question is to find an effective procedure by which we can decide, for each given finite set of plates, whether we can cover up the whole plane (or, equivalently, an infinite quadrant thereof) with copies of the plates subject to the restriction that adjoining edges must have the same color.”

(Wang, 1961)



Undecidability of the Domino Problem

Theorem[Berger 1964] The **Domino Problem** is **undecidable**.

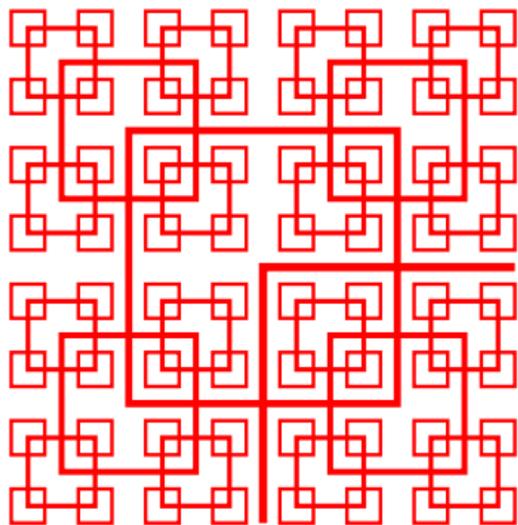
Remark To prove it one needs **aperiodic** tile sets.

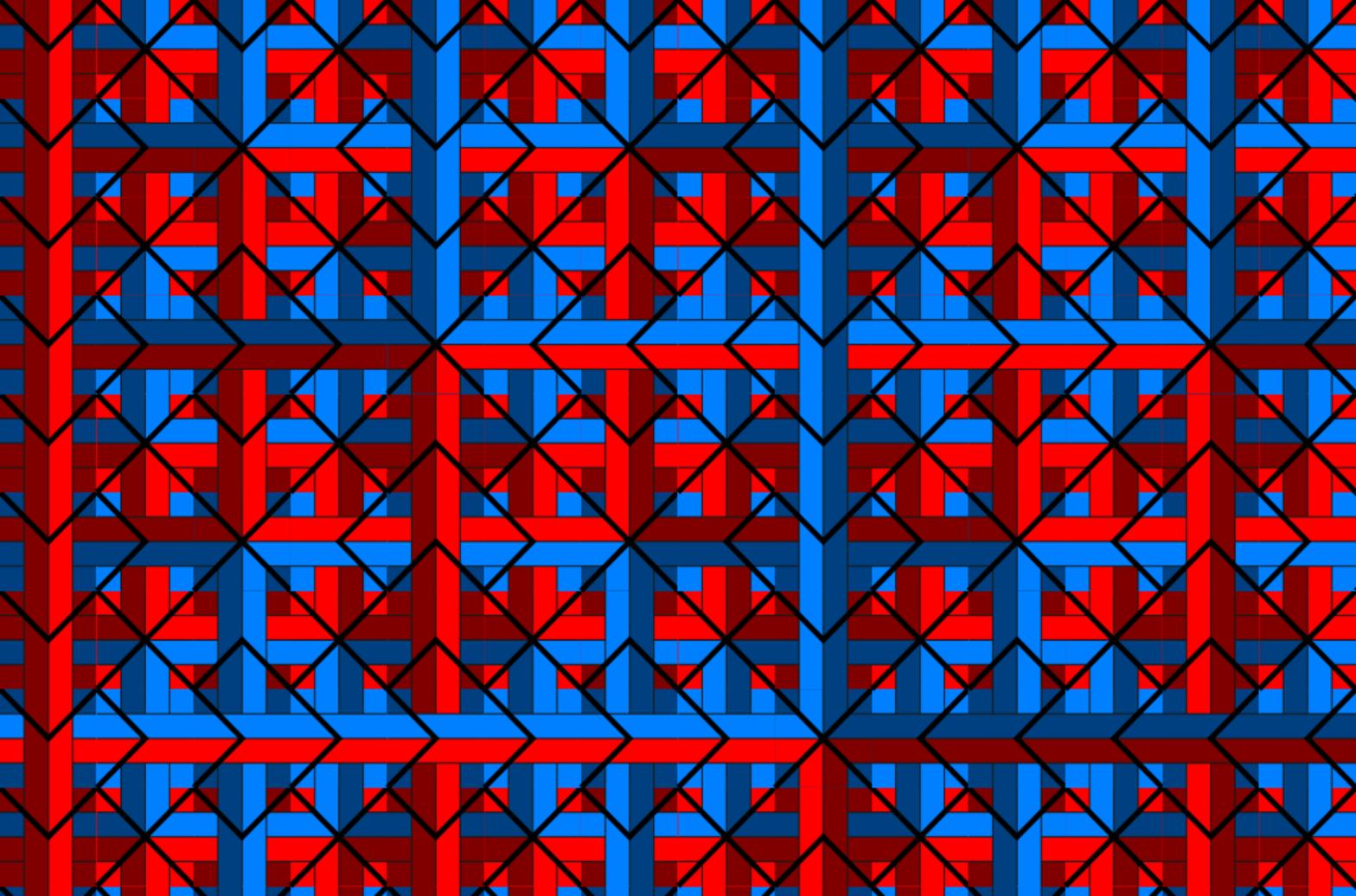
Idea of the proof

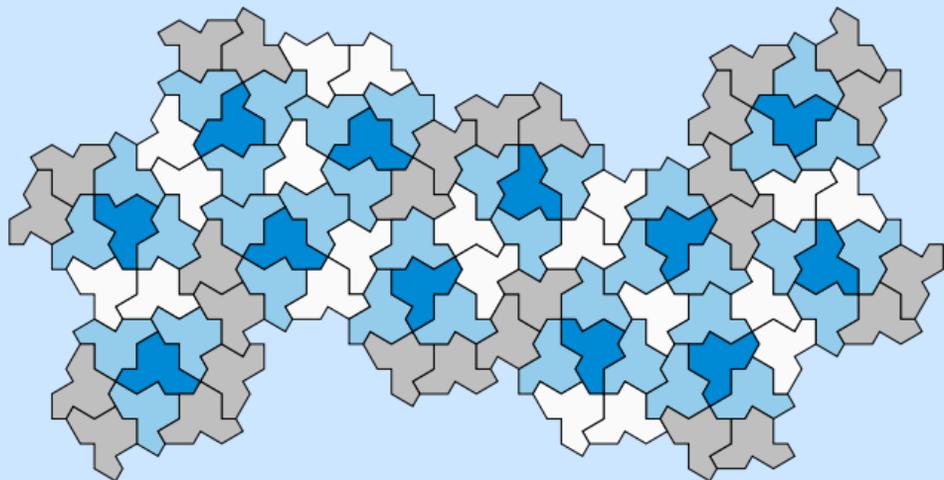
Enforce an (aperiodic) **self-similar hierarchical structure** using local rules.

Insert a **Turing machine** computation **everywhere** using the structure.

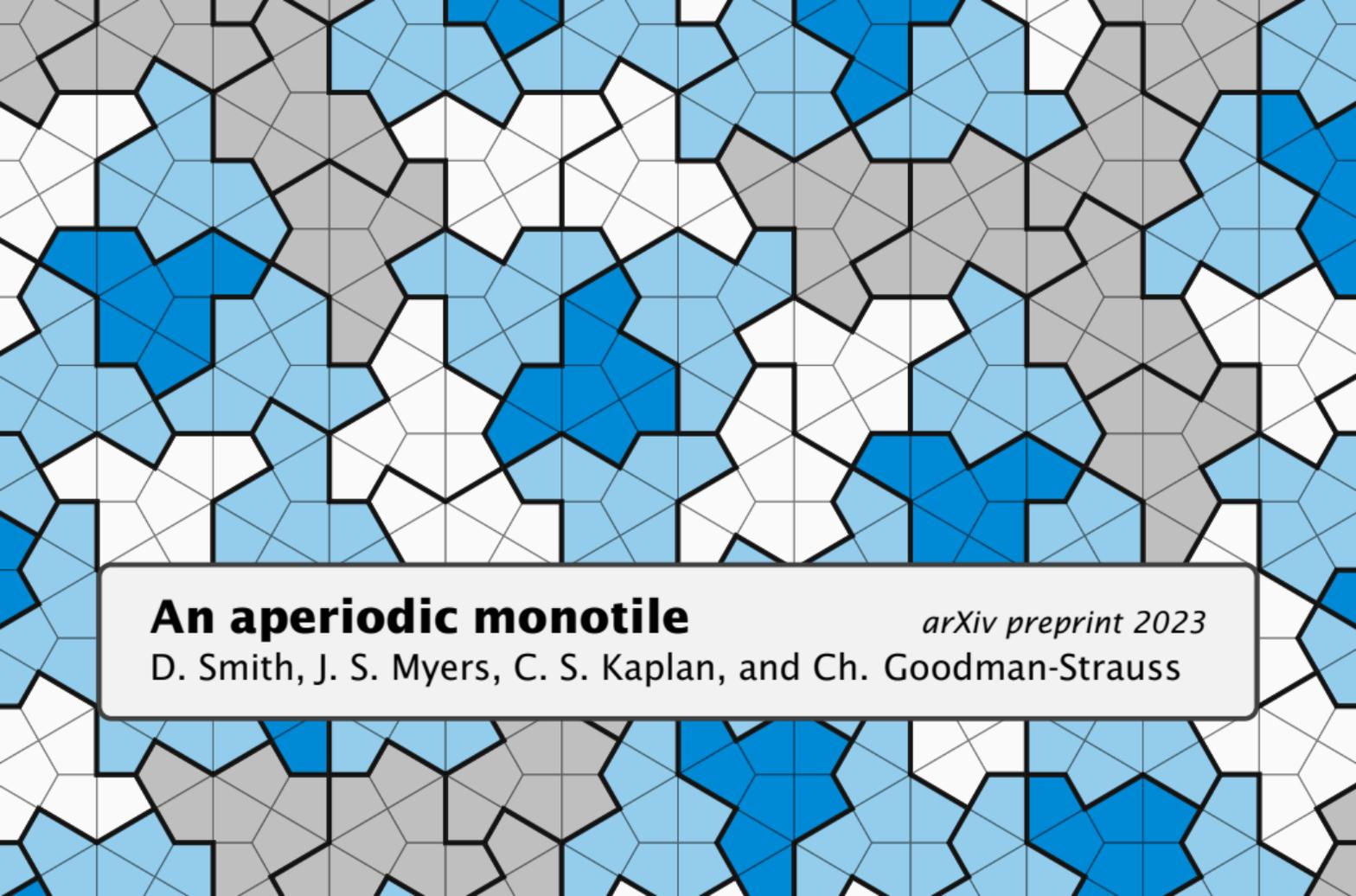
Remark Plenty of different proofs!







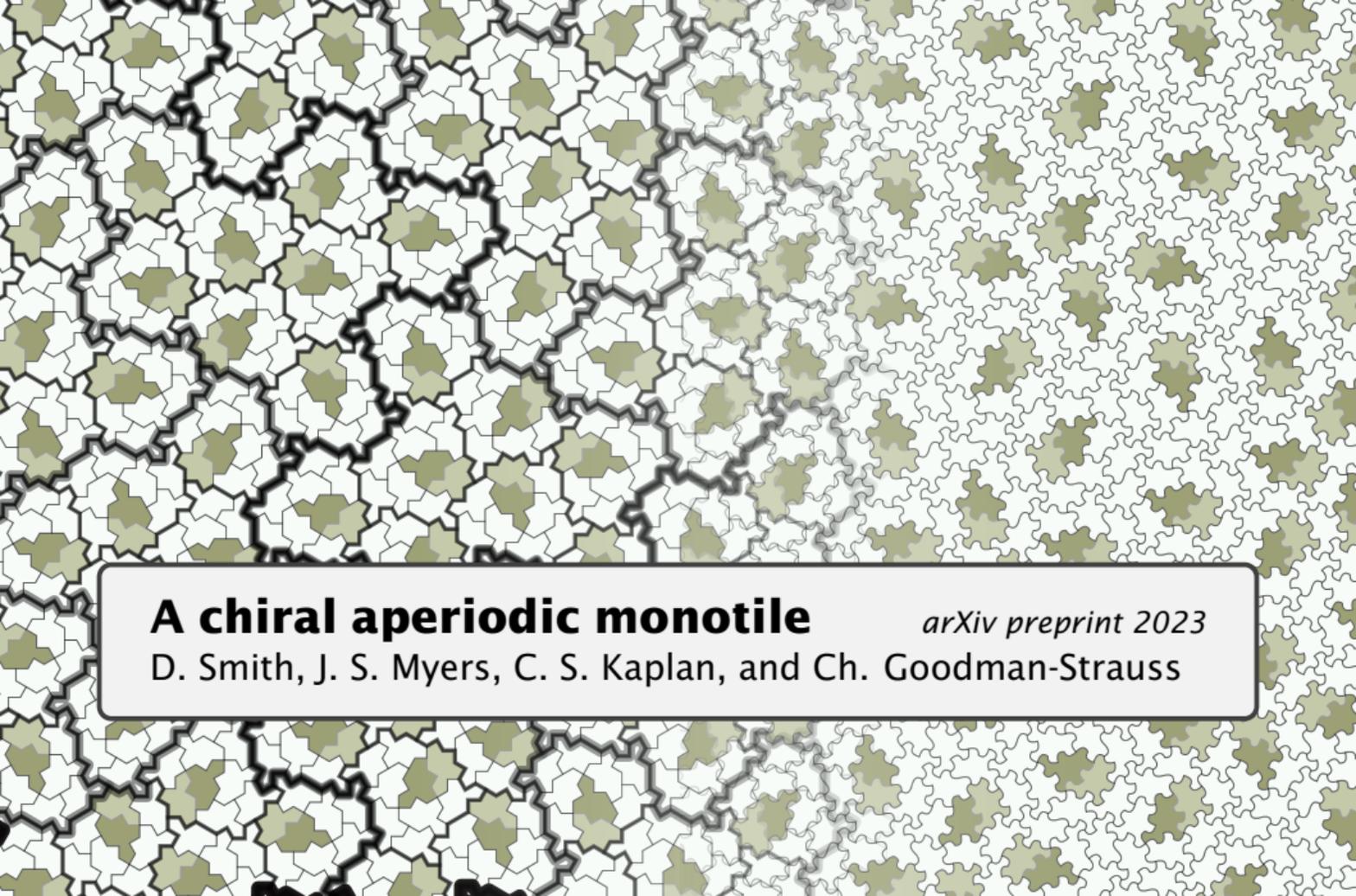
2. Some exciting recent tiling results



An aperiodic monotile

D. Smith, J. S. Myers, C. S. Kaplan, and Ch. Goodman-Strauss

arXiv preprint 2023



A chiral aperiodic monotile

arXiv preprint 2023

D. Smith, J. S. Myers, C. S. Kaplan, and Ch. Goodman-Strauss

A COUNTEREXAMPLE TO THE PERIODIC TILING CONJECTURE

RACHEL GREENFELD AND TERENCE TAO

ABSTRACT. The periodic tiling conjecture asserts that any finite subset of a lattice \mathbb{Z}^d which tiles that lattice by translations, in fact tiles periodically. In this work we disprove this conjecture for sufficiently large d , which also implies a disproof of the corresponding conjecture for Euclidean spaces \mathbb{R}^d . In fact, we also obtain a counterexample in a group of the form $\mathbb{Z}^2 \times G_0$ for some finite abelian 2-group G_0 . Our methods rely on encoding a “Sudoku puzzle” whose rows and other non-horizontal lines are constrained to lie in a certain class of “2-adically structured functions”, in terms of certain functional equations that can be encoded in turn as a single tiling equation, and then demonstrating that

A counterexample to the periodic tiling conjecture

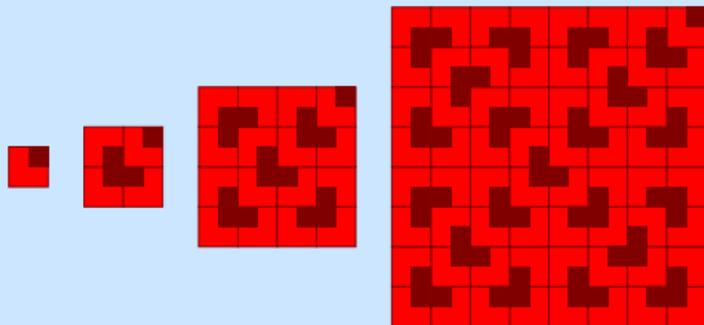
R. Greenfeld, and T. Tao

arXiv preprint 2022

In 1960, Hao Wang [W60, W75] studied the problem of tiling the plane by translated copies of finitely many squares a color attached to each side of each of them, aka *Wang squares*, where one square lies next to another only if the common edges colors match. This is a variant of Hilbert’s famous *Entscheidungsproblem*.

$$S : \begin{matrix} \begin{matrix} \blacksquare & \blacksquare \\ \blacksquare & \blacksquare \end{matrix} \mapsto \begin{matrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{matrix}, \begin{matrix} \blacksquare & \blacksquare \\ \blacksquare & \blacksquare \end{matrix} \mapsto \begin{matrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{matrix}, \begin{matrix} \blacksquare & \blacksquare \\ \blacksquare & \blacksquare \end{matrix} \mapsto \begin{matrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{matrix}, \begin{matrix} \blacksquare & \blacksquare \\ \blacksquare & \blacksquare \end{matrix} \mapsto \begin{matrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{matrix}. \end{matrix}$$

$$\Sigma = \left\{ \begin{matrix} \blacksquare & \blacksquare \\ \blacksquare & \blacksquare \end{matrix}, \begin{matrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{matrix}, \begin{matrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{matrix}, \begin{matrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{matrix} \right\}$$



3. My project at IRIF

Aperiodic Substitutive Tilings

- (1) State of the Art on **aperiodic substitutive tilings** from both construction techniques and proof techniques point of views
will include Berger's lost 103 tiles aperiodic tileset
- (2) **Mechanical Proofs for Tilings**: investigate how the decidability of **Büchi arithmetics** (*à la Walnut*) can simplify some proofs in tilings, in particular for **minimal substitutive tilesets** where the language is **automatic**.

