Formalization of an FRP language with references

SeSTeRce Day

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LIFO - University of Orleans

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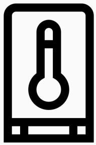

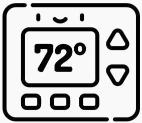

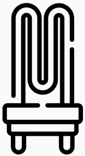

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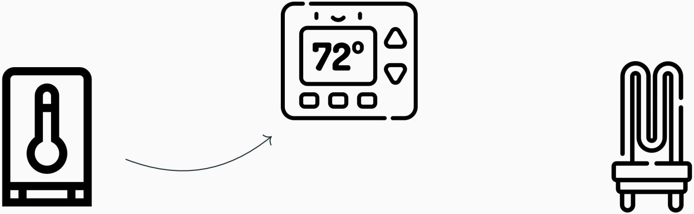

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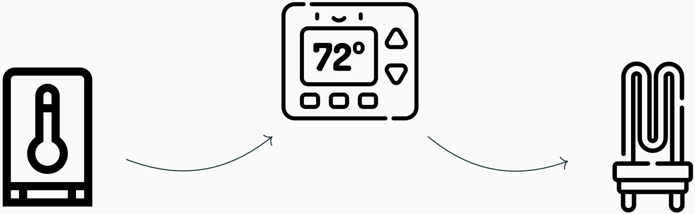

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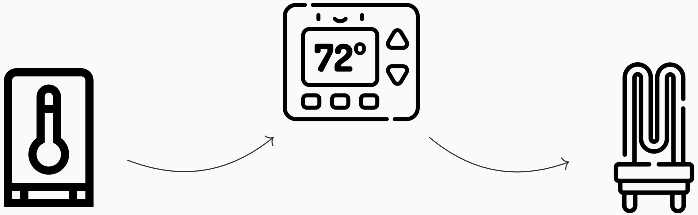

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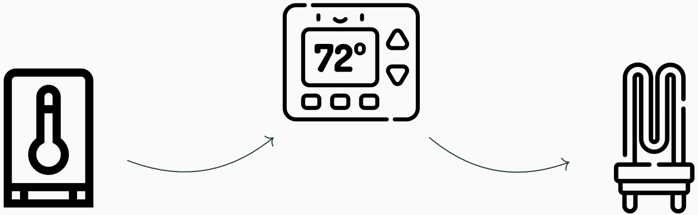

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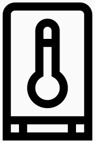
input OFF, TEMP(float);
do







watching OFF;
```



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input OFF, TEMP(float);
do
  signal SWITCH = bot in



      if needs_to_switch(?TEMP) then emit SWITCH
                                else nothing end
watching OFF;
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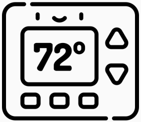

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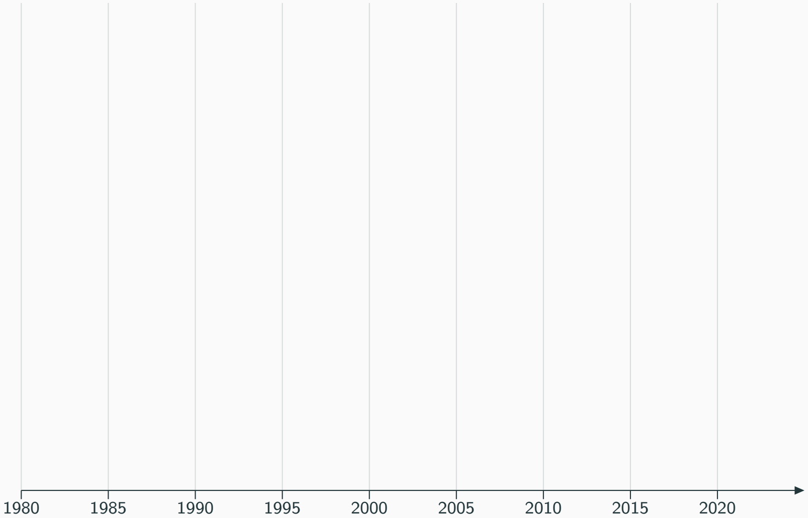
input OFF, TEMP(float);
do
  signal SWITCH = bot in
      present SWITCH then // ...
                     else nothing end
    ||
      if needs_to_switch(?TEMP) then emit SWITCH
                                else nothing end
watching OFF;
```

Timeline



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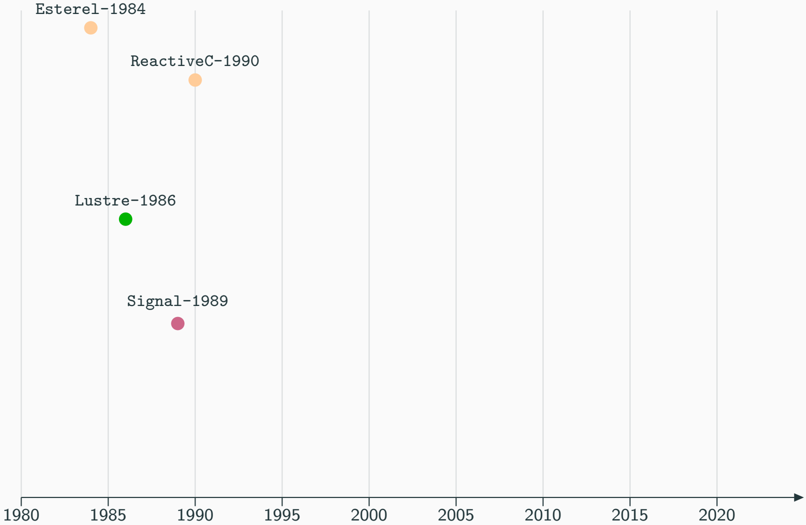

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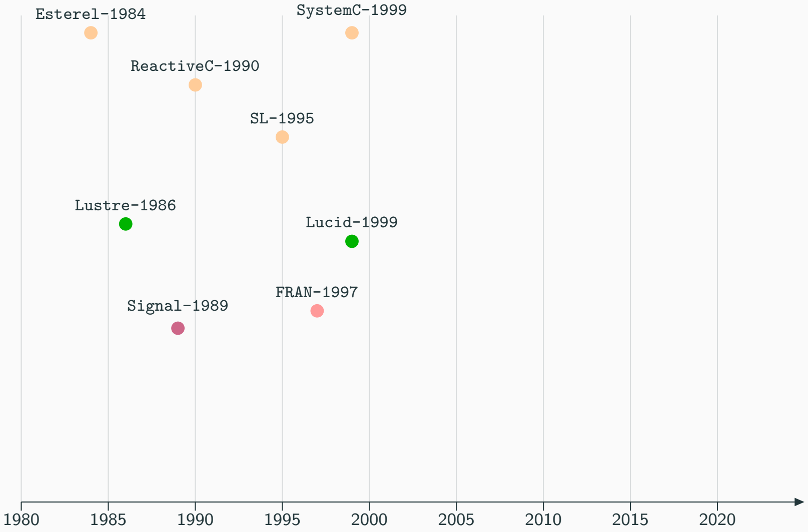
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Functional Reactive Animation [EH97]
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Principle
Data flows are split into two categories:

- $\mathit{Behavior}\ A : \mathit{Time} \to A$
- $\mathit{Event}\ A : \mathit{Time} \times A$

The program has a behavior which is a composition of behaviors and events allow us to modify the behavior.

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```
slowByTwo :: Behavior Time
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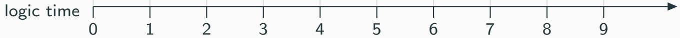
```

time	0	1	2	3	4	5	6	7	8	···
slowByTwo	0	0	1	1	2	2	3	3	4	···



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Examples

```
switchWhenFour :: Behavior Int
switchWhenFour = time untilB (constEv 4 slowByTwo)
```

logic time	0	1	2	3	4	5	6	7	8	9
time	0	1	2	3	4	5	6	7	8	···
slowByTwo	0	0	1	1	2	2	3	3	4	···
switchWhenFour	0	1	2	3	2	2	3	3	4	···

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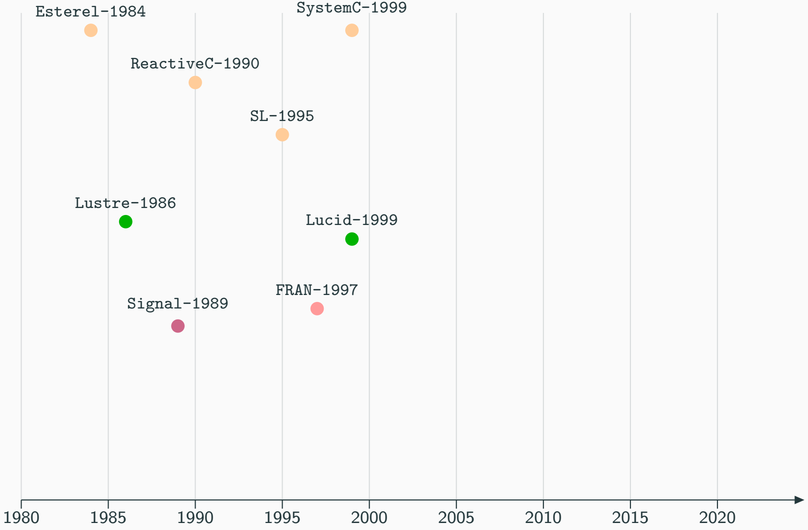

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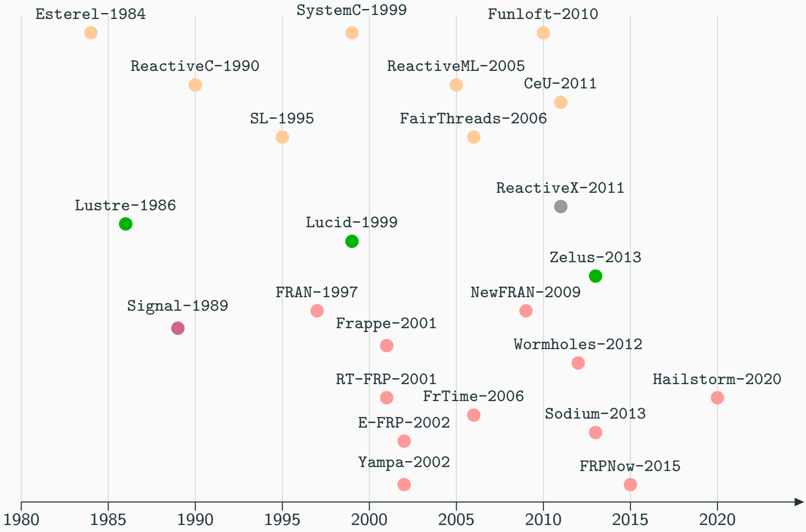
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Reactive programming describes a design paradigm that relies on asynchronous programming logic to handle real-time updates to otherwise static content. It provides an efficient means – the use of automated data streams – to handle data updates to content whenever a user makes an inquiry.

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My Definition
Reactive programming is a programming paradigm concerned with data streams handled with a synchronous or an asynchronous style in order to preserve the coherence of the program.

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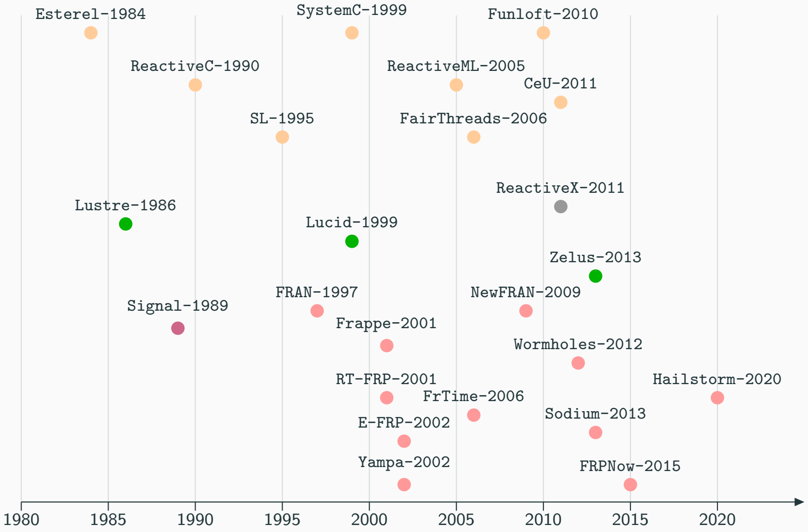
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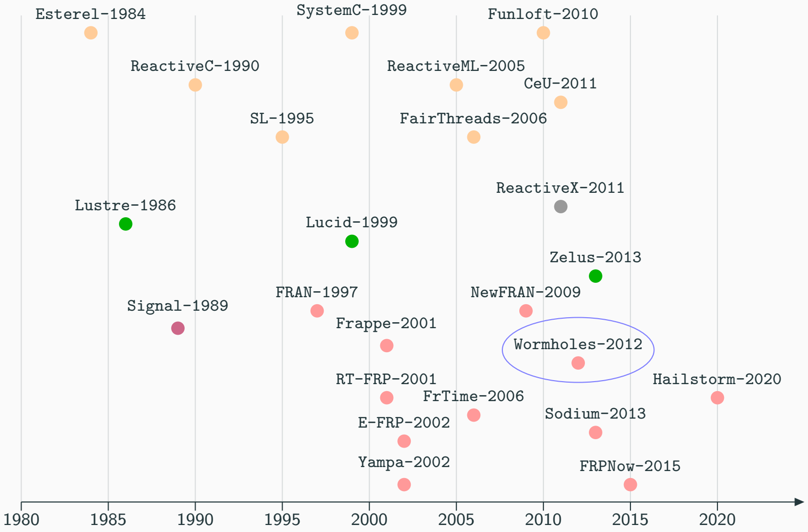

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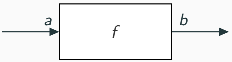
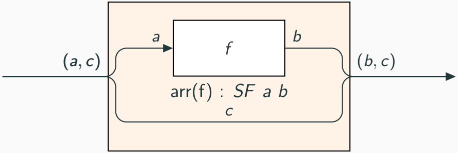
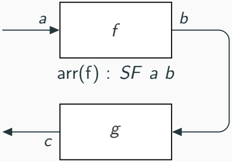
Arrow's syntax



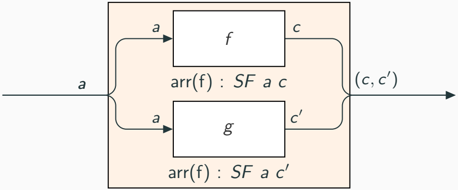
arr : $(a \to b) \to SF\ a\ b$



first : $SF\ a\ b \to SF\ (a, c)\ (b, c)$



arr(g) : $SF\ b\ c$



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Syntax

$$\begin{array}{llll} \text{Expression} & t, t_1, t_2 & ::= & x \mid () \mid (t_1, t_2) \mid \texttt{fst}\ t \mid \texttt{snd}\ t \mid \lambda x.t \mid t_1\ t_2 \\ & & & \mid \texttt{arr}(t) \mid \texttt{first}(t) \mid t_1 \ggg t_2 \mid \texttt{rsf}[r] \\ & & & \mid \texttt{wormhole}[r_{read}, r_{write}](t_i; t) \end{array}$$

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Resource
A resource signal function is a non-local one-way communication channel. Reference can be simulated by two resources: one for reading the other for writing.

The Wormholes syntax

Hypothesis

- `minutes` is a getter resource for the current number of minutes;
- `hours` is a getter resource for the current number of hours;
- `console` is a setter resource for display in the terminal.

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displayTime = rsf[minutes] >>> arr (\n → ((),n)) >>>
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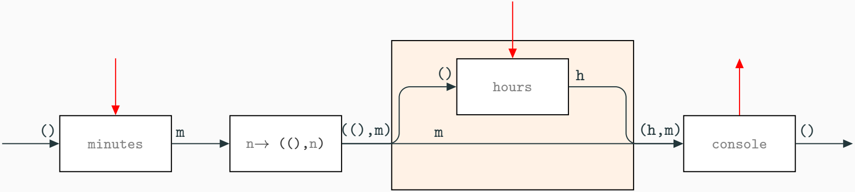

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```
counter = rsf[ctime] >>> arr(\x.((),x)) >>>
            wormhole[cpt_r, cpt_w](0,
               first(rsf[cpt_r])>>>
               arr(\(cpt,ct) → (cpt+ct,cpt) ) >>>
               first(rsf[cpt_w]) >>> arr (fst))
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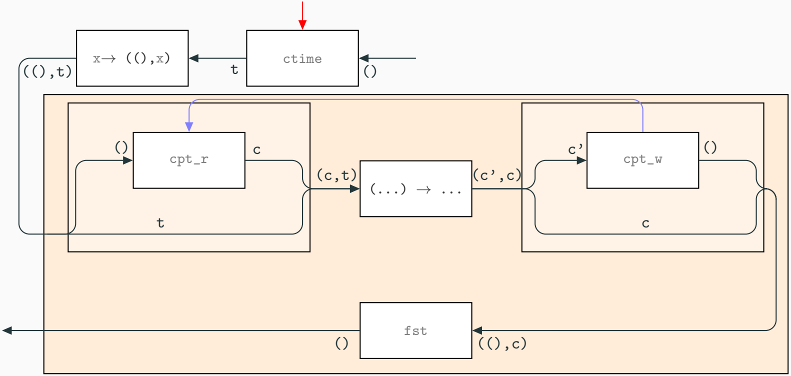

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Typing

Types

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\begin{array}{rrcl}
\text{Resource Type} & t & ::= & \langle \tau_{in}, \tau_{out} \rangle \\
\text{Types} & \tau, \tau_1, \tau_2 & ::= & \mathit{unit} \mid \tau_1 \times \tau_2 \mid \tau_1 \rightarrow \tau_2 \\
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Counting used resources
Reactive function type carries the set of used resources. Consequently, the typing serves as a safeguard for the correct use of the language.

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Example

`rsf[minutes]` will be typed as follows: $unit \overset{\{minutes\}}{\rightsquigarrow} int$.

Rules

$$\frac{\Gamma, \mathcal{R} \vdash f : \tau_1 \rightarrow \tau_2}{\Gamma, \mathcal{R} \vdash \mathtt{arr}(f) : \tau_1 \overset{\emptyset}{\rightsquigarrow} \tau_2}\ \texttt{Ty-Arr} \qquad \frac{\Gamma, \mathcal{R} \vdash sf : \tau_1 \overset{R}{\rightsquigarrow} \tau_3}{\Gamma, \mathcal{R} \vdash \mathtt{first}(sf) : \tau_1 \times \tau_2 \overset{R}{\rightsquigarrow} \tau_3 \times \tau_2}\ \texttt{Ty-First}$$

$$\frac{\begin{array}{cc} \Gamma, \mathcal{R} \vdash sf_1 : \tau_1 \overset{R_1}{\rightsquigarrow} \tau_3 & R_1 \cup R_2 = R \\ \Gamma, \mathcal{R} \vdash sf_2 : \tau_3 \overset{R_2}{\rightsquigarrow} \tau_2 & R_1 \cap R_2 = \emptyset \end{array}}{\Gamma, \mathcal{R} \vdash sf_1 \ggg sf_2 : \tau_1 \overset{R}{\rightsquigarrow} \tau_2}\ \texttt{Ty-Comp}$$

$$\frac{}{\Gamma, \mathcal{R} \cup \{r : \langle \tau_{in}, \tau_{out} \rangle\} \vdash \mathtt{rsf}[r] : \tau_{in} \overset{\{r\}}{\rightsquigarrow} \tau_{out}}\ \texttt{Ty-Rsf}$$

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Typing

Rules

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A small-step operational semantics for non-reactive expression. Rules are not given in the paper but it is defined as a lazy evaluation model.

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A big-step operational semantics for reactive expression.

Temporal transition
A unique transition to move from the current instant to the next instant.

Functional transition

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A big-step operational semantics for reactive expression. The functional transition is defined as follows: $(V, t_v, sf) \Rrightarrow (V', t_v', sf', W)$

- V, V' are resource environments
- t_v, t_v' are stream values
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- Progress and preservation for the evaluation transition

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Why ?
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- `FT-First` asks an implicit normalisation of the stream value;
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- The concept of value provoke an interlacing between evaluation transition and functional transition.

The `FT-First` rule

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The stream do not have to be reduced in most case, except in this case where it needs to be normalized.

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The configuration below is stuck with the current version of the rules.

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Modification done

We add a rule for lift evaluation transition into functional transition. A side effect of that is a simplification of other rules.

$$\frac{t \mapsto t' \quad (V, t_s, t') \Rrightarrow (V_1, t'_s, sf, W)}{(V, t_s, t) \Rrightarrow (V_1, t'_s, sf, W)}\ \texttt{FT-Eval}$$

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Because of the fact that reactive terms are values, each time we want to use a functional transition we have to apply multiple times the evaluation transition on subterms.

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Can we avoid this ?
The modification chosen was to define reactive terms as values only if their subterms are also values and let the evaluation transition pass through the reactive terms.

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- use the HOAS/PHOAS $\longrightarrow$ not chosen because never used
- define our equivalence

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Adding the remaining terms of the Wormholes's syntax brings with him the feared renaming problem.

Example

$$\texttt{wormhole}[r_r, r_w](i; rsf[r_r]) \iff \texttt{wormhole}[r_{read}, r_w](i; rsf[r_{read}])$$

Possibility
Several representation can possibly saved us:

- use the locally nameless representation $\longrightarrow$ break because of the `Ty-wh` rule
- use the HOAS/PHOAS $\longrightarrow$ not chosen because never used
- define our equivalence

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Thanks for your attention !

Do you have any questions ?