UNIVERSITE D'ORLEANS
Faculté des Sciences
LIFO

Laboratoire d'Informatique Fondamentale d'Orléans
4, rue Léonard de Vinci, BP 6759
F-45067 Orléans Cedex 2
FRANCE

Rapport de Recherche
www : http://www.univ-orleans.fr/SCIENCES/LIFO/

Graphs with chordless cycles of bounded length

Irena Rusu
Université d'Orléans, LIFO

Rapport N^o **98-06**

Graphs with chordless cycles of bounded length

Irena Rusu

Université d'Orléans, L.I.F.O., B.P. 6759,
45067 Orléans Cedex 2, France

Abstract

The perfection of *weakly triangulated graphs*, i.e. antihole-free graphs whose induced cycles are isomorphic either to C_3 or to C_4, was proved by Hayward [3] and generated intense studies to efficiently solve the classical NP-complete problems which become polynomial on perfect graphs. If we replace, in the definition above, the C_4 by an arbitrary C_p (p even, at least equal to 6), we obtain new classes of graphs whose perfection is shown in this paper. In fact, we prove a more general result: for any even integer $p \geq 6$, the graphs whose cycles are isomorphic either to C_3 or to one of $C_p, C_{p+2}, \ldots, C_{2p-6}$ are perfect.

Keywords: perfectness, weakly triangulated graphs, dominating pair

1 Introduction

A *perfect graph* is a graph $G = (V, E)$ whose chromatic number χ and density number ω are equal for the graph itself, as well as for all its induced subgraphs. A *hole* is simply a chordless cycle of length at least five and an *antihole* a complement of such a graph. With these definitions, Berge's strong perfect graph conjecture [1] (abbreviated SPGC) claims that a graph is perfect if and only if G contains no odd hole and no odd antihole (as induced subgraphs).

The SPGC is still unsettled, but has been proved for particular classes of graphs (one of the most important of them being the class of *weakly triangulated graphs* [3], in which no hole or antihole is induced), while it is still open for some other particular classes (for instance, the *antihole-free graphs*, in which no antihole is induced). Speaking about the antihole-free graphs, which seem to be quite difficult to approach, we can notice that weakly triangulated graphs are exactly the antihole-free graphs whose only chordless cycles are C_3 and C_4 (where C_k is the classical notation for a chordless cycle of length $k \geq 3$). Then, a natural question arises: what about the antihole-free graphs in which only the cycles C_3 and C_p are allowed (for a fixed even integer $p \geq 6$)? Do they satisfy the SPGC? Do they have even pairs, i.e. pairs (x, y) of vertices such that any chordless path joining x, y is of even length? Do they have nice characterizations yielding efficient recognition algorithms? For weakly triangulated graphs the answers to all these questions are affirmative.

The aim of this note is to prove that each of the classes $\mathcal{C}^p$, obtained by considering the graphs whose cycles are isomorphic either to C_3 or to one of $C_p, C_{p+2}, \ldots, C_{2p-6}$, is a class of perfect graphs (observe that the condition for the graph to be antihole-free is, in fact, not necessary since it is implied by the absence of C_4 in the graph).

2 Main result

As usual, we use the notation $N(x)$ to denote the set of vertices (also called *neighbours of* x) which are adjacent in $G = (V, E)$ to a given vertex x, and the notation $N(S)$ (where $S \subseteq V$) to denote $\cup_{x\in S} N(x) - S$. We call *dominating pair* in G a pair (x, y) of vertices such that $N(x) \cup \{x\} \supseteq N(y)$.

The announced result will be an easy consequence of the following lemma, which establishes that any graph in a class $\mathcal{C}^p$ has either a special pair of vertices, or a special vertex.

Lemma 1 *Let $p \geq 6$ be a fixed even integer and $G = (V, E)$ be a graph containing no chordless cycles but $C_3, C_p, C_{p+2}, \ldots, c_{2p-6}$. Then at least one of the affirmations below is true:*

1. G contains an adjacent dominating pair;

2. G possesses a vertex contained in at most two maximal cliques.

Proof. Let $a_1a_2\ldots a_k$ (without loss of generality we suppose $k \geq 2$) be a longest chordless path of G, chosen such that $|N(a_k) - N(a_{k-1}) - \{a_{k-1}\}|$ is as large as possible. Since the path cannot be extended, every vertex $w \in N(a_k) - \{a_{k-1}\}$ must have a neighbour on the path. Two possibilities exist for a given w:

i) either $wa_{k-1} \in E$;

ii) or $k > p-2$ and there exists $j \in \{2, 0, -2, \ldots, -(p-8)\}$ such that $wa_{k-1}, wa_{k-2}, \ldots, wa_{k-p+j+1} \notin E$, while $wa_{k-p+j} \in E$.

We partition the neighbourhood of a_k in four sets $N(a_k) = \{a_{k-1}\} \cup S \cup T \cup Q$, where $S \cup T = N(a_k) \cap N(a_{k-1})$, $Q = N(a_k) - N(a_{k-1}) - \{a_{k-1}\}$, $T = N(Q) \cap N(a_k)$, $S = N(a_k) \cap N(a_{k-1}) - T$. Notice that, because of *ii)* here above, every vertex in Q is adjacent to some a_{k-p+j}, $j \in \{2, 0, -2, \ldots, -(p-8)\}$ (but non-adjacent to $a_{k-2}, a_{k-3}, \ldots, a_{k-p+j+1}$). If Q is empty, then the affirmation 1. in the lemma is satisfied by the pair $\{a_{k-1}, a_k\}$. In the contrary case, we can prove that Q is a clique. To this end, we use the following small result:

(1) If $u, v \in N(a_k)$ such that they both have neighbours on a (not necessarily chordless) path of length at most $p-5$ included in $V - N(a_k) - \{a_k\}$ then $uv \in E$.

If this is not the case, then let x, y be two neighbours of u, respectively v on the indicated path such that the distance $d(x, y)$ (in number of edges) between x, y is minimum. The chordless cycle induced by u, a_k, v and a chordless path between x, y is of length $d(x, y) + 4 \leq p-1$, a contradiction.

Remark 1 The path $a_{k-p+2}a_{k-p+1}\ldots a_{k-2p+7}$ satisfies the hypothesis of (1).

Now, Q is a clique, otherwise (1) (with Remark 1) is contradicted by $q, q' \in Q$ such that $qq' \notin E$. We can easily deduce that T is a clique too (otherwise a C_4 or a C_5 may be built using a_{k-1}, two non-adjacent vertices of T and two neighbours in Q of these vertices). Moreover,

(2) Every vertex in S is adjacent to a_{k-2} (or the affirmation 1. holds).

If this is not the case, consider $s \in S$ such that $sa_{k-2} \notin E$. Then $sa_{k-3}, sa_{k-4}, \ldots, sa_{k-p+2} \notin E$ (a chordless cycle $sa_{k-1}a_{k-2}\ldots a_{k-i}$ of bad length would be found using the smallest i such that $sa_{k-i} \in E$). By (1), it is sufficient to consider $q \in Q$ and s (then $sq \notin E$), to deduce that $sa_{k-p+1}, sa_{k-p}, \ldots, sa_{k-2p+7} \notin E$ (whenever the indicated vertices exist). Also, the remaining vertices a_{k-2p+l} ($l < 7$) on the path are non-adjacent to s since in this case we would obtain a too long cycle $sa_{k-1}a_{k-2}\ldots a_{k-2p+l}$.

Consequently, the path $a_1a_2\ldots a_{k-1}s$ is chordless and has the same length as the initial path $a_1a_2\ldots a_k$. Now, if $N(s) \subseteq N(a_{k-1}) \cup \{a_{k-1}\}$, then we have the affirmation 1. in the lemma. Otherwise, let $u \in N(s) - (N(a_{k-1}) \cup \{a_{k-1}\})$. Obviously $ua_k \notin E$. Moreover, u satisfies *ii)* (instead of w), otherwise the path $a_1a_2\ldots a_{k-1}su$ is chordless and longer than the initial path. By (1), if we take $q \in Q$, s and a suitable path of the subgraph given by $u, a_{k-p+2}, a_{k-p+1}, \ldots a_{k-2p+8}$ we have a contradiction. So (2) is established.

(3) Every vertex in S is adjacent to every vertex in T.

Let $s \in S$ (so $sa_{k-2} \in E$) and suppose there exists $t \in T$ such that $st \in E$. Then $ta_{k-2} \notin E$ (in this case $a_ksa_{k-2}t$ would be a C_4) and $ta_{k-3}, ta_{k-4}, \ldots, ta_{k-p+2} \notin E$ (otherwise a too small chordless cycle may be found using $\{a_{k-1}, a_{k-2}, \ldots, a_{k-p+2}, t\}$). But t could have some neighbours in $\{a_{k-p+1}, a_{k-p}, \ldots, a_{k-2p+7}\}$. Let, if it exists, a_{k-p+l} ($l < 2$) be the neighbour which is the closest to a_{k-p+2}, so l is odd (since the cycle $a_{k-1}a_{k-2}\ldots a_{k-p+l}t$ must be even).

On the other hand, for s (by (1), using some $q \in Q$ and Remark 1) we have that $sa_{k-p+2}, sa_{k-p+1}, \ldots, sa_{k-2p+7} \notin E$. So let, if it exists, a_{k-p+j} ($j > 2$) be the neighbour of s which is the closest to a_{k-p+2}. Then j is even, since the cycle $a_ksa_{k-p+j}a_{k-p+j-1}\ldots a_{k-p+n}q$ (where $q \in Q$ whose neighbour the closest to a_{k-p+2} is a_{k-p+n}) must be even.

But then the chordless cycle $a_ksa_{k-p+j}a_{k-p+j-1}\ldots a_{k-p+l}t$ has length $(j-l)+4$, which is odd, a contradiction. We deduce that either t or s has no neighbour a_r with $r \geq a_{k-2p+7}$ (with the only exception a_{k-2} for s). We show that s has no such neighbour. If it does, then we can consider, according to the preceding reasoning, that t has no neighbour a_r with $r \geq a_{k-2p+7}$). Take a_{k-p+j} as before and notice that the cycle cycle $a_{k-1}sa_{k-p+j}a_{k-p+j-1}\ldots a_{k-p+n}qt$ (with q and a_{k-p+n} as before) has odd length, but no chord. We have a contradiction.

So, if s has some neighbours on the path $a_1a_2\ldots a_k$, then it must be adjacent to a_{k-2p+6}. We consider the two cases:

i) $sa_{k-2p+6} \notin E$ (or a_{k-2p+6} simply doesn't exist).

Then $a_1a_2\ldots a_{k-2}sa_k$ is a chordless path with the same length as the initial path, so we can use for s, a_k the same reasoning we performed for a_{k-1}, a_k to deduce that $N(a_k) = \{s\} \cup S' \cup T' \cup Q'$, where S', T', Q' have the same meanings and properties as S, T, Q but with respect to s instead of a_{k-1}. Recall that the initial path was chosed such that $|N(a_k) - N(a_{k-1}) - \{a_{k-1}\}|$ was as large as possible, and notice that $N(a_k) - N(a_{k-1}) - \{a_{k-1}\} \subseteq N(a_k) - N(s) - \{s\}$. We deduce that the equality holds, i.e. $Q = Q'$. But then, by the definitions of T, T', we have that $T = T'$, and so s is adjacent to every vertex in T, since it is adjacent to every vertex in T'.

ii) $sa_{k-2p+6} \in E$.

Consider again the vertex $t \in T$ such that $st \notin E$ and let $q \in Q$ such that $tq \in E$. The cycle $a_{k-1}tqa_{k-p+n'}a_{k-p+n'-1} \dots a_{k-2p+6}s$ (where $a_{k-p+n'}$ is the neighbour of q the closest to a_{k-2p+6}, $2 \geq n' \geq -(p-8)$) is a cycle of length $(n'+p-6)+5$, which is at most equal to $p+1$ (since $n' \leq 2$). If it is of length p, then $n' = 1$, so $qa_{k-p+1} \in E$; but then the cycle $a_kqa_{k-p+1}a_{k-p} \dots a_{k-2p+6}s$ has length $1+p-6+4 = p-1$, a contradiction. If it is of length $p+1$, then $n' = 2$ so this cycle must contain a unique chord which will form a C_3 and a C_p (otherwise a chordless cycle smaller than p could be find). As $a_{k-p+n'} = a_{k-p+2}$ and we already now that $ta_{k-p+2} \notin E$ (see before), the only possibility is $qa_{k-p+1} \in E$. But this contradicts the choice of $a_{k-p+n'}$ as the neighbour of q the closest to a_{k-2p+6}.

So (3) is proved. Then we denote $W = \{a_{k-1}\} \cup S$ and we have $N(a_k) = W \cup T \cup Q$, where no edge is missing neither between W and T, nor between W and a_{k-2}. Moreover W, T, Q are cliques (to see this for W, it is sufficient to consider, if any, two non-adjacent vertices in W, a_k, a_{k-2} to obtain a C_4). We can also suppose that $T \neq \emptyset$, since in this case we already have the affirmation 2. in the lemma.

Now, if every vertex in T is adjacent to every vertex in Q, then we are done: for every vertex $t \in T$, the pair (t, a_k) is a dominating pair. Consider therefore a vertex $u \in T$ for which there exist vertices $q, q' \in Q$ such that $uq \in E, uq' \notin E$.

(4) $a_1a_2 \dots a_{k-1}u$ *is a chordless path.*

Once more, we can notice that none of the edges $ua_{k-2}, ua_{k-3}, \dots, ua_{k-2p+7}$ exists in G. Indeed, by (1) (using q') u has no neighbours on the subpath with the extremities a_{k-p+2}, a_{k-2p+7}. Now, suppose that a_{k-p+n}, a_{k-p+m} ($n, m \leq 2$ and even) are respectively the neighbours of q, q' the closest to a_{k-p+3} and let (if it exists) a_{k-j} ($j \in \{2, 3, \dots, p-3\}$) the neighbour of u the closest to a_{k-p+2}. The chordless cycle $ua_{k-j}a_{k-j-1} \dots a_{k-p+n}q$ must be even and imposes that j is odd (since n is even), while the cycle $a_kua_{k-j}a_{k-j-1} \dots a_{k-p+m}q'$ implies that j is even (since m is even). We have a contradiction. So $a_1a_2 \dots a_{k-1}u$ is a chordless path, since any neighbour a_r of u with $r < k-2p+7$ would imply the existence of a too long chordless cycle.

(5) $N(u) - N(a_{k-1}) - \{a_{k-1}\} \subset Q$.

Suppose this is not true and let $z \in N(u) - N(a_{k-1}) - \{a_{k-1}\} - Q$. Then $z \notin N(a_k)$ and there exists some $j \in \{-2, 0, \dots, -(p-8)\}$ such that $za_{k-1}, za_{k-2}, \dots, za_{k-p+j+1} \notin E$, while $za_{k-p+j} \in E$). Again by (1), using u, q' and a suitable path of the subgraph given by $z, a_{k-p+2}, a_{k-p+1}, \dots, a_{k-2p+8}$ we have a contradiction.

Because of (5), we know that either (a_k, u) is a dominating pair (and affirmation 1. in the lemma holds) or u has a neighbour r out of $N(a_k)$, which is necessarily adjacent to

a_{k-1}. In this last case we continue as follows:

(6) The path $a_1a_2\dots a_{k-1}r$ is chordless.

Notice firstly that, because of (1), r has no neighbours in $\{a_{k-p+2}, a_{k-p+1}, \dots, a_{k-2p+8}\}$ (otherwise by considering u, q' and a suitable path of the graph induced by r and the preceding vertices we have, by (1), that $uq' \in E$). Once more, suppose that a_{k-p+n}, a_{k-p+m} are respectively the neighbours of q, q' the closest to a_{k-p+3} and let (if it exists) a_{k-j} ($j \in \{2, 3, \dots, p-3\}$) the neighbour of r the closest to a_{k-p+2}. Notice that $rq, rq' \notin E$ (otherwise we have a C_4 with a_k, a_{k-1}). The cycle $ura_{k-j}a_{k-j-1}\dots a_{k-p+n}q$ imposes that j is even (since n is even), while the cycle $a_kura_{k-j}a_{k-j-1}\dots a_{k-p+m}q'$ implies that j is odd (since m is even). We have a contradiction. Moreover, $ra_{k-2p+7} \notin E$, otherwise the chordless cycle $ura_{k-2p+7}a_{k-2p+8}\dots a_{k-p+h}q$ (where a_{k-p+h} is the neighbour of q the closest to a_{k-2p+7}) has length at most $p-1$.

It remains to show that $ra_{k-2} \notin E$ (then the indicated path must be chordless, otherwise a too long chordless cycle could be found). Suppose the contrary. The path $a_1a_2\dots a_{k-2}rua_k$ is longest than the initial path $a_1a_2\dots a_k$ (a contradiction), except if $ra_{k-2p+6} \in E$. Now, take the chordless cycle $a_ka_{k-1}ra_{k-2p+6}a_{k-2p+7}\dots a_{k-p+h}q$. It has length $h-(-p+6)+5 = h+p-1 \leq p+1$ since $h \leq 2$. If the length is exactly p, then $h = 1$ and the cycle $ura_{k-2p+6}a_{k-2p+7}\dots a_{k-p+h}q$ has length $h-(-p+6)+4 = p-1$, a contradiction. If the length is exactly $p+1$, we have a too long chordless cycle.

(7) The pair (a_{k-1}, r) is a dominating pair.

If this is not the case, then let $z \in N(r) - N(a_{k-1}) - \{a_{k-1}\}$ and notice that $za_k \notin E$ (otherwise, z must be in Q and $a_ka_{k-1}rz$ is a C_4). Now, $a_1a_2\dots a_{k-1}rz$ cannot be a chordless path since it is longer than the initial path. Then z must have some neighbour a_{k-p+j}, $j \in \{2, 0, -2, \dots, -p+8\}$. Let a_{k-p+l}, a_{k-p+g} ($-p+8 \leq l, g \leq 2$) two neighbours of z, respectively q such that $|l-g|$ is minimum (we can suppose $l \geq g$). The cycle $urza_{k-p+l}a_{k-p+l-1}\dots a_{k-p+g}q$ has length $l-g+5$ (which is at most $2-(-p+8)+5 = p-1$) and at most two chordes, zu and zq. Then we must necessarily have $l = g$ and $zu, zq \in E$ (otherwise a too small chordless cycle exists); but then $a_ka_{k-1}rzq$ is a C_5 or contains a C_4.

Because of (7), the affirmation 1. in the lemma is verified once again and the proof is finished.∎

It is easy now to establish the result related to perfectness:

Theorem 1 *For any fixed even integer $p \geq 6$, the graphs containing no chordless cycle but C_3, $C_p, C_{p+2}, \dots, C_{2p-6}$ are perfect.*

Proof. By contradiction, suppose there exists a minimal imperfect graph $G = (V, E)$ with this property. Then, by Lemma 1, either G has a dominating pair of vertices or a vertex v of G is contained in at most two maximal cliques. In the first case, G or $\overline{G}$

has a star-cutset, i.e. a disconnecting set S one of whose vertices is universal in S, and this contradicts the star-cutset lemma (see [2]); in the second one, a result of Padberg [4], stating that in a minimal imperfect graph G any vertex is contained in exactly $\omega(G)$ cliques of size $\omega(G)$, implies that $\omega(G) \leq 2$, thus G is isomorphic to some C_{2k+1} ($k \geq 2$), another contradiction. ∎

Obviously, the property established in Lemma 1 is not sufficient to characterize any of the classes $\mathcal{C}^p$, nor their union. This property could be *useful* to prove that a graph in any of these classes always has even pairs, but it is not *sufficient* to deduce the existence of an even pair (the graph $\overline{C_6}$ shows that the condition "C_4-free" must be used).

3 Concluding remarks

Beside its interest related to perfectness, this result seems to require a particular attention because of the contradictory behaviour of the classes $\mathcal{C}^p$, on one hand, and the class of weakly triangulated graphs, on the other hand. As already noticed, the classes $\mathcal{C}^p$ contain the graphs whose only cycles are C_3 and C_p; these graphs are defined similarly to the weakly triangulated graphs: only the size of the long (i.e. more than 3) cycle changes. However, these new classes of graphs have, *all* of them, a property (given by Lemma 1) which is *not* true for weakly triangulated graphs. The example of (autocomplementary) weakly triangulated graph W we propose is due to Hayward [3]. It has no dominating pair (adjacent or not) and no vertex whose neighbourhood is a union of cliques.

The vertices of W are the elements of $X \cup Y$, where $X = \{x_0, x_1, \ldots, x_{11}\}$, $Y = \{y_0, y_1, \ldots, y_{11}\}$, and the edges are given by several rules. Firstly, the only edges induced by X in W are $x_{3k}x_{3k+1}$, $x_{3k+1}x_{3k+2}$ ($k = 0, 1, 2, 3$). Secondly, the only edges induced by Y in $\overline{W}$ are $y_{3k}y_{3k+1}$, $y_{3k+1}y_{3k+2}$ ($k = 0, 1, 2, 3$). Furthermore, taking all the indices modulo 12, we have for any $k = 0, 1, 2, 3$:

the only edge of W between
$\{y_{3k}, y_{3k+1}, y_{3k+2}\}$ and $\{x_{3k}, x_{3k+1}, x_{3k+2}\}$ is $y_{3k}x_{3k}$,

the only edge of $\overline{W}$ between
$\{y_{3k}, y_{3k+1}, y_{3k+2}\}$ and $\{x_{3k+3}, x_{3k+4}, x_{3k+5}\}$ is $y_{3k}x_{3k+3}$,

the only edge of $\overline{W}$ between
$\{y_{3k}, y_{3k+1}, y_{3k+2}\}$ and $\{x_{3k+6}, x_{3k+7}, x_{3k+8}\}$ is $y_{3k}x_{3k+7}$,

the only edge of W between
$\{y_{3k}, y_{3k+1}, y_{3k+2}\}$ and $\{x_{3k+9}, x_{3k+10}, x_{3k+11}\}$ is $y_{3k+1}x_{3k+9}$.

The proof that this graph is weakly triangulated and has no dominating pair may be found in Hayward's paper. We only show here that, for any vertex, a chordless path on three vertices exists in its neighbourhood, so its neighbourhood is not a union of cliques. Indeed, any of the vertices $x_{3k}, x_{3k+1}, x_{3k+2}, y_{3k}, y_{3k+1}, y_{3k+2}$ contains in its neighbourhood the chordless path $y_{3k+10}y_{3k+8}y_{3k+11}$ ($k = 0, 1, 2, 3$).

Acknowledgements. The author is indebted to Stefan Hougardy for bringing this problem to her attention and for useful discussions concerning the subject.

References

[1] C. Berge - Färbung von Graphen, deren sämtliche bzw. deren ungerade Kreise starr sind, Wiss. Z. Martin Luther Univ., Halle Wittenberg (1961), 114–115.

[2] V. Chvátal - Star-cutsets and perfect graphs, *J. Combin. Theory* Ser. B **39** (1985), 189–199.

[3] R. B. Hayward - Weakly triangulated graphs, *J. Combin. Theory* Ser. B **39** (1985), 200–208.

[4] M. W. Padberg - Perfect zero-one matrices, *Math. Programming* 6 (1974), 180-196.

Publications récentes du LIFO
LIFO's recent publications

97-7 E. MELIN, B. RAFFIN, X. REBEUF, B. VIROT
A Unifying Approach of Data and Task Parallelism Based on Structural Clock Communication Paradigm (extended version)

97-8 S. ANANTHARAMAN, Editor
Proceedings of UNIF'97, International Workshop on Unification

97-9 F. BENTAYEB, D. LAURENT
Inversion de l'algrèbre relationnelle et mises à jour

97-10 I. PARFENOFF
An efficient parallel algorithm for maximum matching in P_4-tidy graphs

97-11 F. ROUSSEL, I. RUSU
An O(m2+mn) algorithm to recognize Meyniel graphs

97-12 M. BAHMA, G. HAINS
Prevision des performances de la θ-jointure et de la multi-jointure pour des machines SN

97-13 F. LOULERGUE, G. HAINS
Design of an event-oriented functionnal parallel language: the finite case

97-14 S. LIMET, F. SAUBION
A proof method for logic programming

97-15 E. MELIN, B. RAFFIN, X. REBEUF, B. VIROT
An Automatic Distribution of Sequential Code Fitting Irregular Data Accesses

Pour récupérer les rapports de recherche du LIFO / To get LIFO's research reports :

- Par WWW : http://www.univ-orleans.fr/SCIENCES/LIFO/Recherche/Publis/RR/index.html
- Par FTP : ftp://ftp-lifo.univ-orleans.fr/pub/RR
- Par courrier papier adressé à / By surface mail adressed to :
  Arnaud Lallouet, LIFO, Université d'Orléans
  4, rue Léonard de Vinci, BP 6759
  F-45067 Orléans cedex 2, FRANCE
  en mentionnant votre adresse.
- Par courrier électronique à / By e-mail to : Arnaud.Lallouet@lifo.univ-orleans.fr