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Rapport de Recherche

Visibly Pushdown Languages and Term Rewriting

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Visibly Pushdown Languages and Term Rewriting (Extended abstract)

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Abstract. To combine tree languages with term rewriting, we introduce a new class of tree languages, that both extends regular languages and restricts context-free languages, and that is closed under intersection (unlike context-free languages). To do it, we combine the concept of visibly pushdown language, with top-down pushdown tree automata, and we get the visibly pushdown tree automata. Then, we use them to express the sets of descendants for a sub-class of growing term rewrite systems, and thanks to closure under intersection, we get that joinability and (restricted) unifiability are decidable.

Keywords : tree languages, term rewriting.

1 Introduction

Tree languages allow to express, manipulate, and decide properties over infinite sets of first-order terms. This is why tree languages have been combined with several other domains, like term rewriting [2, 8], logic programming [9], concurrency [11, 4] (process algebra), protocol verification [3],...

We want to combine tree languages with term rewriting. However, to get something interesting, it is desirable that the considered tree languages have both a big expressivity and good properties. Since these two requirements are contradictory, one will look for a middle-way in between. Regular tree languages have good properties, but have a poor expressivity. Context-free tree languages are much more expressive, but have fewer good properties, in particular they are not closed under intersection. Some other expressive tree languages have the same disadvantages [13, 4, 10], or do not extend regular languages [6].

In this paper, we introduce a new class of tree languages, that both extends regular languages and restricts context-free languages, and that is closed under intersection. To do it, we combine the concept of visibly pushdown language, already used for string languages [1], with the top-down pushdown tree automata of I. Guessarian [5]. We call these new languages *Visibly Pushdown Tree Languages (VPTL)*, and show that they are closed under union and intersection. As a sub-class of context-free languages, membership and emptiness are decidable.

Next, we define the *Visibly Context-free Term Rewrite Systems (VCF-TRS)* as a sub-class of growing term rewrite systems. In [7] it is proved that the set of predecessors of a given regular tree language by a linear (weakened to left-linear in [12]) growing rewrite system, is also regular. Consequently reachability is decidable. However, nothing has been proved about descendants (successors). In this paper, we prove that the descendants of a given VPTL (which may be regular) by a VCF-TRS, is also a VPTL. Since VPTL's are closed under intersection, joinability is decidable in VCF-TRS's, whereas it is undecidable in growing rewrite systems. As a consequence, it is decidable whether two linear terms with disjoint variables are unifiable modulo a confluent VCF-TRS. Thanks to expressivity and closure under intersection, other applications of VPTL's should arise.

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